Calculus I: MAC2311
Spring 2024
Final Exam A
04/27/2024
Time Limit: 120 Minutes

Name: ______________________

Section: ______________________

UF-ID: ______________________

Scantron Instruction: This exam uses a scantron. Follow the instructions listed on this page to fill out the scantron.

A. Sign your scantron **on the back** at the bottom in the white area.

B. Write **and code** in the spaces indicated:

1) Name (last name, first initial, middle initial)
2) UFID Number
3) 4-digit Section Number

C. Under *special codes*, code in the test numbers 4, 1:

1 2 3 • 5 6 7 8 9 0
• 2 3 4 5 6 7 8 9 0

D. At the top right of your scantron, fill in the *Test Form Code* as A .

• B C D E

E. This exam consists of 22 multiple choice questions. Make sure you check for errors in the number of questions your exam contains.

F. The time allowed is 120 minutes.

G. WHEN YOU ARE FINISHED:

1) Before turning in your test check for **transcribing errors**. Any mistakes you leave in are there to stay!

2) You must turn in your scantron to your proctor. **Be prepared to show your GatorID with a legible signature.**

It is your responsibility to ensure that your test has **22 MC questions**. If it does not, show it to your proctor immediately. You will not be permitted to make up any problems omitted from your test after the testing period ends. There are a total of 110 points available on this exam.

Instructions: 22 multiple choice questions. Complete the scantron sheet provided with your information and fill in the appropriate spaces to answer your questions. Only the answers on the scantron sheet will be graded. Each problem is worth five (5) points for a total of 110 points.

1. Evaluate $\int \frac{(\ln x)^2}{x}\, dx$.

$u = \ln x$

$du = \frac{1}{x}\, dx$

$\int u^2\, du$

$\frac{u^3}{3} + C = \frac{(\ln x)^3}{3} + C$

(A) $\frac{(\ln x)^2}{x^2} + C$

(C) $3(\ln x)^2 + C$

(C) $3(\ln x)^3 + C$

(D) $\frac{(\ln x)^3}{3} + C$

(E) $\frac{(\ln x)^3}{x} + C$

2. Use a right-endpoint Riemann sum to approximate the area below $f(x) = 2x^2 - 3x$ on the interval $[0, 4]$ with $n = 4$ rectangles.

$0 \quad 1 \quad 2 \quad 3 \quad 4$

$\Delta x = \frac{4-0}{4} = 1$

(A) 40

(B) 36

(C) 30

(D) 18

(E) 14

$R_4 = \Delta x(f(1) + f(2) + f(3) + f(4))$

$R_4 = 1((-1) + 2 + 9 + 20)$

$R_4 = 30$

3. Find the derivative of $F(x) = \int_2^{x^2+x} \sin^2(t)\,dt$.

(A) $F'(x) = \sin^2(x^2+x)(2x+1)$

(B) $F'(x) = \cos^2(x^2+x)(2x+1)$

(C) $F'(x) = \sin^2(x^2+x)(2x)$

(D) $F'(x) = \cos^2(x^2+x)(2x)$

$$F'(x) = \sin^2(x^2+x)\cdot(2x+1) - 0$$

4. What is the area under the curve $f(x) = \sqrt{x+4}$ on $[0, 5]$?

(A) $\frac{10}{3}$

(B) $\frac{19}{3}$

(C) 9

(D) 10

(E) $\frac{38}{3}$

$$\int_0^5 (x+4)^{\frac{1}{2}}\,dx = \frac{2}{3}(x+4)^{\frac{3}{2}}\Big|_0^5$$

$$= \frac{2}{3}\left[9^{3/2} - 4^{3/2}\right] = \frac{2}{3}\left[27-8\right]$$

$$= \frac{38}{3}$$

5. Let $f(x)$ and $g(x)$ be continuous functions such that $\int_3^0 f(x)\,dx = 5$, $\int_0^1 g(x)\,dx = 2$, and $\int_1^3 g(x)\,dx = 12$. Calculate $\int_0^3 \left(f(x) + 2g(x) + x^2\right)\,dx$.

