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MAC 2311 Spring 2010

Exam 1A

A. Sign your scantron sheet in the white area on the back **in ink**.

B. Write **and code** in the spaces indicated:

1) Name (last name, first initial, middle initial)

2) UF ID number

3) Discussion Section number

C. Under "special codes", code in the test number 1, 1.

● 2 3 4 5 6 7 8 9 0

● 2 3 4 5 6 7 8 9 0

D. At the top right of your answer sheet, for "Test Form Code" encode A.

● B C D E

E. This test consists of 10 five-point and 2 two-point multiple choice questions, three one-point bonus questions and two sheets (4 pages) of partial credit questions worth 26 points. The time allowed is 90 minutes.

F. WHEN YOU ARE FINISHED:

1) Before turning in your test check for **transcribing errors**. Any mistakes you leave in are there to stay.

2) You must turn in your scantron and tearoff sheets to your discussion leader or proctor. **Be prepared to show your picture ID with a legible signature.**

3) The answers will be posted on the MAC 2311 homepage after the exam. Your discussion leader will return your tearoff sheet with your exam score in discussion. Your score will also be posted on Elearning within one week.

NOTE: Be sure to bubble the answers to questions 1–15 on your scantron.

Questions 1 - 10 are worth 5 points each.

1. If $f(x) = \dfrac{\frac{1}{2} - \frac{2}{x}}{x - 4}$, evaluate $\lim_{x \to 4} f(x)$.

a. 1 b. $\frac{1}{8}$ c. $\frac{1}{2}$ d. 0 e. The limit does not exist.

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2. Find each value of x on the interval $[0, 2\pi)$ for which $\tan x \geq \sec x$.

a. $\left(\frac{\pi}{2}, \pi\right] \cup \left(\frac{3\pi}{2}, 2\pi\right)$ b. $\left[0, \frac{\pi}{2}\right) \cup \left[\pi, \frac{3\pi}{2}\right)$ c. $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

d. $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right)$ e. There are no such values of x.

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3. A population of rare insects on an isolated island has been observed to double every three years. If researchers counted 400 insects at the beginning of the study, how many years would it take for the population to grow to 1800? Hint: First find a formula which gives the population of insects on the island after t years of study.

a. $\dfrac{3\ln 2}{\ln(9/2)}$ years b. $\dfrac{3}{\ln 9}$ years c. $\dfrac{\ln(9/2)}{3\ln 2}$ years

d. $\dfrac{3\ln(9/2)}{\ln 2}$ years e. $\dfrac{\ln(3/2)}{\ln 2}$ years

4. Let $f(x) = \dfrac{1}{x-1}$ and $g(x) = \dfrac{2x}{x+1}$. Find $(g \circ f)(x)$ and its domain.

a. $(g \circ f)(x) = \dfrac{2}{x}$ domain: $(-\infty, 0) \cup (0, \infty)$

b. $(g \circ f)(x) = \dfrac{x+1}{x-1}$ domain: $(-\infty, 1) \cup (1, \infty)$

c. $(g \circ f)(x) = \dfrac{x+1}{x-1}$ domain: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

d. $(g \circ f)(x) = \dfrac{2}{x}$ domain: $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

e. $(g \circ f)(x) = \dfrac{2x}{x^2-1}$ domain: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

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5. Let $f(x) = \dfrac{3x+6}{4-x^2}$.

Which of the following statements is/are true?

P. $\lim_{x \to 2^-} f(x) = \infty$.

Q. $f(x)$ can be made continuous at $x = -2$ by defining $f(-2) = \dfrac{3}{4}$.

R. $f(x)$ has horizontal asymptote $y = 0$.

S. $f(x)$ has vertical asymptotes $x = -2$ and $x = 2$.

a. P, Q and R

b. R and S only

c. Q and R only

d. P and R only

e. P, Q and S

6. An object moves in a straight line so that its position in feet after t seconds is given by $s(t) = 2t^2+3$. Find a formula for the average velocity of the object on the interval $[1, 1+h]$ for $h \neq 0$, and the instantaneous velocity after 1 second.

