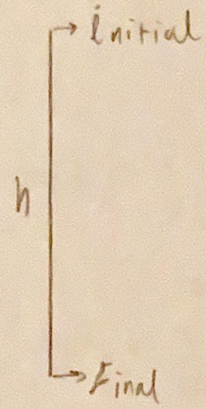
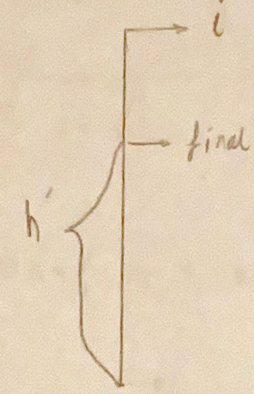
Solutions

1.





$$E_i = E_f$$
$$mgh = \frac{1}{2} m v^2$$

if $\frac{1}{2} m v^2 = mgh$

$$\Rightarrow \frac{\frac{1}{2} m v^2}{9} = \frac{mgh}{9}$$

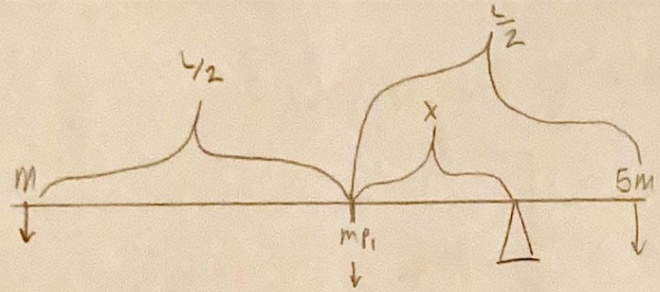
$$E_i = E_f$$

$$mgh = mgh' + \underbrace{\frac{1}{2} m \left(\frac{v}{3}\right)^2}_{\frac{\frac{1}{2} m v^2}{9} = \frac{mgh}{9}}$$

$$\cancel{mg}h = \cancel{mg}h' + \frac{\cancel{mg}h}{9}$$

$$\boxed{h' = \frac{8h}{9}}$$

2.



what is X?

$$\tau = 0 = m\cancel{g}\cdot(L/2+x)\cdot\cancel{\sin\theta} + m_{p_1}\cdot\cancel{g}\cdot x\,\cancel{\sin\theta} - 5m\cancel{g}(L/2-x)\cdot\cancel{\sin\theta}$$

$$0 = \frac{m\cdot L}{2} + mX + m_{p_1}\cdot X - \frac{5mL}{2} + 5mX$$

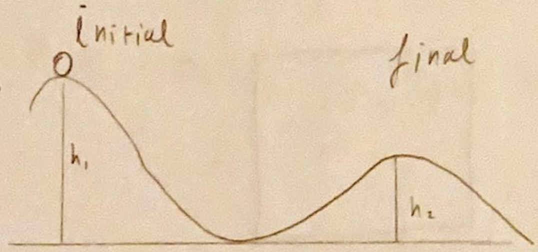
$$= -2mL + 6mX + m_{p_1}X$$

$$2mL = 6mX + m_{p_1}X$$

$$2mL = X\left[6m + m_{p_1}\right]$$

$$\boxed{X = \frac{2mL}{6m+m_{p_1}} = \frac{4mL}{2\left[6m+m_{p_1}\right]}}$$

3.



$$E_1 = E_f$$

$$mgh_1 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh_2$$

kinetic
rotational
energy

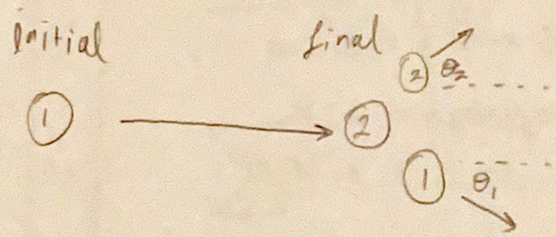
$$\omega = \frac{v}{R} \qquad I_{\text{solid sphere}} = \frac{2}{5} m R^2$$

$$mgh_1 = \frac{1}{2} mv^2 + \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{5} m \cancel{R^2} \cdot \frac{v^2}{\cancel{R^2}} + mgh_2$$

$$mgh_1 = \frac{7}{10} mv^2 + mgh_2$$

$$\cancel{m}gh_1 - \frac{7}{10} \cancel{m} v^2 = \cancel{m}gh_2 \longrightarrow \boxed{h_2 = h_1 - \frac{7v^2}{10g}}$$

4.



