1) $x(t) = 4t - t^2$, $0 \le t \le \sqrt{3}$

$y(t) = \frac{8\sqrt{2}}{3} t^{3/2}$

Formula: $L = \int_a^\beta \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$\downarrow$ Plug in formula

$$L = \int_0^{\sqrt{3}} \sqrt{(4-2t)^2 + \left(\frac{24\sqrt{2}}{6} t^{1/2}\right)^2}\, dt$$

$\downarrow$ simplify

$$\int_0^{\sqrt{3}} \sqrt{4t^2 + 16t + 16}\, dt$$

$\downarrow$ complete the square

$$\int_0^{\sqrt{3}} \sqrt{4(t+2)^2}\, dt$$

$\downarrow$ simplify

$$\int_0^{\sqrt{3}} \sqrt{4} \cdot \sqrt{(t+2)^2}\, dt = \int_0^{\sqrt{3}} 2(t+2)\, dt = \boxed{\int_0^{\sqrt{3}} 2t+4\, dt}$$

2) $f(x) = 8x + \cos x$

* use T_2 to approximate $f(.1)$

① Find T_2 using $\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$:

Plug in $n=0$, $n=1$, and $n=2$ with $a=0$

$$T_2(x) = \frac{f^{(0)}(0)}{0!}(x)^0 + \frac{f^{(1)}(0)}{1!}x^1 + \frac{f^{(2)}(0)}{2!}x^2$$

Find needed derivatives and plug in center 0; Plug those into $T_2(x)$

$$T_2(x) = 1 + 8x - \frac{\cancel{\cos x}\ 1}{2!}x^2$$

Plug in $x = .1$

$$= 1 + 8(.1) - \frac{1}{2}(.1)^2$$

convert decimals to fractions

$$= 1 + 8\left(\frac{1}{10}\right) - \frac{1}{2}\left(\frac{1}{10}\right)^2 = \boxed{1.795}$$

3) $x(t) = te^{t}$

$y(t) = e^{t}$

$$\boxed{\text{Formula: } \frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right)}{\frac{dx}{dt}}}$$

2nd Derivative formula

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{e^{t}}{te^{t}+e^{t}}\right)}{te^{t}+e^{t}}$$

$\downarrow$ Continue differentiating

$$\frac{d^2y}{dx^2} = \frac{-\frac{1}{(t+1)^2}}{te^{t}+e^{t}}$$

We need a t-value to plug in... set y-coordinate of given point equal to $y(t)$ to find t:

$$e = e^{t} \xrightarrow[\text{gives}]{} \boxed{t=1}$$

Plug in $t=1$ into $\frac{d^2y}{dx^2}$ to obtain $\boxed{-\frac{1}{8e}}$

4) Find sum of: $\underbrace{\left(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\dots\right)}_{A} - \underbrace{\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\dots\right)}_{B}$

* Find power series representation of A and B first, and then find their sum:

$$A: \left(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\dots\right) = \sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}(1)^{2n+1} \xrightarrow[\text{Let } x=1]{} \sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}x^{2n+1}$$

This matches the series rep for arctan(x), so to find A, plug in $x=1$ into arctan(x) formula:

$$A = \arctan(1) = \sum_{n=0}^{\infty}\frac{(-1)^n(1)^{2n+1}}{(2n+1)} = \frac{\pi}{4}$$

Now Find B:

$$B: \left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\dots\right) = \sum_{n=0}^{\infty}\frac{(-1)^n(1)^n}{n!} = \sum_{n=0}^{\infty}\frac{(-1)^n}{n!} \longrightarrow \sum_{n=0}^{\infty}\frac{x^n}{n!}$$

Let $x=-1$

Now find the function whose power series representation in this

We know $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$\downarrow$ Plug in $x=-1$ on Both sides to find B

$$e^{-1} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = B$$

Now we have the sum:

$$A - B = \boxed{\frac{\pi}{4} - e^{-1}} \xrightarrow{\text{rewrite}} \frac{e\pi - 4}{4e}$$

5) Convert $r = 4\cos\theta - 2\sin\theta$ into Cartesian Coordinates.

