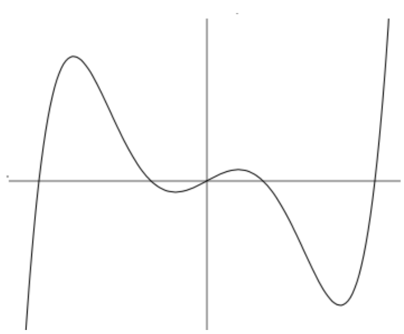
Review for MAC 1140 Exam 3

1. (L14) Which of the following equations is 'odd'?

   (a) $f(x) = \dfrac{2}{x-1}$ (b) $f(x) = \dfrac{1}{\sqrt{x^2+5}}$ (c) $f(x) = x^3 - x$ (d) $f(x) = |x|$

   (ans. b and d are even, c is odd).

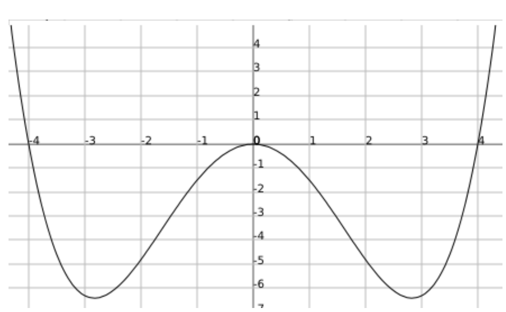
2. (L14) Match the graph with its polynomial function, $y =$ ________



   (a) $2x^3 - 3x + 1$ (b) $-\dfrac{1}{3}x^3 + x^2 - \dfrac{4}{3}$ (c) $\dfrac{1}{5}x^5 - 2x^3 + \dfrac{9}{5}x$ (d)$-\dfrac{1}{5}x^5 - 2x^3 + \dfrac{9}{5}x$

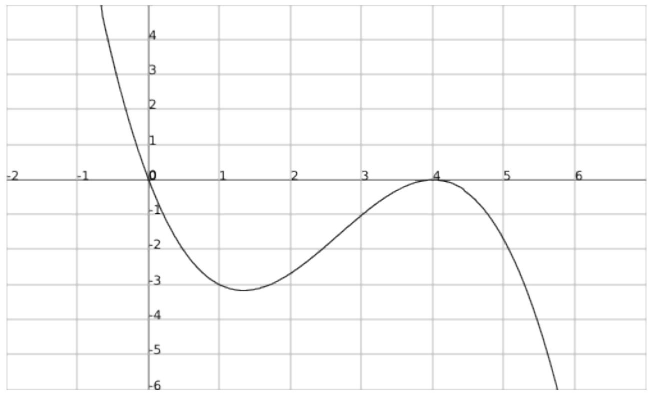
   (ans. (c))

3. Given the graph below, is the degree of the polynomial even or odd? Is the leading coefficient a_n positive or negative? Construct a possible polynomial that matches the graph and the given points.



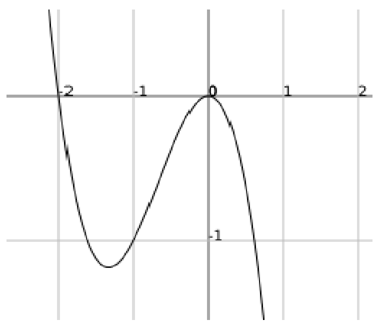
   (ans. $f(x) = x^2(x+4)(x-4)$, answer is not unique)

4. (L14) Sketch $h(x) = -\frac{1}{3}x(x-4)^2$, determine the end behavior and intercepts, zeros, and multiplicities.



(ans. rises to the left, falls to the right; $x-$intercept at $x = 0(1), 4(2); y - int : y = 0$)

5. (L14) Sketch $g(x) = -5x^2 - x^3$, how many turning points does the graph have?



(ans. 2 turning points)

6. (L14) Find all zeros of $g(x) = 2(x^3 - 9x)(x+3)^3(x^2+4)^3$ and their multiplicities and determine if graph touches or cross at the zeros; also find the intercepts and end behavior.

