MAC 2233 FINAL EXAM A
FALL 2007

A. Sign your scantron sheet in the white area on the back in ink.

B. Write and code in the spaces indicated:

1) Name (last name, first initial, middle initial)
2) UF ID number
3) Discussion section number

C. Under "special codes", code in the test ID number 6, 1.

1 2 3 4 5 • 7 8 9 0
• 2 3 4 5 6 7 8 9 0

D. At the top right of your answer sheet, for "Test Form Code" encode A .

• B C D E

E. This test consists of twenty-two five-point multiple choice questions, and two five-point bonus questions.

The time allowed is 2 hours.

F. WHEN YOU ARE FINISHED:

1) Before turning in your test check for transcribing errors. Any mistakes you leave in are there to stay.
2) You must turn in your scantron to your discussion leader. Be prepared to show your picture I.D. with a legible signature.
3) The answers will be posted on the MAC2233 homepage after the exam.

1. If $f(x) = \begin{cases} \dfrac{\frac{1}{3} - \frac{1}{x}}{x - 3} & x \neq 3 \\ 0 & x = 3 \end{cases}$, find $\lim_{x \to 3} f(x)$.

a. 1   b. $\frac{1}{9}$ c. 0   d. $\frac{2}{3}$ e. The limit does not exist.

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2. Let $f(x) = x + 3$ and let $g(x) = \dfrac{|x|}{x}$.

If $h(x) = g(f(x))$, then $\lim_{x \to -3^-} h(x) =$ ____________.

a. -1 b. 2   c. -2 d. 1   e. 0

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3. Find the value of k which will make $g(x) = \begin{cases} x^2 + kx & x < 1 \\ e^x & x \geq 1 \end{cases}$

continuous at $x = 1$.

a. $k = e + 1$ b. $k = e$ c. $k = \dfrac{e+1}{2}$

d. $k = e - 2$ e. $k = e - 1$

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4. Write the equation of the tangent line to $f(x) = \dfrac{x}{\sqrt{x^2 - 3}}$ at $x = 2$.

a. $y = -3x - 4$ b. $y = -x - 2$ c. $y = 2$

d. $y = -3x + 8$ e. $y = -x + 4$

5. Let $f(x) = \begin{cases} -1 & x \le 0 \\ 3 - x & 0 < x < 2 \\ \dfrac{1}{3-x} & x > 2 \end{cases}$.

Which of the following statements is/are true?

A. $\lim_{x \to 0^+} f(x) = f(0)$.

B. $f(x)$ has nonremovable discontinuities at $x = 0$ and $x = 3$.

C. $f(x)$ can be made continuous at $x = 2$ by defining $f(2) = 1$.

a. A and B only   b. B only   c. C only

d. B and C only   e. A, B and C

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6. Per capita consumption of diet carbonated drinks in the United States can be modeled by $f(x) = 0.02x^2 + 0.8x + 3.94$ for $0 \le x \le 20$, where x is the number of years since 1988 and $f(x)$ is the number of gallons consumed per year per person. Find the rate at which consumption is changing from $x = 0$ to $x = 10$ years, and at $x = 10$ years.

	Average rate of change from $x = 0$ to $x = 10$	**Rate of change at $x = 10$ years**
a.	increasing by 10 gal/person	increasing by 5.2 gal/person
b.	increasing by 1.8 gal/person	increasing by 1.2 gal/person
c.	increasing by 1.0 gal/person	increasing by 1.2 gal/person
d.	increasing by 1.8 gal/person	increasing by 1.4 gal/person
e.	increasing by 1.0 gal/person	increasing by 1.4 gal/person

7. If $f(x) = \dfrac{x^2 - x}{x^2 + 3x - 4}$, let $p = \lim_{x \to 1^+} f(x)$ and $q = \lim_{x \to -4^-} f(x)$. Then

a. $p = -\infty$ and $q = \infty$

b. $p = +\infty$ and $q = -\infty$

c. $p = \dfrac{1}{5}$ and $q = +\infty$

d. $p = \dfrac{1}{5}$ and $q = -\infty$

e. $p = -\infty$ and $q = \dfrac{4}{5}$

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8. A company is manufacturing a new product. The demand function for the product is $p(x) = 24 - 0.02x$ where p is the price per item and x is the weekly production level. If production is decreasing by 10 items per week, find the rate at which revenue is changing with respect to time when weekly production is 200 items.

a. Revenue is decreasing by \$160 per week.

b. Revenue is increasing by \$16 per week.

c. Revenue is increasing by \$120 per week.

d. Revenue is decreasing by \$12 per week.

e. Revenue is increasing by \$200 per week.

