Review for MAC 1140 Exam 2 Spring 2011

1. Solve the following inequalities:

   (a) $2x + 5 \le 4 + 3x$ (b) $-3 < 1 - 2(x + 5) \le 5$

   (c) $2 + |2 - x| \ge 0$ (d) $4 - \frac{8}{3}|\frac{2x-3}{4} + \frac{1}{2}| > 0$

   (ans: a. $[1, \infty)$, b. $[-7, -3)$, c. $(-\infty, \infty)$, d. $(-\frac{5}{2}, \frac{7}{2})$.)

2. Find x so that the distance between the points $(x, 3)$ and $(-3, 5)$ is 5.

   (ans: $x = -3 \pm \sqrt{21}$)

3. Find the center C and radius r of the circle $3x^2 + 3y^2 + 12x - 6y = 1$.
   (ans: $C(-2, 1)$ and $r = \frac{4\sqrt{3}}{3}$)

4. Find (a) the standard form and (b)the general form of the equations of the circle whose 2 end points of a diameter are $(1, -2), (9, 6)$.

   (ans:$(x - 5)^2 + (y - 2)^2 + 32$, $x^2 - 10x + y^2 - 4y - 3 = 0$)

5. Find (a) the standard form and (b)the general form of the equations of the circle whose center is $(-1, -1)$ with radius 3.

   (ans:$(x + 1)^2 + (y + 1)^2 = 9, x^2 + 2x + y^2 + 2y - 7 = 0$)

6. Find (a) the standard form and (b)the general form of the equations of the circle with center $(5, -3)$ passing through $(1, 0)$.

   (ans:$(x - 5)^2 + (y + 3)^2 = 25$).

7. Which of the following functions are even, odd or neither?
Any symmetry?

$f(x) = |x-3|,\ g(x) = |x| - 3,\ h(x) = x - 3,\ \ p(x) = (x-3)^2 + 3,$
$q(x) = \frac{1}{\sqrt{x^2+5}},\ \ k(x) = x^3 - 2x^2,\ \ l(x) = x^4 - x^2,\ \ r(x) = x^3 + x^5.$
$f_1(x) = 4x^4 - 2x^2 + 1, f_2(x) = 3x^5 + 5x^3 + 1.$

(ans: Even: g, l, q, f_1; Odd: r; Neither: f, h, p, k, l, f_2).

8. Given 2 points $A(-3,5), B(3,-2)$ on the line.
(a) Find the midpoint M and the length of the segment AB.
(b) Determind whether the point $(69,-79)$ is on the line passing through the points A and B.
(ans. $M = (0, 3/2); d(AB) = \sqrt{85}$, yes.)

9. Find the equation of the line passing through $(-6,-3)$ that is parallel to the line through $(-1,2)$ and $(\frac{1}{2}, 4)$. Also find the $y-int$.
(ans: $4x - 3y = -15; y - int = 5$)

10. Given $g(x) = (x-3)^2$. Which of the following is/are true?
A. The range is $[3, \infty)$
B. The function is increasing through out its domain
C. The vertex is at $(3,0)$
D. There are no $x-$int.
(ans: only C)

11. (a) If $f(x) = \dfrac{x+2}{2x+1}$ and $g(x) = \dfrac{x}{x-2}$, find $(f \circ g)(5)$.
(b) If $f(x) = \dfrac{1}{x^2}$ and $g(x) = \sqrt{1-x}$, find $(f \circ g)(x)$, $\left(\frac{f}{g}\right)(x)$ and their domains. Find $(f \circ g)(0)$
(ans: $(f \circ g)(x) = \frac{3x-4}{3x-2}$, $f(g(x))(5) = \frac{11}{13}$, $f(g(x)) = \frac{1}{1-x}$, domain:$(-\infty, 1)$.; $\left(\frac{f}{g}\right)(x) = \frac{1}{x^2\sqrt{1-x}}$, Domain:$(-\infty, 0) \cup (0, 1)$; $(f \circ g)(0) = 1$)

12. The profit function P for a company selling x items is $P(x) = -3x^2 + 96x - 368$. What value of x will maximize the profit?

    (ans: $x = 16$).

13. The height s of an object aftger t seconds is given by $s = -16t^2 + 128t + 50$. Find the maximum height of the object and the time it takes the object to reach this height.

    (ans: 306 ft, 4 sec.)

14. (a) Given $h(x) = 4 - \frac{x}{3}$, explain why does $h(x)$ have an inverse function and find $h^{-1}(-2)$.

    (b) Given $g(x) = \sqrt{x} - 1$, explain why does $g(x)$ have an inverse function and graph $g^{-1}(x)$.