(A) 20 (B) 23 (C) 28 (D) 32 (E) 50

$\int_3^0 f(x)\,dx = 5 \rightarrow \int_0^3 f(x)\,dx = -5$

$\underbrace{\int_0^1 g(x)\,dx}_{2} + \underbrace{\int_1^3 g(x)\,dx}_{12} = \underbrace{\int_0^3 g(x)\,dx}_{14}$

$\int_0^3 (f(x) + 2g(x) + x^2)\,dx$

$= \int_0^3 f(x)\,dx + 2\int_0^3 g(x)\,dx + \int_0^3 x^2\,dx$

$= -5 + 2(14) + \frac{x^3}{3}\Big|_0^3$

$= -5 + 28 + 9 = \boxed{32}$

6. $F(x) = \frac{\ln x}{x^2}$ is an antiderivative of which of the following functions?

(A) $f(x) = \frac{1-2\ln x}{x^4}$ (B) $f(x) = \frac{1-2x\ln x}{x^2}$

(C) $f(x) = \frac{(\ln x)^2}{x^3}$ (D) $f(x) = \frac{1-2\ln x}{x^3}$

Take the derivative of F(x):

$F'(x) = \frac{x^2\left(\frac{1}{x}\right) - (\ln x)2x}{(x^2)^2}$

$F'(x) = \frac{x - 2x(\ln x)}{x^4}$

$F'(x) = \frac{1 - 2(\ln x)}{x^3}$

7. A particle's position at time t is given by $s(t) = 5t^2 - \ln(t) + 1$ for $t > 0$. Which expression below gives the particle's **displacement** from $t = 1$ to $t = 2$?

(A) $\int_1^2 (5t^2 - \ln(t) + 1)\, dt$ (B) $\int_1^2 \left(10t - \frac{1}{t}\right) dt$ (C) $\int_1^2 \left(\frac{5}{3}t^3 - t\ln(t)\right) dt$ (D) $\int_1^2 \left(10 + \frac{1}{t^2}\right) dt$

$$\text{Displacement} = \int_1^2 v(t)\,dt = \int_1^2 \left(10t - \frac{1}{t}\right) dt$$

$$v(t) = s'(t) = 10t - \frac{1}{t}$$

8. Evaluate $\int_1^2 (3x+1)^2\, dx$.

(A) 31 (B) 38 (C) 46 (D) 52

$$\int_1^2 (3x+1)^2\,dx = \int_1^2 (9x^2 + 6x + 1)\,dx$$

$$= 3x^3 + 3x^2 + x \Big|_1^2 = 24 + 12 + 2 - (3 + 3 + 1)$$

$$= 38 - 7 = \boxed{31}$$

9. Find $f(x)$ given $f'(x) = 1 + 2e^x + 2\sin(x)$ and the initial condition $f(0) = 3$.

(A) $f(x) = x + 2e^x - 2\cos(x) + 3$ (B) $\cancel{f(x) = x + 2e^x + 2\cos(x) + 2}$

(C) $f(x) = x + 2e^x + 2\cos(x) + 3$ (D) $\cancel{f(x) = x + 2e^x - 2\cos(x) + 2}$

$f(x) = \int f'(x) = \int (1 + 2e^x + 2\sin(x))\,dx$

use $f(0) = 3$

$f(x) = x + 2e^x - 2\cos(x) + C$

$\therefore 3 = 0 + 2e^0 - 2\cos(0) + C$

$3 = 2 - 2 + C$

$\therefore C = 3$

$\boxed{f(x) = x + 2e^x - 2\cos(x) + 3}$

10. Which of the following is the right-endpoint Riemann sum approximation of $f(x) = \frac{3}{x^2}$ on the interval $[3, 12]$ using $n = 3$ rectangular subintervals?

(A) $\sum_{i=1}^{3} \left(\frac{3}{(3+3i)^2}\right) 3$ (B) $\cancel{\sum_{i=0}^{3} \left(\frac{3}{(3+3i)^2}\right) 2}$

(C) $\cancel{\sum_{i=0}^{3} \left(\frac{3}{(3i)^2}\right) 3}$ (D) $\cancel{\sum_{i=1}^{3} \left(\frac{3}{(3i)^2}\right) 2}$