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9. If $f(x) = xe^{2x}$, find each x-value at which the graph of $f(x)$ has a horizontal tangent line.

a. $x = 0$ only

b. $x = -1$ only

c. $x = -\dfrac{1}{2}$ only

d. $x = -1$ and $x = 0$

e. $x = -\dfrac{1}{2}$ and $x = 0$

10. The demand and cost functions for a certain product are given by $p(x) = 60 - 0.1x$ and $C(x) = 12x + 1200$, $0 \leq x \leq 400$.

Which one of the following statements is **not** true?

a. The profit from the sale of 100 items is \$2600.

b. Since marginal profit when $x = 100$ is \$28 per unit, the profit from the sale of 101 items is approximately \$2628.

c. Since $P''(100) < 0$ for the profit function $P(x)$, marginal profit is decreasing at $x = 100$.

d. In order to sell 300 items, the price of the product must be \$30.

e. Revenue is maximized when 200 items are sold.

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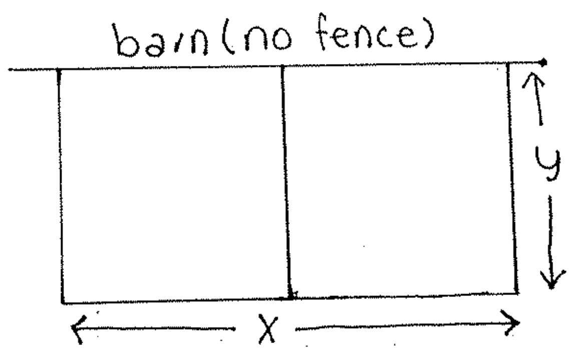
11. Two adjacent corrals are to be constructed next to a barn using 480 feet of fencing. If no fence is needed next to the barn, find the values of x and y which will maximize the total area enclosed.



a. $x = 240$ ft. and $y = 80$ ft.

b. $x = 120$ ft. and $y = 120$ ft.

c. $x = 60$ ft. and $y = 140$ ft.

d. $x = 240$ ft. and $y = 120$ ft.

e. $x = 30$ ft. and $y = 150$ ft.

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12. Find the slope of the tangent line to the curve $x \ln y + x^2 - 4y = x + 2$ at the point $(3, 1)$.

a. -1 b. 5 c. 2 d. $\frac{5}{3}$ e. $-\frac{1}{3}$

13. Find each vertical and horizontal asymptote of the function

$$f(x) = \frac{x^2 - 1}{x^3 - x^2}.$$

a. $x = -1$, $x = 0$, $x = 1$ and $y = 0$

b. $x = 0$, $x = 1$ and $y = -1$

c. $x = 0$ and $y = 0$ only

d. $x = 0$, $x = 1$ and $y = 0$ only

e. $x = 0$ and $y = -1$ only

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14. Given $f(x) = \frac{x^2 + 12}{x - 2}$, $f'(x) = \frac{x^2 - 4x - 12}{(x-2)^2}$, and $f''(x) = \frac{32}{(x-2)^3}$.

Which of the following statements is/are true? Be sure to consider domain.

A. $f(x)$ has a relative maximum at $x = -2$ and a relative minimum at $x = 6$.

B. $f(x)$ is both increasing and concave up on $(6, \infty)$.

C. $f(x)$ has an inflection point at $x = 2$.

a. A only

b. B only

c. B and C only

d. A and B only

e. A, B, and C

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15. Find the absolute minimum and maximum values of $f(x) = 2x^3 + 3x^2 - 12x$ on the interval $[0, 2]$.

a. 0, 4

b. −7, 0

c. −7, 4

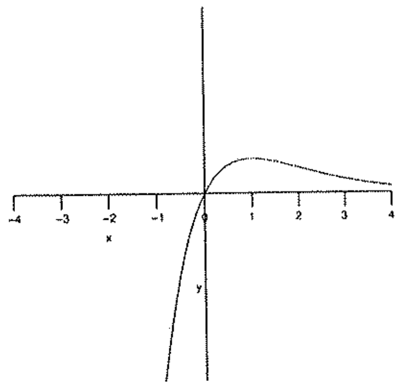
d. 0, 20

e. 4, 20

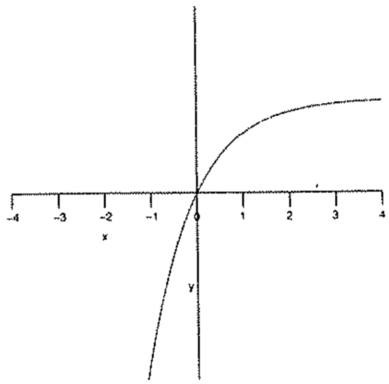
16. If $f(x) = \dfrac{x}{e^x}$, then $f'(x) = \dfrac{1-x}{e^x}$ and $f''(x) = \dfrac{x-2}{e^x}$.