	Average Velocity	**Instantaneous Velocity**
a.	$4+2h$ feet/sec	6 feet/sec
b.	$2h^2+4$ feet/sec	4 feet/sec
c.	$4+2h$ feet/sec	4 feet/sec
d.	$2+4h+2h^2$ feet/sec	2 feet/sec
e.	$2h^2+4$ feet/sec	6 feet/sec

---

7. Let $f(x) = \begin{cases} e^{x-3} & x < 2 \\ r - 2x & x \geq 2 \end{cases}$. Find the value of r which will make $f(x)$ continuous for all real numbers. $r =$____________.

a. $2+e$ b. $\frac{4}{e}$ c. $2-\frac{1}{e}$ d. $4-e$ e. $4+\frac{1}{e}$

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8. Let $p = \lim_{x \to -\infty} \frac{|2x+4|}{x-1}$ and $q = \lim_{x \to \infty} \frac{3}{2+e^x}$. Then

a. $p = -\infty$ and $q = \infty$

b. $p = -2$ and $q = 0$

c. $p = 2$ and $q = \frac{3}{2}$

d. $p = -2$ and $q = \frac{3}{2}$

e. $p = 2$ and $q = 0$

9. Let $f(x) = \sqrt{4-x}$. Find $f^{-1}(x)$ and its domain. Hint: consider the relationship between range and domain of inverse functions. You may want to sketch your functions.

a. $f^{-1}(x) = 4 - x^2$ domain: $(-\infty, \infty)$

b. $f^{-1}(x) = \dfrac{1}{\sqrt{4-x}}$ domain: $(-\infty, 4)$

c. $f^{-1}(x) = \sqrt{4-x^2}$ domain: $[-2, 2]$

d. $f^{-1}(x) = 4 - x^2$ domain: $[0, \infty)$

e. $f^{-1}(x) = \sqrt{4-x^2}$ domain: $[0, 2]$

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10. According to the Intermediate Value Theorem, the function $f(x) = 2\cos x - 2x - 1$ must have a zero on which one of the following intervals?

a. $\left(\frac{\pi}{2}, \pi\right)$ b. $\left(\frac{3\pi}{2}, 2\pi\right)$ c. $\left(0, \frac{\pi}{2}\right)$ d. $\left(\pi, \frac{3\pi}{2}\right)$

e. none of these

The following problems are worth 2 points each.

11. Evaluate $\lim_{x\to 0^+} \dfrac{1-\cos x}{\sin^2 x}$. Hint: use a trig identity to rewrite the function.

a. ∞ b. 0 c. 1 d. $-\infty$ e. $\frac{1}{2}$

12. Evaluate $\lim_{x \to -\infty} e^{\frac{x^2+1}{x}}$. Note that $\frac{x^2+1}{x}$ is the exponent.

a. 0   b. $-\infty$ c. $\dfrac{1}{e}$ d. ∞ e. 1

Bonus!!

State whether each of following is true or false (one point each).

13. $f(x) = \dfrac{x}{\sin x}$ is an odd function.

a. True   b. False

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14. $\lim_{x \to 5^-} \ln(5 - x) = -\infty$.

a. True   b. False

---

15. The graph of the function $f(x) = \dfrac{|x|}{x}$ has a hole at $x = 0$.

a. True   b. False

MAC 2311 TEST IA PART II
SPRING 2010

Sect # ________________ Name ______________________________

UFID __________________ Signature ______________________________

SHOW ALL WORK TO RECEIVE FULL CREDIT.

1. Let $f(x) = \sqrt{2x+3}$.

   a. **Use limits** to find the slope of the tangent line to $f(x)$ at $x = 3$.

   b. Write the equation of the tangent line to $f(x)$ at $x = 3$.

$y =$ ________________________

2. Solve each of the following:

a. Find the solution set: $2\ln x - \ln(x+2) = 0$

b. Let $f(x) = \cos 2x$ and let $g(x) = 3\sin x + 2$. Find each value of x on the interval $[0, 2\pi]$ at which $f(x) = g(x)$.

$x =$ ____________________

3. Evaluate the following:

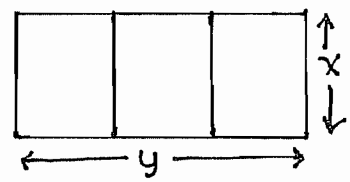
a. $\cot\left(\arcsin\dfrac{2}{3}\right) =$ ____________. Hint: draw a triangle.

b. $\cos^{-1}\left(\cos\dfrac{5\pi}{4}\right) =$ ____________

Sect # ______________ Name ______________________

4. A farmer wishes to enclose a rectangular piece of land to create three equal garden areas as shown below using 160 yards of fencing.

a. Find a function $A(x)$ which gives the total area enclosed in terms of x. Find the domain of $A(x)$.



$A(x) =$ ______________________

domain(interval notation) ______________________

b. Use your function to find the dimensions that will enclose an area of 800 square yards.