* Formulas to know: $r^2 = x^2 + y^2$

$r\cos\theta = x$

$r\sin\theta = y$

Start with: $r = 4\cos\theta - 2\sin\theta$

$\downarrow$ multiply both sides by r

$r^2 = 4r\cos\theta - 2r\sin\theta$

$\downarrow$ Plug in formulas above

$x^2 + y^2 = 4x - 2y$

$\downarrow$ rewrite

$(x^2 - 4x) + (y^2 + 2y) = 0$

$\downarrow$ complete the square of both terms

$(x^2 - 4x + 4) - 4 + (y^2 + 2y + 1) - 1 = 0$

$\downarrow$ rewrite into equation of circle

$(x-2)^2 + (y+1)^2 = 5$

r^2

So we have Circle w/ center $(2,-1)$ and $r = \sqrt{5}$

6) Find Taylor Series of e^x, centered at 2:

Start with: $$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$\downarrow$ Plug in $(x-2)$ for x

$$e^{x-2} = \sum_{n=0}^{\infty} \frac{(x-2)^n}{n!}$$

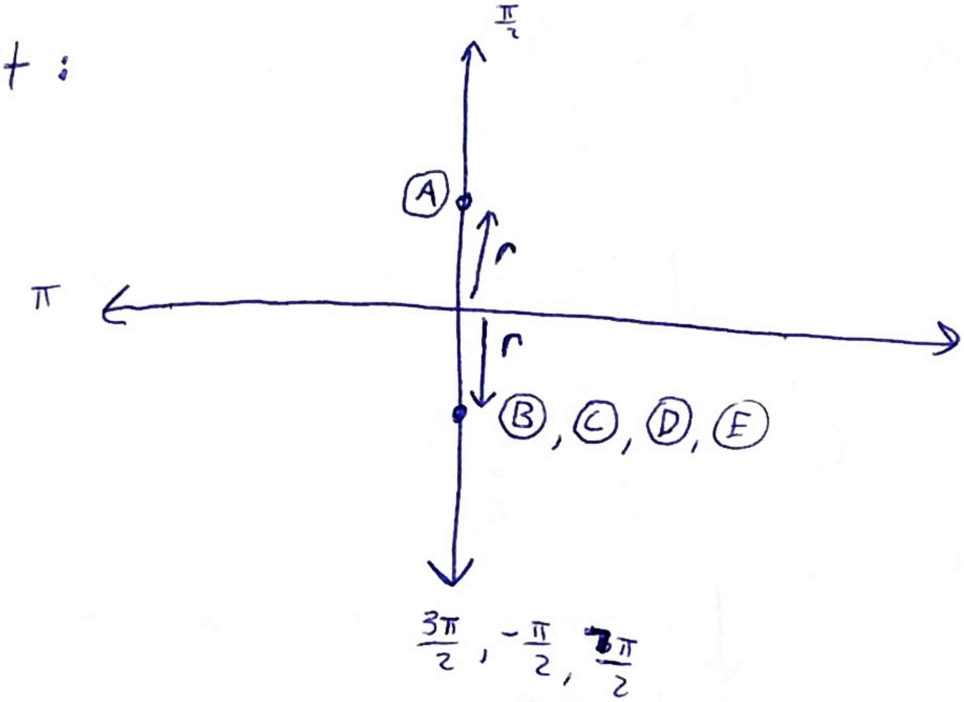
$\downarrow$ rewrite left side to get e^x alone

$$e^x \cdot e^{-2} = \sum_{n=0}^{\infty} \frac{(x-2)^n}{n!}$$

$\downarrow$ Solve for e^x

$$\boxed{e^x = \sum_{n=0}^{\infty} \frac{e^2 (x-2)^n}{n!}}$$

7) Plot each point on a polar graph to see if each choice gives the same point:



Choice A doesn't match others.

8) $\int_0^{.1} e^{-x^2}\,dx = \sum_{n=0}^{\infty} c_n x^n$

① First find the power series representation of the above definite integral:

Start with: $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$$\downarrow$$

$$e^{-x^2} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}$$

$\downarrow$ Integrate both sides

$$\int_0^{.1} e^{-x^2}\,dx = \left.\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{n!(2n+1)}\right]_0^{.1}$$

$\downarrow$ Plug in upper/lower bounds of integral into series representation

$$\int_0^{.1} e^{-x^2}\,dx = \sum_{n=0}^{\infty} \frac{(-1)^n (.1)^{2n+1}}{n!(2n+1)} - 0$$

Now rewrite the decimal as a fraction ($.1$ as $\frac{1}{10}$)