Which of the following best represents the graph of $y = f(x)$?

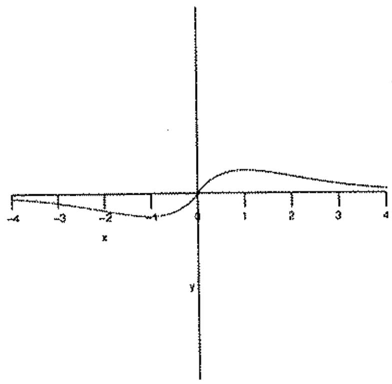
a.



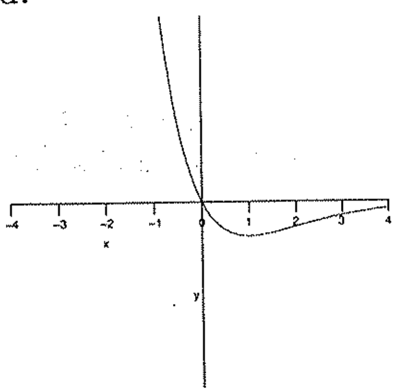
b.



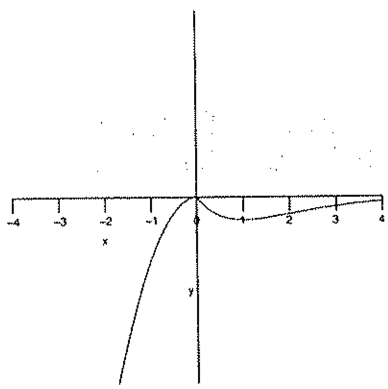
c.



d.



e.



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17. If $f(x) = \ln\sqrt{\dfrac{e^{2x}}{x^2+1}}$, find $f'(1)$.

a. $-\dfrac{1}{2}$ b. e c. 0 d. $\dfrac{1}{2}$ e. $\dfrac{e}{2}$

18. Evaluate $\int x^3 e^{1-x^4}\, dx$.

a. $e^{1-x^4} + C$ b. $-\dfrac{e^{1-x^4}}{4} + C$ c. $\dfrac{e^{2-x^4}}{2-x^4} + C$

d. $\dfrac{x^4}{4} e^{1-x^4} + C$ e. $-4e^{1-x^4} + C$

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19. The slope of the tangent line to the curve $y = f(x)$ at any point is given by $f'(x) = \dfrac{x - 3x^2}{x^2}$. If $f(1) = 2$, find $f(3)$.

a. $-\dfrac{8}{3}$ b. $\ln 3 + 2$ c. $\ln 3 - 4$ d. $-\dfrac{14}{3}$ e. $\ln 3 - 9$

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20. Find the area of the region under the graph of $f(x) = \dfrac{e^x}{\sqrt{e^x + 3}}$ on $[0, \ln 6]$.

a. 2 b. $\ln\dfrac{9}{4}$ c. 1 d. $\ln\dfrac{3}{2}$ e. 6

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21. Evaluate $\int_1^e \dfrac{\sqrt[3]{\ln x}}{x}\, dx$.

a. $\dfrac{1}{3}$ b. $\dfrac{3}{4}$ c. $-\dfrac{1}{4}$ d. $\dfrac{2}{3}$ e. 1

22. The demand function for a product is given by $D(x) = 50 - x$ and the supply function is $S(x) = x^2 + 20$. Find the Producers' Surplus at the equilibrium price.

a. \$83.33 b. \$75 c. \$62.50 d. \$12.50 e. \$55

BONUS!!! (5 points each)

23. Evaluate $\lim_{h \to 0} \frac{e^{3(x+h)^2} - e^{3x^2}}{h}$. Hint: consider the definition of derivative.

a. e^{3x^2} b. $\frac{1}{3}e^{3x^2}$ c. $3e^{3x^2}$ d. $\frac{1}{6x}e^{3x^2}$ e. $6xe^{3x^2}$

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24. Find the average value of $f(x) = \begin{cases} x+1 & x \leq 0 \\ e^{3x} & x > 0 \end{cases}$ on $[-1, 1]$.

a. $\frac{3}{2}e^3 - \frac{5}{4}$ b. $\frac{1}{6}e^3 + \frac{1}{12}$ c. $\frac{1}{2}e^3 - \frac{1}{4}$ d. $\frac{1}{6}e^3 + \frac{1}{2}$ e. $\frac{1}{3}e^3 - \frac{1}{2}$

HAVE A HAPPY HOLIDAY!!!