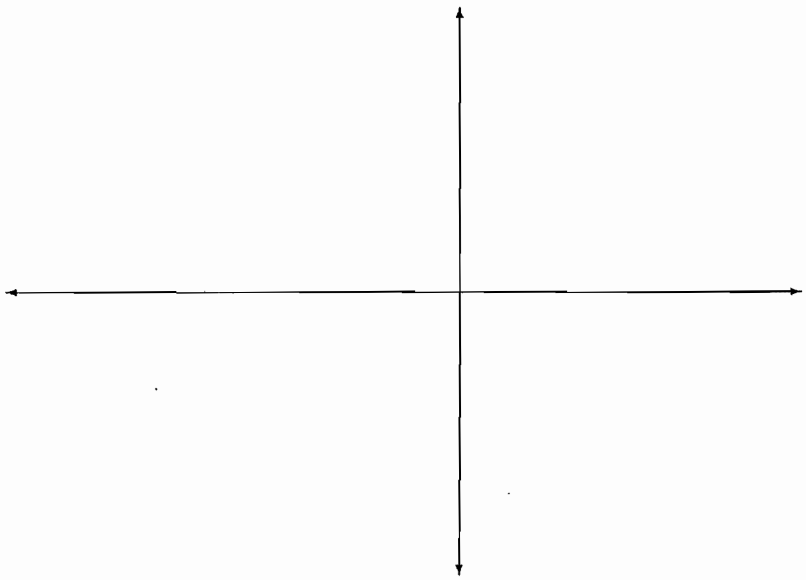
$x =$ ____________ $y =$ ____________

5. It can be shown that for θ in $\left(0, \frac{\pi}{2}\right)$, $\theta \cos\theta \leq \sin\theta \leq \theta$.

Use the Squeeze Theorem and this inequality to find $\lim_{\theta \to 0^+} \frac{\sin\theta}{\theta}$.

6. Consider the function $f(x) = \begin{cases} \tan^{-1} x & x < 0 \\ 3 - |x| & 0 \leq x < 3 \\ x^2 - 9 & x > 3 \end{cases}$

(a) Sketch the graph of $f(x)$.



(b) Find the following limits if possible.

$\lim_{x \to -\infty} f(x) =$ ____________ $\lim_{x \to 0} f(x) =$ ____________

$\lim_{x \to 0^-} f(x) =$ ____________ $\lim_{x \to 3^-} f(x) =$ ____________

$\lim_{x \to 0^+} f(x) =$ ____________ $\lim_{x \to 3^+} f(x) =$ ____________

(c) List all discontinuities of $f(x)$ and state whether they are jump, infinite or removable.

(d) Find each interval on which $f(x)$ is continuous.

MAC 2311 Spring 2010

Exam 1B

A. Sign your scantron sheet in the white area on the back **in ink**.

B. Write **and code** in the spaces indicated:

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2) UF ID number

3) Discussion Section number

C. Under "special codes", code in the test number 1, 2.

●	2	3	4	5	6	7	8	9	0
1	●	3	4	5	6	7	8	9	0

D. At the top right of your answer sheet, for "Test Form Code" encode B.

A ● C D E

E. This test consists of 10 five-point and 2 two-point multiple choice questions, three one-point bonus questions and two sheets (4 pages) of partial credit questions worth 26 points. The time allowed is 90 minutes.

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NOTE: Be sure to bubble the answers to questions 1–15 on your scantron.

Questions 1 - 10 are worth 5 points each.

1. If $f(x) = \dfrac{\frac{1}{4} - \frac{4}{x}}{x - 16}$, evaluate $\lim_{x \to 16} f(x)$.

   a. The limit does not exist. b. $\dfrac{1}{4}$ c. 0 d. $\dfrac{1}{64}$ e. 1

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2. Find each value of x on the interval $[0, 2\pi)$ for which $\tan x \leq \sec x$.

   a. $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right)$ b. $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ c. $\left[0, \frac{\pi}{2}\right) \cup \left[\pi, \frac{3\pi}{2}\right)$

   d. $\left(\frac{\pi}{2}, \pi\right] \cup \left(\frac{3\pi}{2}, 2\pi\right)$ e. $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right)$

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3. A population of rare insects on an isolated island has been observed to double every four years. If researchers counted 600 insects at the beginning of the study, how many years would it take for the population to grow to 1500? Hint: First find a formula which gives the population of insects on the island after t years of study.