So the series we're working with is:

$$\sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{1}{10}\right)^{2n+1}}{n!\,(2n+1)}$$

$$\Big\downarrow \text{ rewrite}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{10^{2n+1} \cdot n! \cdot (2n+1)}$$

So we know $|\text{error}| \leq b_{n+1}$, and we're given ~~to~~ $\underline{n=2}$, so our ~~error~~ inequality is:

$$|\text{error}| \leq b_{2+1}$$

$$\downarrow$$

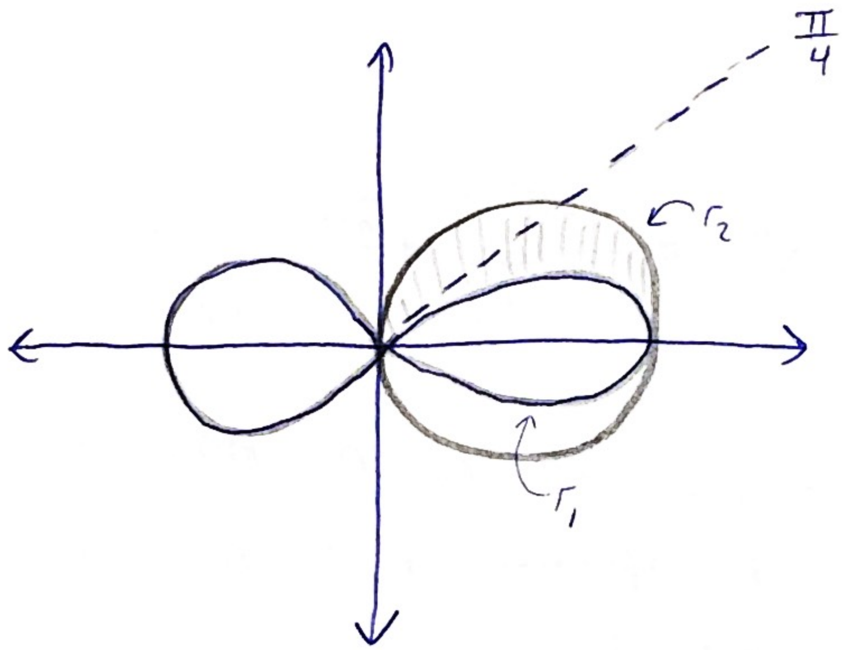
$$|\text{error}| \leq b_3$$

$$\Big\downarrow \text{ to find } b_3\text{, plug in } n=3 \text{ into absoblute value of series}$$

$$\boxed{|\text{error}| \leq \frac{1}{10^7 \cdot 7 \cdot 3!}}$$

9) Given: $r_1^2 = 9\cos(2\theta)$, $r_2 = 3$

First draw the polar curves:



We want the shaded region, which is given by:

$$\int_0^{\frac{\pi}{4}} \frac{1}{2}(r_2)^2 d\theta - \int_0^{\frac{\pi}{4}} \frac{1}{2}(r_1)^2 d\theta$$

$\downarrow$ Plug in

$$\int_0^{\frac{\pi}{4}} \frac{1}{2}(3)^2 d\theta - \int_0^{\frac{\pi}{4}} \frac{1}{2}(9\cos 2\theta)\, d\theta$$

$\downarrow$ simplify

$$\boxed{\frac{9\pi}{4} - \frac{1}{2}\int_0^{\frac{\pi}{4}} 9\cos(2\theta)\, d\theta}$$

10) $f(x) = \tan^{-1}(x)$, find $f^{(2019)}(0)$

* Set taylor series equal to Mclaurin series for $\arctan x$:

$$\frac{f^{(2019)}(0)}{(2019)!}x^{2019} = \frac{(-1)^n}{(2n+1)}x^{2n+1}$$

↳ we need to figure out an "n" value to plug into right side. Set exponents of x equal to each other and solve for n:

$$2019 = 2n+1$$

↓ solve for n

$$n = 1009$$

Now plug in $n = 1009$ on the right side:

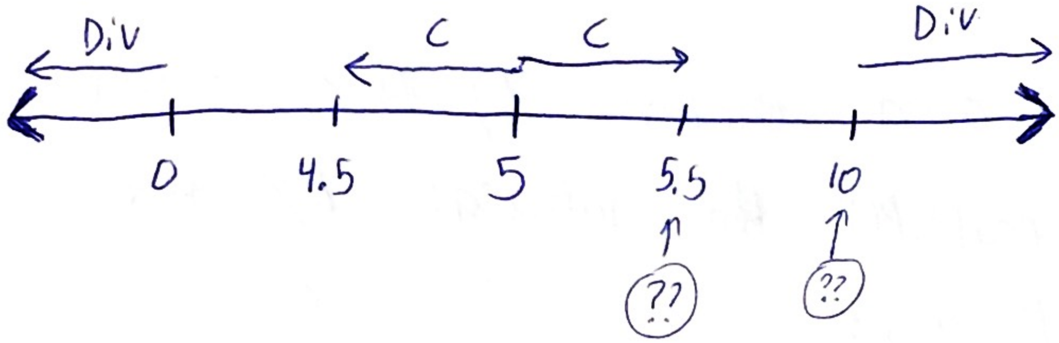
$$\frac{f^{(2019)}(0)}{(2019)!} = \frac{-1}{(2(1009)+1)}$$

↓ solve for $f^{(2019)}(0)$

$$f^{(2019)}(0) = \frac{-(2019)!}{(2019)} = \frac{-\cancel{(2019)}\cdot 2018!}{\cancel{(2019)}} = \boxed{-(2018!)}$$

11) Given: $\sum_{n=0}^{\infty} c_n (x-5)^n$, Converges for $x = 4.5$
Diverges for $x = 0$

① Draw numberline:



② Now find x-value for f and g, and then go to those x-values on the numberline:

$f: \sum c_n (1)^n \longrightarrow \sum c_n (1)^n = \sum c_n (x-5)^n$

$\downarrow$

$1 = x - 5 \longrightarrow \boxed{x = 6}$

not enough info at $x=6$...

$g: \sum (-1)^n c_n \longrightarrow \sum (-1)^n c_n = \sum c_n (x-5)^n$

$\downarrow$

$-1 = x - 5 \longrightarrow \boxed{x = 4}$

not enough info at $x=4$...

12) $r = \sin(4\theta)$

The area in first Quadrant is given by the integral $\boxed{\frac{1}{2}\int_0^{\frac{\pi}{4}} r^2 d\theta}$

To find the area enclosed by the entire Polar graph, multiply the integral by the number of Petals:

↳ for $r = a\sin(\underline{n}\theta)$, there will be $\underline{2n}$ Petals,

so the area is given by:

$$2n\left(\frac{1}{2}\int_0^{\frac{\pi}{4}} r^2 d\theta\right)$$

↓ Here, $n=4$

$$\boxed{8\left(\frac{1}{2}\int_0^{\frac{\pi}{4}} r^2 d\theta\right)}$$

$f(x) = (x-1)^{1/2}$, centered at $a=10$

n	$f^{(n)}(x)$	$f^{(n)}(10)$	$\frac{f^{(n)}(10)}{n!}$
0	$(x-1)^{1/2}$		
1	$\frac{1}{2}(x-1)^{-1/2}$		
2	$\frac{1}{2}\left(-\frac{1}{2}\right)(x-1)^{-3/2}$		
3	$\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(x-1)^{-5/2}$		
4	$\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)(x-1)^{-7/2}$		
5	$\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\left(-\frac{7}{2}\right)(x-1)^{-9/2}$		
$\vdots$	$\vdots$		
n	$\frac{(-1)^{n+1}\, 1\cdot 3\cdot 5\cdots(2n-1)}{2^n}(x-1)^{-\frac{2n-1}{2}}$	$\frac{(-1)^{n+1}\, 1\cdot 3\cdot 5\cdots(2n-1)}{2^n}(10-1)^{-\left(\frac{2n-1}{2}\right)}$	$\frac{(-1)^{n+1}\, 1\cdot 3\cdot 5\cdots(2n-1)}{2^n\, 3^{2n-1}\, n!}$

$$\underbrace{(10-1)^{1/2}}_{n=0} + \underbrace{\frac{1}{2}(10-1)^{-1/2}(x-10)^1}_{n=1} + \sum_{n=2}^{\infty} \frac{(-1)^{n+1}\, 1\cdot 3\cdot 5\cdots(2n-3)}{2^n\cdot 3^{2n-1}\cdot n!}(x-10)^n$$

B) Find $T_2(x)$: Plug in $\underline{n=2}$ into our series

$$T_2(x) = 3 + \frac{1}{6}(x-10)^1 - \frac{1}{2^2 \cdot 3^3 \cdot 2!}(x-10)^2$$

c) Approximate $\sqrt{9.1}$

We need to find an x-value to plug into $T_2(x)$.