$$X: \quad \vec{P}_i = \vec{P}_f$$

$$m_1 v_{1i} + \cancel{m_2 v_{2,i}} = m_1 v_{1f} \cdot \cos\theta_1 + m_2 v_{f2} \cdot \cos\theta_2$$

$$v_{2,i} = 0$$

$$v_{1i} = v_{1f} \cdot \cos\theta_1 + \frac{m_2}{m_1} \cdot v_{f2} \cdot \cos\theta_2$$

$$y: \quad m_1 v_{i1,y} + m_2 v_{i2,y} = 0 = -m_1 v_{1f} \cdot \sin\theta_1 + m_2 v_{2f} \cdot \sin\theta_2$$

$$v_{2f} = \frac{m_1}{m_2} \cdot v_{1f} \cdot \frac{\sin\theta}{\sin\theta_2}$$

$$V_{1,i} = V_{1f} \cdot \cos\theta_1 + \frac{\cancel{m_2}}{\cancel{m_1}} \cdot \left[\frac{\cancel{m_1}}{\cancel{m_2}} \cdot V_{1f} \cdot \frac{\sin\theta_1}{\sin\theta_2}\right] \cdot \cos\theta_2$$

$$V_{1i} = V_{1f} \cdot \cos\theta_1 + V_{1f} \cdot \sin\theta_1 \cdot \frac{\cos\theta_2}{\sin\theta_2}$$

$$\boxed{V_{1i} = V_{1f} \cdot \cos\theta_1 + \frac{V_{1f} \cdot \sin\theta_1}{\tan\theta_2}}$$

$* \ \frac{\sin}{\cos} = \tan$

$\therefore \frac{\cos}{\sin} = \frac{1}{\tan}$

5. $a = -\frac{3.63\,m}{s^2}$

$V_i = \frac{15.6\,m}{s}$, $V_f = 0$

$\Delta x = ?$

$V_f^2 = V_i^2 + 2a\Delta x$

$\Delta x = \frac{-V_i^2}{2a}$

$\Delta x = \frac{-\frac{243.36\,m^2}{s^2}}{2 \cdot \left(-\frac{3.63\,m}{s^2}\right)}$

$\Delta x = 33.52\,m$

radius of wheel $= 0.27\,m$

circumference $= 2\pi r$
$= 1.70\,m$

full ratations $= \frac{\Delta x}{\text{circumference}}$

$= \frac{33.52\,m}{1.70\,m}$

$= 19.72$

$\boxed{\therefore \text{FULL rotations} = 19}$

6. Kepler's 3rd Law:

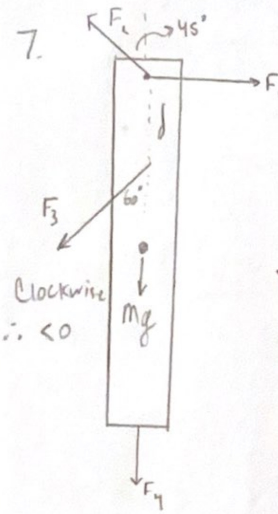
$$T_{\text{Space ship}}^2 = \frac{4\pi^2 r^3}{GM_{\text{planet}}}$$

$$r^3 = \frac{T_{ss}^2 \cdot GM_{\text{Planet}}}{4\pi^2}$$

$$\boxed{r = \sqrt[3]{\frac{T^2 \cdot GM}{4\pi^2}}}$$

• Because it is geosynchronous, the spaceship orbits around the planet just how earth orbits around the sun. $\therefore$ $T_{\text{spaceship}} = T_{\text{Earth}}$ & $M_{\text{planet}} = M_{\text{sun}}$ we can use Kepler's 3rd law without modifying it any further.

7.



Clockwise $\therefore < 0$

$F_1 = F_2 = F_3 = F_4 = 5.00\,N$

$I_{\text{thin rod}} = \frac{1}{3} ML^2$

$$\sum \tau = I\alpha = -F_3 \cdot d \cdot \sin(60°) - \cancel{F_2 \cdot 0} + \cancel{F_1 \cdot 0} + \cancel{mg \cdot \tfrac{L}{2} \cdot \sin(0)}$$
$$+ \cancel{F_4 \cdot L \cdot \sin 0}$$

0 0 0 0

$$d = \frac{I\alpha}{-F_3 \cdot \sin(60°)} \qquad d = \frac{\frac{1}{3} \cdot 2.30\,kg \cdot [0.900\,m^2]}{-5.00\,N \cdot \sin(60°)}$$

$$\boxed{d = 2.02\,m}$$

8.