   a. $\dfrac{\ln(5/2)}{4 \ln 2}$ years b. $\dfrac{\ln(5/2)}{\ln 2}$ years c. $\dfrac{4 \ln(5/2)}{\ln 2}$ years

   d. $\dfrac{4}{\ln 5}$ years e. $\dfrac{4 \ln 2}{\ln(5/2)}$ years

4. Let $f(x) = \dfrac{1}{x-2}$ and $g(x) = \dfrac{3x}{x+1}$. Find $(g \circ f)(x)$ and its domain.

a. $(g \circ f)(x) = \dfrac{x+1}{x-2}$ domain: $(-\infty, 2) \cup (2, \infty)$

b. $(g \circ f)(x) = \dfrac{3}{x-1}$ domain: $(-\infty, 1) \cup (1, \infty)$

c. $(g \circ f)(x) = \dfrac{3x}{x^2 - x - 2}$ domain: $(-\infty, -1) \cup (-1, 2) \cup (2, \infty)$

d. $(g \circ f)(x) = \dfrac{x+1}{x-2}$ domain: $(-\infty, -1) \cup (-1, 2) \cup (2, \infty)$

e. $(g \circ f)(x) = \dfrac{3}{x-1}$ domain: $(-\infty, 1) \cup (1, 2) \cup (2, \infty)$

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5. Let $f(x) = \dfrac{2x+6}{9-x^2}$.

Which of the following statements is/are true?

P. $\lim_{x \to 3^-} f(x) = \infty$.

Q. $f(x)$ can be made continuous at $x = -3$ by defining $f(-3) = \dfrac{1}{3}$.

R. $f(x)$ has vertical asymptotes $x = -3$ and $x = 3$.

S. $f(x)$ has horizontal asymptote $y = 0$.

a. P and S only b. Q and S only c. R and S only

d. P, Q and S e. P, Q and R

6. Let $f(x) = \begin{cases} r + 3x & x \le 2 \\ e^{x-3} & x > 2 \end{cases}$. Find the value of r which will make $f(x)$ continuous for all real numbers. $r =$__________.

a. $6 + e$ b. $\frac{1}{e} - 6$ c. $-3 - \frac{1}{e}$ d. $-6 - e$ e. $4 + \frac{1}{e}$

---

7. An object moves in a straight line so that its position in feet after t seconds is given by $s(t) = 3t^2 + 1$. Find a formula for the average velocity of the object on the interval $[1, 1+h]$ for $h \neq 0$, and the instantaneous velocity after 1 second.

	Average Velocity	**Instantaneous Velocity**
a.	$6 + 3h$ feet/sec	6 feet/sec
b.	$3h^2 + 6$ feet/sec	6 feet/sec
c.	$3h^2 + 6$ feet/sec	3 feet/sec
d.	$6 + 3h$ feet/sec	3 feet/sec
e.	$3 + 6h + 3h^2$ feet/sec	9 feet/sec

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8. Let $p = \lim_{x \to -\infty} \frac{|4x + 3|}{x - 1}$ and $q = \lim_{x \to \infty} \frac{5}{2 + e^x}$. Then

a. $p = -4$ and $q = \frac{5}{2}$

b. $p = -\infty$ and $q = \infty$

c. $p = -4$ and $q = 0$

d. $p = 4$ and $q = 0$

e. $p = 4$ and $q = \frac{5}{2}$

9. Let $f(x) = \sqrt{1-x}$. Find $f^{-1}(x)$ and its domain. Hint: consider the relationship between range and domain of inverse functions. You may want to sketch your functions.

a. $f^{-1}(x) = \sqrt{1-x^2}$ domain: $[0, 1]$

b. $f^{-1}(x) = \sqrt{1-x^2}$ domain: $[-1, 1]$

c. $f^{-1}(x) = 1 - x^2$ domain: $[0, \infty)$

d. $f^{-1}(x) = \dfrac{1}{\sqrt{1-x}}$ domain: $(-\infty, 1)$

e. $f^{-1}(x) = 1 - x^2$ domain: $(-\infty, \infty)$

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10. According to the Intermediate Value Theorem, the function $f(x) = 3\cos x - 2x - 2$ must have a zero on which one of the following intervals?

a. $\left(\frac{3\pi}{2}, 2\pi\right)$ b. $\left(\pi, \frac{3\pi}{2}\right)$ c. $\left(\frac{\pi}{2}, \pi\right)$ d. $\left(0, \frac{\pi}{2}\right)$

e. none of these

The following problems are worth 2 points each.

11. Evaluate $\lim\limits_{x \to -\infty} e^{\frac{x^2+2}{x}}$. Note that $\frac{x^2+2}{x}$ is the exponent.

a. $-\infty$ b. 0 c. ∞ d. 1 e. $\dfrac{2}{e}$

12. Evaluate $\lim_{x \to 0^+} \frac{1 - \cos x}{\sin^2 x}$. Hint: use a trig identity to rewrite the function.

a. $-\infty$ b. 1 c. $-\infty$ d. 0 e. $\frac{1}{2}$

Bonus!!

State whether each of following is true or false (one point each).