   (ans.zeros: $x = -3(4)touch, 0(1)corss, 3(1)cross)$, $x-$int: $x = -3, 0, 3$; $y-$int: $y = 0$; end behavior: rises on both sides.

7. (L14) Use synthetic division to show that $(x-1), (x-2)$ are factors of $f(x) = 2x^4 - 3x^3 - 4x^2 + 3x + 2$ and factor $f(x)$ completely.

   (ans. $f(x) = (x-1)(x-2)(x+1)(2x+1)$)

8. (L15) Perform the indicated operations. write your answer in standard form.

   (a) $(4-3i) - (4+3i)$ (b) $(i^{23} - 1)^2$ (c) $\frac{1+i}{i^3}$ (d) $\frac{3-i}{1-2i}$

   (ans. $(a) - 6i \quad (b) 2i \quad (c) i - 1 \quad (d) 1 + i$)

9. (L15)

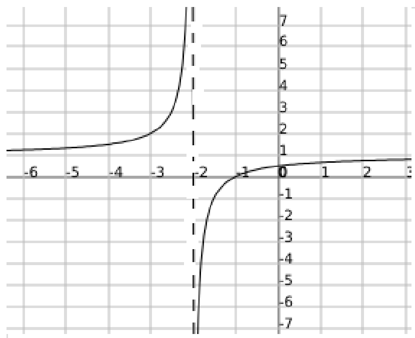
   (a) Find the conjugate $\overline{z}$ of the following, then find $z + \overline{z}$, $z - \overline{z}$, $z \cdot \overline{z}$, $z \div \overline{z}$ for
   (i) $z = 2 + i\sqrt{5}$ (ii) $z = -3i$ (iii)$z = 5$

   (b) Evaluate:
   (i) $\sqrt{(2i-6)(6+2i)}$ (ii) $\sqrt{-12} \cdot \sqrt{-75}$ (iii) $\sqrt{-12} - \sqrt{-75}$ (iv) $\sqrt{-12} \cdot \sqrt{75}$

(ans.(a)(i)$\overline{z} = 2 - i\sqrt{5}$,$z + \overline{z} = 4$, $z - \overline{z} = 2i\sqrt{5}$, $z \cdot \overline{z} = 9$, $z \div \overline{z} = -\frac{1}{9} + +\frac{4\sqrt{5}}{9}i$
(ii)$\overline{z} = 3i$, $z + \overline{z} = 0$, $z - \overline{z} = 6i$, $z \cdot \overline{z} = 9$, $z \div \overline{z} = -1$
(iii) $\overline{z} = 5$, $z + \overline{z} = 10$, $z - \overline{z} = 0$, $z \cdot \overline{z} = 25$, $z \div \overline{z} = 1$

(b)(i)$2i\sqrt{10}$ (ii)-30 (iii)$-3\sqrt{3}i$ (iv)$30i$)

10. (L17) Sketch the graph of $1 - f(x+2)$ if $f(x) = \frac{1}{x}$. Find all vertical and horizontal asymptotes of the graph.



(ans. VA: $x = -2$; HA: $y = 1$, graph does NOT cross its HA.)

11. (L18) Determine the test interval by using a number line or a sign chart. Solve each inequality.

(a) $\dfrac{x+3}{x-1} \leq 2$ ans:$(-\infty, 1) \cup [5, \infty)$

(b) $\dfrac{3x-8}{5-x} > x$ $(-\infty, -2) \cup (4, 5)$

(c) $\dfrac{x-3}{x+3} \leq \dfrac{x+3}{x-3}$ $(-3, 0] \cup (3, \infty)$

(d) $\dfrac{(1-x)^2(x+2)^3(6-x)^2}{(1-x)^5(x+2)^2(x+5)} \leq 0$ $(-5, -2) \cup (1, \infty)$

12. (L18) Solve the inequality.

(a) $x^3 + 2x + 7 \leq 3x^2 + 6x - 5$ ans: $(-\infty, -2] \cup [2, 3]$

(b) $x^3 - x^2 \geq 0$ ans: $\{0\} \cup [1, \infty)$

13. (L18)At what $x-$values does the polynomial change signs $f(x) = x^3 - x^2$ ans: $x = 1$ only

14. (L17) Sketch the graph of the rational function, find domain, holes, asymptotes, intercepts and points where the graph crosses its horizontal asymptote. If any of the above don't exist, write "NONE".