↳ To find x, set function given equal to number we're approximating:

$$\sqrt{x-1} = \sqrt{9.1}$$

$$\downarrow \text{solve for } x$$

$$\boxed{x = 10.1}$$

Plug this x-value into $T_2(x)$:

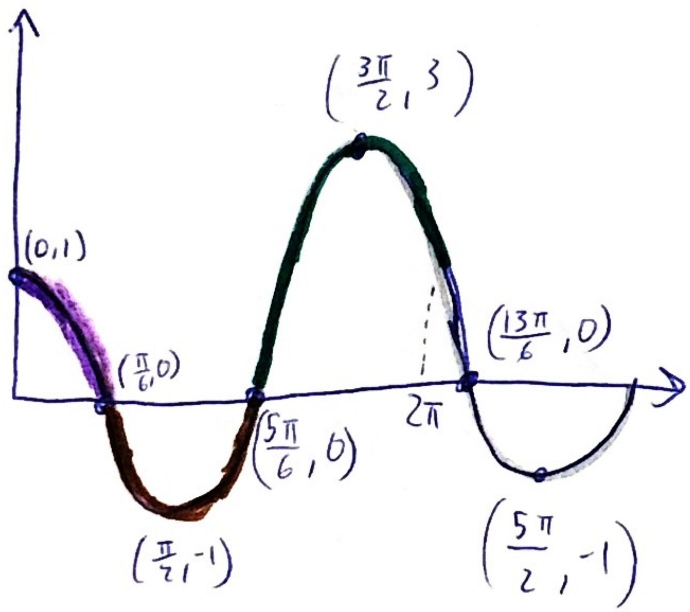
$$\boxed{T_2(x) = 3 + \frac{1}{6}(10.1-10) - \frac{1}{2^2 \cdot 3^3 \cdot 2!}(10.1-10)^2}$$

2) $r = 1 - 2\sin\theta$

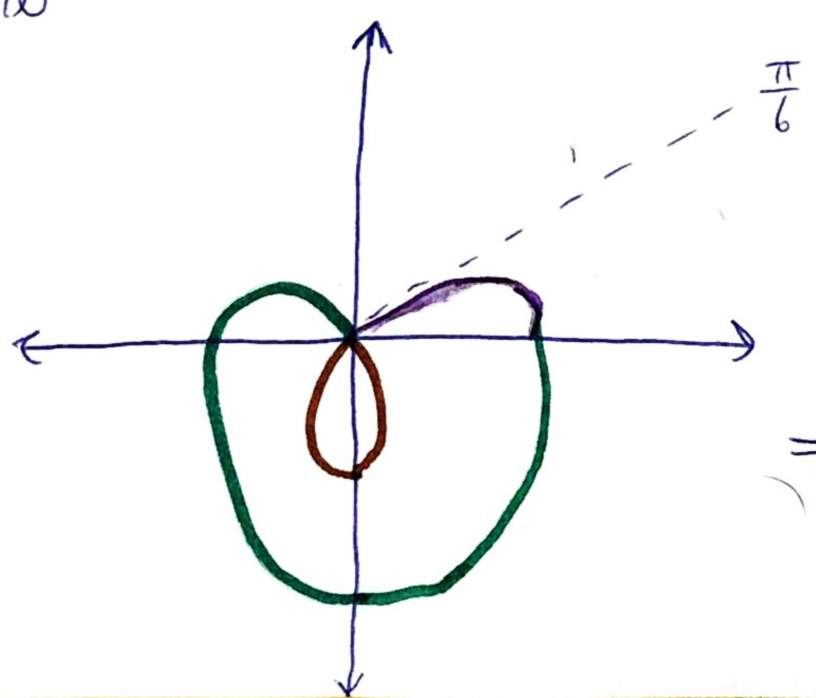
A) Set $r = 0$: ~~$0 = 1 - 2\sin\theta$~~

$0 = 1 - 2\sin\theta \longrightarrow \sin\theta = \frac{1}{2} \longrightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

Draw flat curve:



Draw polar curve:



$\Rightarrow$ Area in Quadrant 1 is given by $\int_0^{\frac{\pi}{6}} \frac{1}{2} r^2 \, d\theta$