13. $\lim_{x \to 2^-} \ln(2 - x) = -\infty$.

a. True b. False

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14. The graph of the function $f(x) = \frac{|x|}{x}$ has a hole at $x = 0$.

a. True b. False

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15. $f(x) = \frac{x}{\sin x}$ is an odd function.

a. True b. False

MAC 2311 TEST IB PART II
SPRING 2010

Sect # ________________ Name ______________________________

UFID __________________ Signature __________________________

SHOW ALL WORK TO RECEIVE FULL CREDIT.

1. Let $f(x) = \sqrt{3x+1}$.

   a. **Use limits** to find the slope of the tangent line to $f(x)$ at $x = 1$.

   b. Write the equation of the tangent line to $f(x)$ at $x = 1$.

$y =$ ______________________

2. Solve each of the following:

a. Find the solution set: $2\ln x - \ln(2x+3) = 0$

b. Let $f(x) = \cos 2x$ and let $g(x) = 2 - 3\sin x$. Find each value of x on the interval $[0, 2\pi]$ at which $f(x) = g(x)$.

$x =$ ______________________

3. Evaluate the following:

a. $\cos^{-1}\left(\cos\frac{4\pi}{3}\right) =$ __________

b. $\tan\left(\arcsin\frac{2}{3}\right) =$ __________. Hint: draw a triangle.

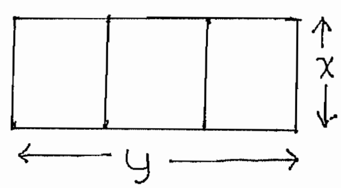
Sect # ______________ Name ______________________

4. It can be shown that for θ in $\left(0, \frac{\pi}{2}\right)$, $\theta\cos\theta \le \sin\theta \le \theta$.

Use the Squeeze Theorem and this inequality to find $\lim_{\theta \to 0^+} \frac{\sin\theta}{\theta}$.

5. A farmer wishes to enclose a rectangular piece of land to create three equal garden areas as shown below using 240 yards of fencing.

a. Find a function $A(x)$ which gives the total area enclosed in terms of x. Find the domain of $A(x)$.



$A(x) =$ ______________________

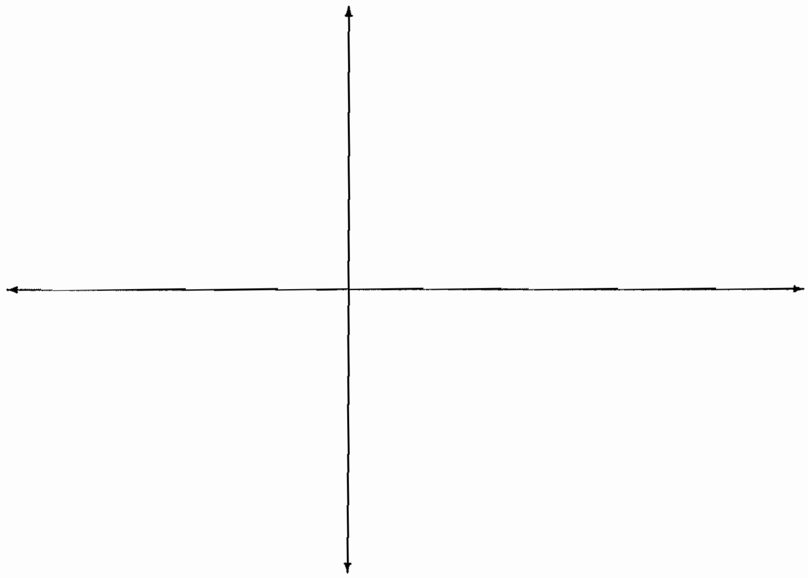
domain(interval notation) ______________________

b. Use your function to find the dimensions that will enclose an area of 1800 square yards.

$x =$ ____________ $y =$ ____________

6. Consider the function $f(x) = \begin{cases} x^2 - 1 & x < -1 \\ \sqrt{x+1} & -1 < x < 0 \\ \tan^{-1} x & x \geq 0 \end{cases}$

(a) Sketch the graph of $f(x)$.



(b) Find the following limits if possible.

$\lim_{x \to -1^-} f(x) =$ __________ $\lim_{x \to 0^-} f(x) =$ __________

$\lim_{x \to -1^+} f(x) =$ __________ $\lim_{x \to 0} f(x) =$ __________

$\lim_{x \to 0^+} f(x) =$ __________ $\lim_{x \to \infty} f(x) =$ __________

(c) List all discontinuities of $f(x)$ and state whether they are jump, infinite or removable.

(d) Find each interval on which $f(x)$ is continuous.