MAC 2312
Fall 2019
EXAM 2

Section # ____________ Name ______________________________

UF ID # ____________ TA Name ______________________________

A. Sign your scantron on the back at the bottom in ink.

B. In pencil, write and encode on your scantron in the spaces indicated:

1) Name (last name, first initial, middle initial)
2) UF ID Number
3) Section Number

C. Under "special codes", code in the test ID number 2, 1.

1 ● 3 4 5 6 7 8 9 0

● 2 3 4 5 6 7 8 9 0

D. At the top right of your answer sheet, for "Test Form Code", encode A.

● B C D E

E. 1) There are fifteen 2-points multiple choice questions, plus two free response questions of 10 points for a total of 40/40 points.
2) The time allowed is 90 minutes.
3) You may write on the test.
4) Raise your hand if you need more scratch paper or if you have a problem with your test. DO NOT LEAVE YOUR SEAT UNLESS YOU ARE FINISHED WITH THE TEST.

F. **KEEP YOUR SCANTRON COVERED AT ALL TIMES.**

G. When you are finished:

1) Before turning in your test, check for transcribing errors. Any mistakes you leave in are there to stay.
2) Bring your test, scratch paper, and scantron to your proctor to turn them in. Be prepared to show your UF ID card.
3) Answers will be posted in E-Learning after the exam.

The Honor Pledge: "On my honor, I have neither given nor received unauthorized aid in doing this exam."

Student's Signature:_______________

3. Match the sequence on the left with the theorem on the right to determine if the sequence is convergent.

i. $\{\pi^{1-2n}\}$

ii. $\left\{\dfrac{\sin^2(n)}{\sqrt{n+10}}\right\}$

iii. $\left\{\left(\dfrac{n-1}{n}\right)^{2n}\right\}$

P. Squeeze Theorem

Q. Absolute Convergent Theorem

R. Geometric Sequence Theorem

S. Related Function Theorem

A. (i) Converges to 0 by R, (ii) Converges to 0 by P; (iii) Converges to e by S

B. (i) Converges to 0 by R, (ii) Converges to 0 by S; (iii) Diverges by P

C. (i) Converges to 0 by P, (ii) Converges to 0 by S; (iii) Converges to e^{-2} by P

D. (i) Converges to π^{-2} by P, (ii) Converges to 0 by P; (iii) Converges to e^{-2} by S

E. (i) Converges to 0 by R, (ii) Converges to 0 by P; (iii) Converges to e^{-2} by S

---

4. Which series below converges conditionally but not absolutely?

P. $\displaystyle\sum_{n=7}^{\infty} \frac{\cos(n\pi)}{(\ln n)^2}$ Q. $\displaystyle\sum_{n=8}^{\infty} \frac{(-1)^n}{n(\ln n)^2}$

A. P only

B. Q only

C. Both P and Q

D. Neither

---

5. Find the sum of the geometric series.

$$\sum_{n=3}^{\infty} \frac{(-3)^{n-1}}{4^{n-2}}$$

A. $\dfrac{9}{7}$ B. $-\dfrac{27}{7}$ C. $-\dfrac{27}{16}$ D. $\dfrac{17}{16}$ E. $\dfrac{7}{4}$

---

10. Using the **Root test** to determine if the series converges.

$$\sum_{n=7}^{\infty}\left(\frac{n^2-n-1}{2n^2+1}\right)^n$$

A. $\lim\limits_{n\to\infty}\sqrt[n]{|a_n|}=1$, divergent.

B. $\lim\limits_{n\to\infty}\sqrt[n]{|a_n|}>1$, convergent.

C. $\lim\limits_{n\to\infty}\sqrt[n]{|a_n|}<1$, divergent.

D. $\lim\limits_{n\to\infty}\sqrt[n]{|a_n|}>1$, divergent.

E. $\lim\limits_{n\to\infty}\sqrt[n]{|a_n|}<1$, convergent.

---

11. Given the series $\displaystyle\sum_{n=5}^{\infty}(-1)^n\frac{2^n n!}{7\cdot 11\cdot 15\cdots(4n+3)}$.

What's the limit of the ratio test, $L=\lim\limits_{n\to\infty}\left|\dfrac{a_{n+1}}{a_n}\right|$?

A. $\frac{1}{3}$ B. $\frac{1}{2}$ C. $\frac{1}{4}$ D. $\frac{1}{e}$ E. $\frac{1}{7}$

---

12. Find the limit of the sequence, if converges.

$$\left\{\frac{1}{3},\ \frac{1}{3}+\frac{1}{9},\ \frac{1}{3}+\frac{1}{9}+\frac{1}{27},\ \cdots\right\}$$

A. $\frac{1}{3}$ B. Div. C. $\frac{1}{2}$ D. 1 E. $\frac{2}{3}$

---

1. Let $\{a_n\}$ be a <u>bounded</u> and monotonically <u>increasing</u> sequence such that $a_1 = 0$ and $a_{n+1} = \dfrac{a_n + 3}{3a_n + 10}$. Find the limit of the sequence.