$\longrightarrow$

$m = 0.125\ kg$

$F = 200N$

$L = 1.20m = \Delta x$

$V_f = ?$

$\Delta t = ?$

$V_i = 0$

$\vec{J} = F \cdot \Delta t = m \Delta V$

$\frac{F}{m} = a$

$\Delta x = v_i \cdot t + \frac{a}{2} \cdot t^2$

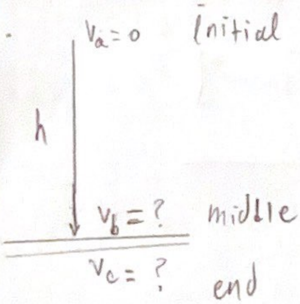
$\Delta x = \cancel{v_i t}^{\,0} + \frac{F}{2m} \cdot t^2 \rightarrow t = \sqrt{\frac{2 m \Delta x}{F}}$

$t = 0.039$ seconds

$F \cdot \Delta t = m[V_f - V_i]$

$V_f = \frac{F \cdot \Delta t}{m} \rightarrow \boxed{V_f = 61.97 \cong 62 \frac{m}{s}}$

9.



$\cancel{m}gh = \frac{1}{2}\cancel{m}(V_b)^2$

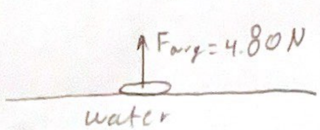
$V_b = \sqrt{2gh}$

$V_b = -7 \frac{m}{s}$ (negative because it is pointing down)

from V_b to V_c

$F_{avg} = 4.80N$ for $1.49s$

Force is > 0 because it is pointing up



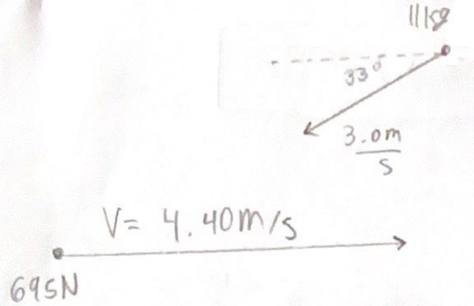
$F_{avg} \cdot \Delta t = m \Delta V = m(V_c - V_b) = \Delta \vec{P}$

$\frac{F_{avg} \cdot \Delta t}{m} + V_b = V_c = -1.70 \frac{m}{s}$

(< 0 because it's pointing down)

$\boxed{V_c = 1.70\ m/s}$

10.



$$\vec{P}_{initial} = \vec{P}_{final}$$

$$M_{Shaun} \cdot V_s - M_{ball} V_b \cdot \cos 33° = (m_s + m_b) \cdot V_f$$

$$m = \frac{695N}{9.8\frac{m}{s^2}}$$

$$m = 70.92\ kg$$

$$70.92kg \cdot 4.40\frac{m}{s} - 11kg \cdot 3.0\frac{m}{s} \cdot \cos 33°$$

$$= (81.92kg) \cdot V_f$$

$$\boxed{V_{final} = 3.47\frac{m}{s}}$$

11.

initial $\longrightarrow$ final

$v_i = 10\frac{m}{s}$ $t = 10s$ $v_f = 40\frac{m}{s}$

$m = 1000\,kg$

what is mechanical power output

$$P = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t} = \frac{\text{change in mechanical energy}}{\Delta t}$$

$$W = \Delta E = \Delta K.E = K.E_f - K.E_i$$

$$= \frac{1}{2}m\cdot(v_f)^2 - \frac{1}{2}m(v_i)^2$$

$$= \frac{1}{2}m\left[v_f^2 - v_i^2\right]$$

$$W = \frac{1}{2}\cdot 1000\,kg \cdot \left[\frac{1600\,m^2}{s^2} - \frac{100\,m^2}{s^2}\right]$$

$$= 750000\,J$$

$$\boxed{P = \frac{W}{t} = \frac{750000\,J}{10\,s} = 75000\,J}$$

12.

$L = 4.5m$

$\delta = 0.04m$

$r = 0.02m$

$k = \frac{490.7N}{m}$

$$\frac{F}{A} = y \cdot \frac{d}{L}$$

$$y = \frac{F \cdot L}{A \cdot d}$$

$$y = \frac{F \cdot L}{\pi r^2 \cdot d}$$

$$y = \frac{35kg \cdot \frac{9.8m}{s^2} \cdot 4.5m}{\pi (0.02\,m)^2 \cdot 0.699m}$$