    (c) For the given functions below with its restricted domain, find $f^{-1}(x)$ and their domain and range .
    (a)$f(x) = x^2 - 2x, x \geq 1$ (b) $f(x) = \sqrt{x^2 + 2x}, x \geq 0$

    (ans:(a) Inverse function exists because h is an 1-to-1 function and $h^{-1}(-2) = 18$
    (b)Inverse function exists because g is an 1-to-1 function and $g^{-1}(x) = (x+1)^2$
    (C) (a)$f^{-1}(x) = 1 + \sqrt{1+x}$,D:$[-1, \infty)$; R:$[1, \infty)$.
    (b) $f^{-1}(x) = -1 + \sqrt{1 + x^2}$,D:$[0, \infty)$; R:$[0, \infty)$)

15. Explain how $g(x) = (x-1)^2 + 2$ can be obtained by a transformation of the graph of $f(x) = x^2$.

    (ans: shift right 1 unit, upward 2 units).

16. Given $f(x) = -3x^2 - 6x - 5$. Find the vertex, $x-$int, $y-$int, domain and range.

    (ans:$Vertex: (-1, -2)$; $x-$int: none; $y - int$: $(0, -5)$; $D: (-\infty, \infty)$; $R: (-\infty, -2]$)

17. A piecewise function f is given:

$$f(x) = \begin{cases} x+1, & x \le -2 \\ -1, & -2 < x < 1 \\ x^2+1, & x \ge 1 \end{cases}$$

and g is a one-to-one function such that $g(1) = -2$.

Find: (a) the x values on which $f(x) \le 0$;
(b) let $a = f(-2)$,$b = g^{-1}(-2)$ and $c = y - int$, find $a - b - c$.

(ans: (a)$(-\infty, 1)$, (b)-1).

18. Find the average rate of change of $f(x) = \frac{1}{1-x}$ on the interval $(-3, -1)$ and from -3 to x.

(ans: $\frac{1}{8}$, and)

19. Let

$$f(x) = \begin{cases} \dfrac{\sqrt{1-x}}{x^2-9} & -4 \le x < 1 \\ \dfrac{1}{x-4}, & 3 < x \end{cases}$$

Find the domain of the function f.

(ans: $D : [-4, -3) \cup (-3, 2) \cup (2, 1) \cup (3, \infty)$)

20. please practice the homework problem number 92 in section 1.4.

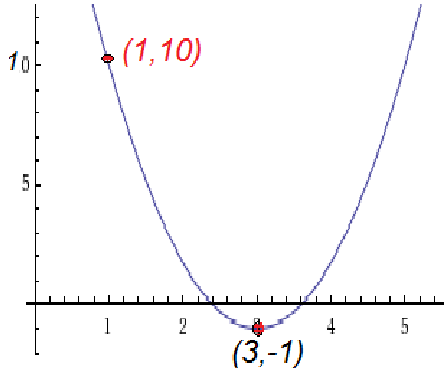
21. Find k so that the line connecting the points $(1, -2), (3, k)$ is perpendicular to the line $2x + y + 4 = 0$.

(ans: $k = -1$).

22. Let f be an odd 1-to-1 function such that $f(1) = 3, f^{-1}(1) = 2$. Find $f^{-1}(3) - f(2) =?$

(ans: 0).

23. Consider the parabola below,



(a) Determine and express the equation of the graph as $f(x) = a(x - h)^2 + k$.

(b) If $(2, 4)$ is a point on the graph of its parent function $y = x^2$, what's the corresponding point on f?

(c) What's the zero of the function f?

You may use the following steps to help you determine the equation of the given graph:

Start with the parent function: ________________.

Horizontal Shift ____ units (right/left) (select one)

Vertical (stretch/compression) (select one)

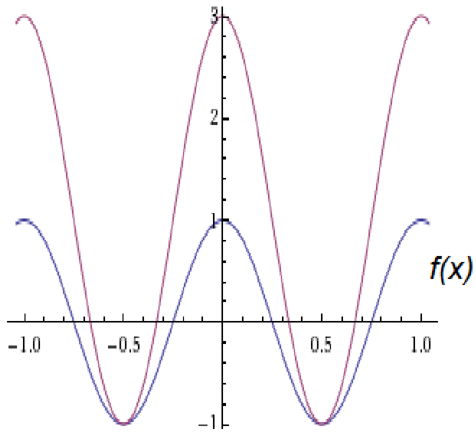
Vertical shift qquad units (up/down) (select one)

(Ans: $y = \frac{11}{4}(x-3)^2 - 1$); $(5, 10)$; $x = 3 \pm \frac{2}{\sqrt{11}}$)

24. If $(1, 4)$ is a point on the graph of $y = h(x)$, find the corresponding point on the graph of $-2h(\frac{1}{2}x)$.

(ans: $(\frac{1}{2}, -8)$).

25. Consider the graph of f shown in blue, find a possible formula for the transformation of f shown in the same coordinate system in red.



(ans: $2f(x) + 1$).