(a) $f(x) = \dfrac{(x-2)(x^2-4x+3)}{(x^2-6x+8)(x-1)^2}$ ans:$D : x \neq 4, 2, 1; Hole : (2, 1/2); VA : x = 4, 1; HA : y = 0$; cross at $(3, 0)$; $x - int : x = 3; y - int : y = -3/4$

(b) $f(x) = \dfrac{3x^2 - 6x}{x^2 + x - 12}$ ans:$VA : x = -4, 3; HA : y = 3$; cross at (4,3);$x - int : x = 0, 2$

15. (L17) Find the range, the zeros of $f(x) = \dfrac{(x-5)}{x+2}$. Does the graph of $f(x)$ cross its horizontal asymptote? ans:$Range : (-\infty, 1) \cup (1, \infty); zeros : x = 5$; no

16. (L16) Determine all zeros of $f(x) = 2x^3 - 9x^2 + 14x - 5$, given that $2 - i$ is a zero. How many real zeros must have? Factor completely as a product of linear factor(s) and irreducible quadratic factor(s). ans:$x = 1/2, x = 2 - i, x = 2 + i; (2x - 1)(x^2 - 4x + 5)$; one real root

17. (L16) True/False: $f(x) = x^3 + x^2 - x - 1$ has no real zeros. ans: False

18. (L16) True/false: $f(x) = ax^4 + bx^3 + c$ has at least one real zero. ans: False

19. (L16) Use discriminant to determine the number and type of solutions to the following equations. Solve each equation in the complex number system.

    (a) $4x(x + 3) = -9$ (b) $3x^2 - 4x + 5 = 0$

    (ans.(a) 1 repeated solution $x = -\frac{3}{2}$, (b)2 distinct complex solutions $x = \frac{2 \pm \sqrt{11}i}{3}$)

20. (L16) Find all solutions of the equations in the complex number system:

    (a) $x^4 - 1 = 0$ (b) $(y + 1)^2 = -16$ (c) $2x^2 - x - 5 = 0$

    (ans.(a) $x = \pm i, \pm 1$ (b) $y = -1 \pm 4i$ (c)$\frac{1 \pm \sqrt{41}}{4}$)

21. Find the 10th degree polynomial that has the following zeros (multiplicities are in parentheses)

    $$x = -1(2), \quad x = -2(3), \quad x = i(2), \quad x = 0(1)$$

    and satisfies the condition $f(1) = -27$.
    (Ans. $f(x) = -\frac{1}{16}x(x + 1)^2(x + 2)^3(x - i)^2(x + i)^2$)

22. Use synthetic division to find thh quotient and remainder when

    $$f(x) = 2x^3 + 4x - 5$$

    is divided by $(x - 3)$.
    (Ans. $Q(x) = 2x^2 + 6x + 22; R(x) = 61$)

23. Use factor theorem to determine whether $(x - 1)$ is a factor of

    $$f(x) = 4x^4 - 3x^3 + 2x^2 - 5x + 8.$$

    (Ans. No).

24. Find all zeros of

    $$f(x) = x^5 + 5x^4 + 5x^3 + 25x^2 - 36x - 180$$

    if -5 is one zero of f.
    (Ans. $x = \pm 3i, \pm 2, -5$).

25. Find the equation of a rational function which has the following features:

    $x - int : x = -3$(where it crosses the $x-$axis) and $x = 0$ (where it touches the $x-$axis).

    VA: $x = -2$ (where it does not change the signs) and $x = 1$ (where it does change the signs)

    Hole: $(-4, 0)$

    (It is known that there are no other zeros of the numerator or denominator except the ones that are listed, and each listed zero has multiplicity not greater than 2. Also, the leading coefficients of the numerator and the denominator are equal 1).
    (Ans. $\frac{(x+3)x^2(x+4)^2}{(x+2)^2(x-1)(x+4)}$).