$n = 3$

$\Delta x = \frac{b-a}{n} = \frac{12-3}{3} = 3$

$x_i = a + \Delta x \cdot i$

$x_i = 3 + 3i$

$R_n = \sum_{i=1}^{n} f(x_i)\,\Delta x$

$R_n = \sum_{i=1}^{3} f(3+3i) \cdot 3$

$R_n = \sum_{i=1}^{3} \left[\frac{3}{(3+3i)^2}\right] \cdot 3$

11. Calculate $\int \frac{\sin x}{\cos^3 x}\, dx$.

(A) $\sec^2 x + C$

(B) $\frac{\sec^2 x}{2} + C$

(C) $\frac{\sec^3 x}{3} + C$

(D) $\frac{\sec^4 x}{4} + C$

Let $u = \cos x$

$du = -\sin x\, dx$

$-du = \sin x\, dx$

$$\int \frac{-du}{u^3}$$

$$= -\int u^{-3}\, du$$

$$= -\frac{u^{-2}}{-2} + C = \frac{1}{2}(\cos x)^{-2} + C$$

$$\boxed{= \frac{1}{2}\sec^2 x + C}$$

12. Consider the following statements for an increasing function, $f(x)$, with $f(x) > 0$:

(i) The Right Riemann Sum, R_n, of $f(x)$ is an *overestimate* of the true area under the curve. ✓
(ii) The Left Riemann Sum, L_n, of $f(x)$ is an *underestimate* of the true area under the curve. ✓
(iii) The greater the number of rectangles (subintervals) in the Riemann sum, the *better* the area approximation is. ✓

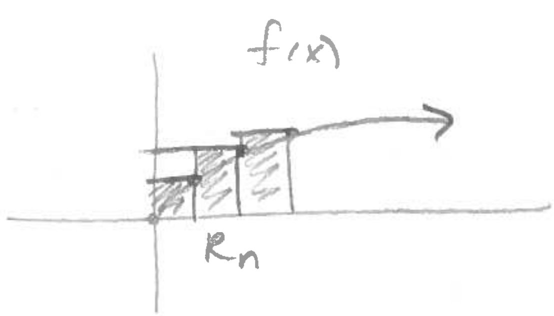
How many of the following statements are **true**?

(A) None

(B) One

(C) Two

(D) Three



13. Compute dy using the function $y = x^3 + 3x$ as x goes from 1 to $\frac{1}{2}$.

(A) -6 (B) -3 (C) 3 (D) 6

$dy = (3x^2 + 3)\,dx$

$dy = (3(1)^2 + 3)(-\frac{1}{2})$

$dy = (6)(-\frac{1}{2}) = \boxed{-3}$

$dx = x_2 - x_1$

$dx = \frac{1}{2} - 1 = -\frac{1}{2}$

14. Suppose that a menacing bird population at Norman Hall is growing at a rate of $f'(t) = 50 + 30t$, where t is measured in years. At time $t = 0$, the bird population is at 200 birds. What is the **total** bird population after 3 years?

(A) 150 (B) 285 (C) 350 (D) 485 (E) 535

Total = 200 + Net change

$\text{Total} = 200 + \int_0^3 (50 + 30t)\,dt$

$\text{Total} = 200 + (50t + 15t^2)\Big|_0^3$

$\text{Total} = 200 + (150 + 135 - 0) = \boxed{485}$

15. Calculate

$$\lim_{x\to 0}(1-2x)^{\frac{1}{x}}.$$

"1^∞"

(A) 0 (B) $\frac{1}{e}$ (C) $\frac{1}{e^2}$ (D) e (E) e^2

$$= e^{\lim_{x\to 0} \ln(1-2x)^{1/x}} = e^{\boxed{\lim_{x\to 0} \frac{1}{x}\cdot\ln(1-2x)}} = \boxed{e^{-2}}$$

$$\lim_{x\to 0}\frac{\ln(1-2x)}{x} \quad \text{"}\frac{\ln(1)}{0}\text{"} = \text{"}\frac{0}{0}\text{"} ✓$$

$$\overset{\mathcal{L}}{=} \lim_{x\to 0}\frac{\frac{1}{1-2x}(-2)}{1} = \frac{\frac{-2}{1}}{1} = -2$$

16. Compute the limit

$$\lim_{x\to\infty}\frac{e^x}{x^2}.$$

"$\frac{\infty}{\infty}$" ✓

(A) 0 (B) ∞ (C) $-\infty$

(D) 1 (E) Does not exist.