$$\boxed{y = 1.76 X 10 \frac{N}{m^2}}$$

$$F = -k \cdot d$$

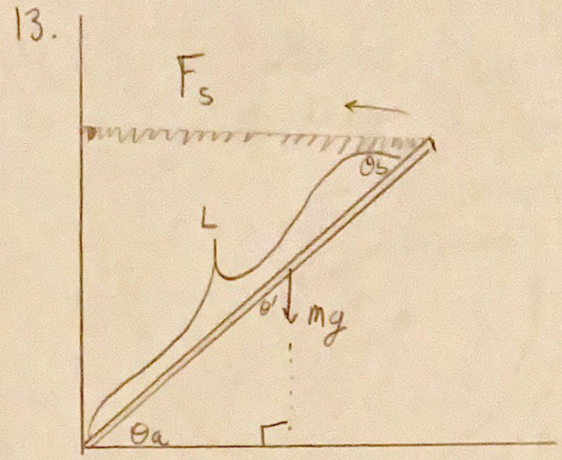
$$-mg = -k \cdot d$$

$$\delta = \frac{-mg}{-k}$$

$$d = \frac{-35kg \cdot \frac{9.8m}{s^2}}{\frac{-490.7N}{m}}$$

$$d = 0.699\ m$$

13.



$\theta_a = \theta_b$

$\sin\theta' = \cos\theta_a$ → * $\sin(90-\theta) = \cos\theta$

$F_s = k \cdot \Delta x$

$= 1752 \frac{N}{m} \cdot 0.2m$

$F_s = 350.4\,N$

$\tau = 0 = 350.4N \cdot \cancel{L} \cdot \sin\theta_b - Mg \cdot \frac{\cancel{L}}{2} \cdot \sin\theta'$

$350.4N \cdot \sin\theta_b - \frac{Mg}{2} \cdot \cos\theta_a = 0$

$350.4N \sin\theta_b = \frac{588N}{2} \cdot \cos\theta_a$ * $\theta_a = \theta_b$

$\frac{\sin\theta_b}{\cos\theta_a} = \tan\theta = \frac{588N}{2 \cdot 350.4N}$

$\theta = \tan^{-1}\left[\frac{588}{2 \cdot 350.4}\right]$

$\boxed{\theta = 39.998^\circ}$

14. $\underline{\omega_{final}} = \frac{33.3 \text{ rev}}{60 \text{ seconds}} \cdot \frac{2\pi}{1 \text{ rev}} = \frac{3.4872 \text{ rad}}{\text{seconds}}$ $\underline{\omega_i} = 0$

$\underline{\theta} = 2 \text{ rev} \cdot \frac{2\pi}{1 \text{ rev}} = 12.57 \text{ radians}$

$\omega_{final}^2 = \cancel{\omega_{initial}^2} + 2\alpha \cdot \theta$

$\therefore \alpha = \frac{(\omega_{final})^2}{2\theta} = \frac{(3.4872)^2}{2 \cdot 12.57} = \frac{0.48 \text{ radians}}{s^2} = \alpha$

$\tau_{required} = I_{\substack{uniform \\ disk}} \cdot \alpha$

$= \frac{1}{2} M R^2 \cdot \alpha$ $\quad * M = 0.16 kg, \ R = 0.15 m$

$\boxed{\tau_{required} = 8.707 \times 10^{-4} \ Nm}$

15. How much work for that torque in 2 revolutions, in mJ?

Work due to Torque $= \tau \cdot \Delta\theta$

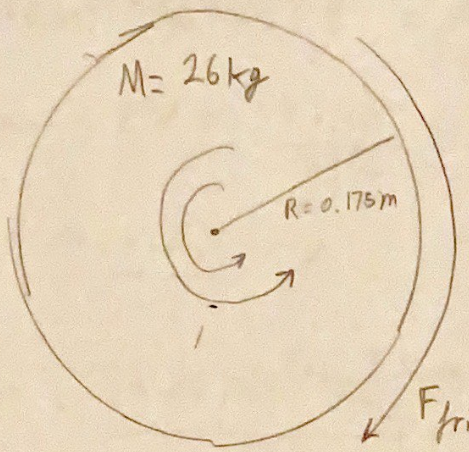
$= 8.707 \times 10^{-4} \ Nm \cdot \left[2 \text{ rev} \cdot \frac{2\pi}{1 \text{ rev}}\right]$

$W_\tau = 0.0109 J \cdot \frac{1000 \ mJ}{1 J}$

$\boxed{W_\tau = 10.9 \ mJ}$

16.

$F_N = 2.4\ N$



$F_{friction} = \mu_k \cdot F_N = 0.2 \cdot 2.4N = 0.48\ N$

$\omega_i = \frac{200.0\ rev}{60\ s} \cdot \frac{2\pi}{1\ rev} = \frac{20.94\ rads}{s}$, $\omega_{final} = ?$

$\tau = -F_{friction} \cdot R \cdot \sin\theta = -0.48N \cdot 0.175m = I\alpha = \frac{1}{2}MR^2 \cdot \alpha$

$\alpha = \frac{-0.48N \cdot 0.175m}{\frac{1}{2} \cdot 26kg \cdot (0.175m)^2}$

$\alpha = \frac{-0.211\ rads}{s^2}$

What is ω_f after 10 seconds?