A. $\dfrac{3+\sqrt{5}}{2}$ B. $\sqrt{2}-1$ C. $\dfrac{-3+\sqrt{13}}{2}$

D. $\dfrac{-3-\sqrt{13}}{2}$ E. $\dfrac{3-\sqrt{5}}{2}$

---

2. Let S_n be the nth partial sum of the series $\sum_{n=1}^{\infty} a_n$. Which statement below is **<u>TRUE</u>**?

A. If $\lim_{n\to\infty}(S_n - S_{n-1}) = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

B. If $\sum_{n=1}^{\infty} a_n$ diverges, then $\lim_{n\to\infty}(S_n - S_{n-1}) \neq 0$.

C. If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty}(S_n - S_{n-1})$ is not necessarily 0.

D. If $\lim_{n\to\infty}(S_n - S_{n-1}) \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges.

E. More than one statements are true.

---

6. Given a series $\sum a_n$. How many of the following statements are **FALSE**?

- If $\lim\limits_{n\to\infty} \left|\dfrac{a_{n+1}}{a_n}\right| = 1$, then the series converges conditionally.
- If $\lim\limits_{n\to\infty} a_n = 0$, then the series converges.
- If $\lim\limits_{n\to\infty} \sqrt[n]{|a_n|} = 0$, then the series converges absolutely.
- If $\lim\limits_{n\to\infty} \sqrt[n]{|a_n|} = 1$, then the series diverges.

A. 1 B. 0 C. 2 D. 3 E. 4

---

7. What's the least number of terms we need to add in order to estimate the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n(n+1)}$ within 0.01?

A. 10 B. 9 C. 11 D. 98 E. 99

---

8. Determine the values of k for which the series $\displaystyle\sum_{n=5}^{\infty} \frac{n^{3/2}}{\sqrt{n^k + 7n}}$ will converge.

A. $0 < k < 5$ B. $k > 4$ C. $0 < k < 4$

D. $k > \dfrac{9}{2}$ E. $k > 5$

---

9. Which series below can be shown **convergent** using **limit comparison test**?

P. $\displaystyle\sum_{n=5}^{\infty} \frac{5 - \sin n}{n^5}$, Q. $\displaystyle\sum_{n=5}^{\infty} \sqrt{n}\tan^2\left(\frac{\pi}{n}\right)$, R. $\displaystyle\sum_{n=5}^{\infty} \sin\left(\frac{\pi}{n^2}\right)$.

A. only P B. only R C. only Q

D. only Q and R E. All

---

13. How many of the following series can we conclude diverges using the Test for Divergence?

P. $\displaystyle\sum_{n=8}^{\infty}\left(\frac{n}{n+1}\right)^n$, Q. $\displaystyle\sum_{n=8}^{\infty}\frac{(-1)^n\ln n}{n}$, R. $\displaystyle\sum_{n=8}^{\infty}\frac{(-1)^n n}{5n+1}$

A. 0 B. 1 C. 2 D. 3

---

14. For each of the following series, determine if it is absolutely convergent, conditionally convergent or divergent.

P. $\displaystyle\sum_{n=7}^{\infty}\frac{(-1)^n e^n}{n!}$, R. $\displaystyle\sum_{n=8}^{\infty}\frac{\cos(n\pi)}{\sqrt{n}(\ln n)^2}$

A. P. absolutely convergent, Q. absolutely convergent

B. P. conditionally convergent, Q. conditionally convergent

C. P. conditionally convergent, Q. absolutely convergent

D. P. absolutely convergent, Q. conditionally convergent

E. P. absolutely convergent, Q. divergent

---

15. Determine if the series converge.

P. $\displaystyle\sum_{n=7}^{\infty}\left(\frac{n}{n+1}\right)^{n^2}$, Q. $\displaystyle\sum_{n=7}^{\infty}\frac{(\cos^2(n))^n}{4n^2}$

A. only P converges

B. both P and Q converge

C. only Q converges

D. both P and Q diverge

THIS PAGE WAS INTENTIONALLY LEFT BLANK.

Section # ____________ **Name** ______________________________

UF ID # ____________ **TA Name** ______________________________

Part II: Free Response There are 2 questions on this portion of the exam. Show **ALL** work clearly in the space provided for the first problem. Your work must be complete, logical and understandable, or it will receive no credit. Please cross out or fully erase any work that you do not want graded. A total of **10** points are available on this portion of the exam.

FR Scores	
1	/5
2	/5
FR Total	/10

The Honor Pledge: "On my honor, I have neither given nor received unauthorized aid doing this exam."

Signature: ____

2. Which of the following series $\sum a_n$ can we conclude converges or diverges using the direct comparison test (DCT)? What b_n do you compare a_n to?

If DCT can not be used, what test can we use?

(You may also use abbreviations: TFD(test for divergent), AST (alternating series test), LCT (limit comparison test), INT (integral test)).

You do not have to show work.

(a) $\displaystyle\sum_{n=7}^{\infty} \frac{8+\cos n}{n^2}$.

i. Circle one: (convergent, divergent)

ii. If DCT: $b_n =$ ________________

iii. If Not DCT, test: ___________.

(b) $\displaystyle\sum_{n=7}^{\infty} \frac{1}{1+n^2}$.

i. Circle one: (convergent, divergent)

ii. If DCT: $b_n =$ ________________

iii. If Not DCT, test: ___________.

(c) $\displaystyle\sum_{n=5}^{\infty} \frac{1}{n^2(\ln n)}$

i. Circle one: (convergent, divergent)

ii. If DCT: $b_n =$ ________________

iii. If Not DCT, test: ___________.

(d) $\displaystyle\sum_{n=5}^{\infty} \frac{1}{n(\ln n)^2}$

i. Circle one: (convergent, divergent)

ii. If DCT: $b_n =$ ________________

iii. If Not DCT, test: ___________.

(e) $\displaystyle\sum_{n=7}^{\infty} \frac{5+(-1)^n}{n-1}$.

i. Circle one: (convergent, divergent),

ii. If DCT: $b_n =$ ________________

iii. If Not DCT, test: ___________.

1. Find the sum of the telescoping series.

$$\sum_{n=5}^{\infty}\left[\sec\left(\frac{1}{n}\right)-\sec\left(\frac{1}{n+2}\right)\right]$$

(a) Determine the nth partial sum s_n of the series.

$s_n =$ ____________________

(b) Determine the sum of the series.

Sum= ____________________