$$\overset{\mathcal{L}}{=} \lim_{x\to\infty}\frac{e^x}{2x} \quad \text{"}\frac{\infty}{\infty}\text{"}$$

$$\overset{\mathcal{L}}{=} \lim_{x\to\infty}\frac{e^x}{2} \quad \text{"}\frac{\infty}{2}\text{"}$$

$$= \boxed{\infty}$$

17. Let $f(x) = \frac{1}{x} + 2\tan(\pi x)$. Calculate $f'(1)$.

$(A)\ 2\pi + 1$ $(B)\ 2\pi - 1$ $(C)\ -2\pi + 1$ $(D)\ -2\pi - 1$

$$f'(x) = \frac{-1}{x^2} + 2\sec^2(\pi x)\cdot\pi$$

$$f'(1) = -1 + 2\sec^2(\pi)\cdot\pi$$

$$f'(1) = -1 + \frac{2}{(\cos\pi)^2}\cdot\pi$$

$$f'(1) = -1 + \frac{2}{(-1)^2}\cdot\pi = \boxed{2\pi - 1}$$

18. Suppose $f(x)$ and $g(x)$ are differentiable functions. Let $h(x) = \frac{f(x)}{g(x)}$. Suppose that $f(1) = 3$, $f'(1) = -2$, $g(1) = 7$, and $g'(1) = 1$. Calculate $h'(1)$.

$(A)\ -\frac{11}{7}$ $(B)\ -\frac{11}{49}$ $(C)\ \frac{11}{49}$ $(D)\ -\frac{17}{49}$ $(E)\ -\frac{17}{7}$

$$h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

$$h'(1) = \frac{g(1)f'(1) - f(1)g'(1)}{(g(1))^2}$$

$$h'(1) = \frac{(7)(-2) - (3)(1)}{7^2} = \frac{-17}{49}$$

19. Find the derivative using implicit differentiation.

$$\frac{d}{dx}\left((x^2+y^2)^3 - y^2 \right) \frac{d}{dx}\left(= 2 \right)$$

(A) $\frac{dy}{dx} = \frac{6x(x^2+y^2)}{y-3y(x^2+y^2)^2}$

(B) $\frac{dy}{dx} = \frac{3x(x^2+y^2)^2}{-y+3y(x^2+y^2)^2}$

(C) $\frac{dy}{dx} = \frac{3x(x^2+y^2)^2}{y-3y(x^2+y^2)^2}$

(D) $\frac{dy}{dx} = \frac{x(x^2+y^2)^2}{y+3(x^2+y^2)^2}$

$$3(x^2+y^2)^2(2x+2yy') - 2yy' = 0$$

$$6x(x^2+y^2)^2 + 6yy'(x^2+y^2)^2 - 2yy' = 0$$

$$6yy'(x^2+y^2)^2 - 2yy' = -6x(x^2+y^2)^2$$

$$y' = \frac{-6x(x^2+y^2)^2}{6y(x^2+y^2)^2 - 2y} = \frac{3x(x^2+y^2)^2}{-3y(x^2+y^2)^2 + y}$$

20. Calculate

$$\lim_{x \to 3^-} \frac{x+10}{x-3}.$$

"$\frac{13}{0}$" one-sided test

left

(A) ∞ (B) $-\infty$ (C) 0 (D) 1 (E) Does not exist.

$(x = 2.9)$ $\frac{x+10}{x-3}$ $\frac{(+)}{(-)} \to \boxed{-\infty}$

21. An object moves along a straight line with position function given by $s(t) = t^3 + 2t$, where $s(t)$ is measured in feet and t in seconds. What is the average velocity in feet per second of the object over the interval $[1, 3]$?

(*A*) 10 (*B*) 12 (*C*) 13 (*D*) 15 (*E*) 17

Average Velocity $= \dfrac{S(3) - S(1)}{3-1} = \dfrac{27+6-(1+2)}{2}$

$= \dfrac{30}{2} = \boxed{15}$

22. Evaluate

$$\lim_{x \to 2} e^{3x^2} \cos(\pi x).$$

Plug in X.

(*A*) 0 (*B*) $-e^6$ (*C*) e^6 (*D*) e^{12} (*E*) $-e^{12}$

$= e^{3(2)^2} \cos(2\pi) = e^{12}(1) = \boxed{e^{12}}$