$\omega_f = \omega_i + \alpha \cdot t$

$\omega_f = \frac{20.94\ rad}{s} - \frac{0.211}{s^2} \cdot 10s$

$$\boxed{\omega_f = \frac{18.83\ rads}{s}}$$

17. $L = m \cdot v_{tran} \cdot R$

$T = 687 \text{ days} \cdot \frac{24 \text{ hours}}{1 \text{ day}} \cdot \frac{3600 \text{ s}}{1 \text{ hour}} = 59356800 \text{ seconds}$

$v_{tran} = r\omega = \frac{r \cdot 2\pi}{T}$

$r = 228 \times 10^6 \text{ km} \cdot \frac{10^3 \text{ m}}{1 \text{ km}} = 228 \times 10^9 \text{ m}$

$L = m \cdot \frac{r \cdot 2\pi}{T} \cdot r$

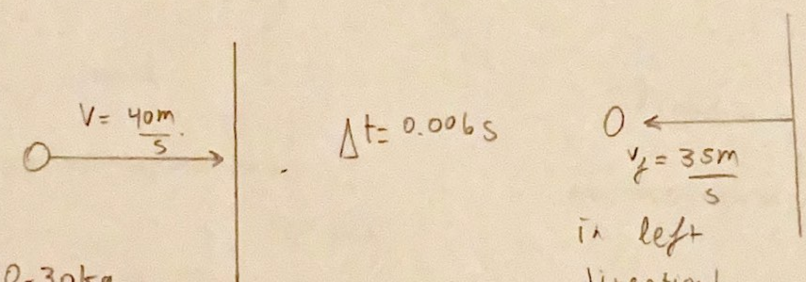
$L = \frac{(6.36 \times 10^{23} \text{ kg}) \cdot 2\pi \cdot (228 \times 10^9 \text{ m})^2}{59356800 \text{ seconds}}$

$\boxed{L = 3.50 \times 10^{39} \frac{\text{kg m}^2}{\text{s}}}$

18.

Initial

final



$m = 0.30 \text{kg}$

in left direction!

$\therefore v_f = -35 \frac{m}{s}$

$\vec{J}_1 = F \cdot \Delta t = m(v_f - v_i)$

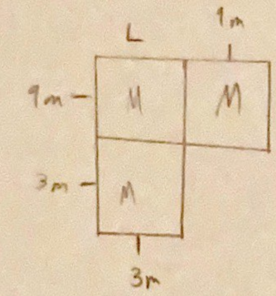
$F = \frac{m(v_f - v_i)}{\Delta t}$

$F = \frac{0.30 \text{kg} \left[-35\frac{m}{s} - 40\frac{m}{s}\right]}{0.006 \text{ s}} = -3750 \text{ N}$ ←

experienced by ball

$\therefore$ $\boxed{\text{3750 N of force on the wall}}$

19. $L = 6$



$$X: \frac{\cancel{M} \cdot 3m + \cancel{M} \cdot 3m + \cancel{M} \cdot 9m}{3\cancel{M}}$$

$$X: \frac{15m}{3} = \underline{5m}$$

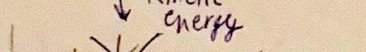
$$y: \frac{\cancel{M} \cdot 3m + \cancel{M} \cdot 9m + \cancel{M} \cdot 9m}{3\cancel{M}}$$

$$y: \frac{21m}{3} = \underline{7m}$$

$\boxed{\text{Answer: } (5m, 7m)}$

20.

initial



$\Delta X_i = 18cm$

$V_{ball} = 0$

final

$\Delta X_f = 12cm$

$V_{ball\ final} = ?$

$K = \frac{28N}{m}$

$M = 0.056\ kg$

No initial kinetic energy

$$U_i + \cancel{K_i} = U_f + K_f$$

$$\frac{1}{2} k(\Delta X_i)^2 = \frac{1}{2} k(\Delta X_f)^2 + \frac{1}{2} m V_f^2$$

$$V_f = \sqrt{\frac{k\left[(\Delta X_i)^2 - (\Delta X_f)^2\right]}{m}} \qquad V_f = \sqrt{\frac{\frac{28N}{m} \cdot \left[0.0324m^2 - 0.0144m^2\right]}{0.056\ kg}} = \frac{3.0\ m}{s}$$