MAC 2233 TEST 3B
FALL 2007

A. Sign your scantron sheet in the white area on the back in ink.

B. Write and code in the spaces indicated:

1) Name (last name, first initial, middle initial)
2) UF ID number
3) Discussion section number

C. Under "special codes", code in the test ID number 3, 2.

1	2	•	4	5	6	7	8	9	0
1	•	3	4	5	6	7	8	9	0

D. At the top right of your answer sheet, for "Test Form Code" encode B.

A	•	C	D	E

E. This test consists of five two-point and nine five-point multiple choice questions, five one-point bonus questions, and two pages (both sides) of partial credit questions worth 25 points.

The time allowed is 90 minutes.

F. WHEN YOU ARE FINISHED:

1) Before turning in your test check for transcribing errors. Any mistakes you leave in are there to stay.
2) You must turn in the tear off sheets and your scantron to your discussion leader. Be prepared to show your picture I.D. with a legible signature.
3) The answers will be posted on the MAC2233 homepage after the exam.

Problems 1 - 5: 2 points each

1. Find each vertical asymptote of $f(x) = \dfrac{x^2 + 2x - 3}{x^3 - x}$.

a. $x = -1$ and $x = 1$ only

b. $x = -1$ and $x = 0$ only

c. $x = 0$ and $x = 1$ only

d. $x = -1$, $x = 0$ and $x = 1$

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2. Evaluate using $f(x)$ from Problem 1: $\lim\limits_{x \to 1^+} f(x) =$_____.

a. 2 b. $-\infty$ c. $+\infty$ d. $\dfrac{3}{2}$

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3. The graph of $f(x) = \dfrac{3x^2 - x}{x^2 + 2}$ crosses its horizontal asymptote at _____.

a. $x = 3$ b. $x = \dfrac{3}{2}$ c. $x = -6$

d. The graph of $f(x)$ does not cross its horizontal asymptote.

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4. If $\ln x + \ln 4 = \ln 8$, then $x =$_____.

a. The equation has no solution. b. 2 c. 4 d. 16

5. Mr. Jones wants to set up a home entertainment system when he retires in 2009. If he invests \$3000 in a certificate of deposit (CD) which earns 6 percent annual interest compounded semiannually (twice per year), how much will be in the account when the CD matures in two years?

a. $3000(1.06)^4 = 3787.43$ dollars

b. $3000(1.03)^4 = 3376.53$ dollars

c. $\dfrac{6000}{(1.03)^2} = 5655.58$ dollars

d. $3000(1.06)^2 = 3370.80$ dollars

Problems 6 - 14: 5 points each

6. Let $f(x) = \dfrac{x-1}{x^2}$. Given that $f'(x) = \dfrac{2-x}{x^3}$ and $f''(x) = \dfrac{2x-6}{x^4}$, then which of the following statements is/are true of $f(x)$? Be sure to consider the domain of f as necessary.

A. $\lim_{x \to 0^+} f(x) = -\infty$.

B. The graph of $f(x)$ has no horizontal asymptotes.

C. $f(x)$ has a critical point at $x = 2$ only.

D. According to the Second Derivative Test, $f(x)$ has a relative maximum at $x = 2$.

a. C and D only b. B and C only c. A and D only

d. A, C and D e. A, B and C

7. The relative extrema of $f(x) = \dfrac{x^3}{x-4}$ occur at which of the following:

	relative maximum(a) at $x =$_____	relative minimum(a) at $x =$_____
a.	6	none
b.	0	6
c.	4	0, 6
d.	6	0
e.	none	6

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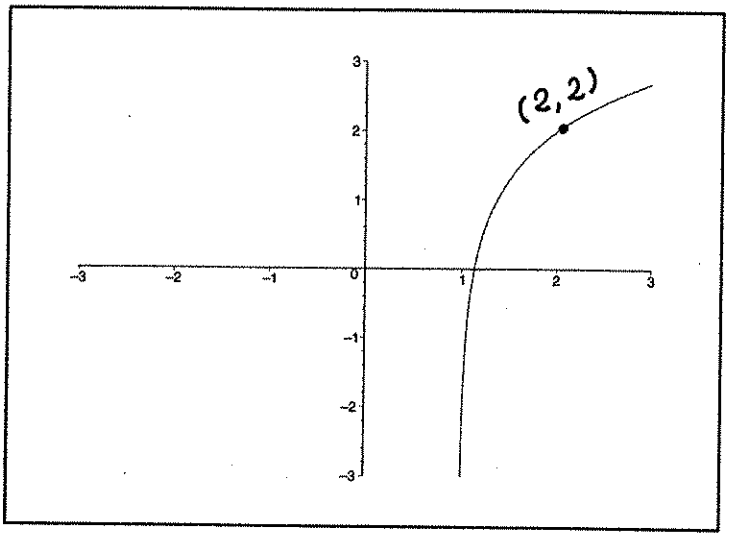
8. Which function below best represents the given graph?



a. $y = \ln(x-2) + 1$

b. $y = e^{x-2} + 1$

c. $y = \ln(x-1) + 2$

d. $y = \ln(x+1) + 2$

e. $y = 3 - e^{x-2}$

9. Find the absolute maximum and minimum values of $f(x) = x(x-6)^2$ on the interval $[1, 4]$.

a. 16, 0 b. 32, 0 c. 25, 16 d. 32, 16 e. 32, 25

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10. If $f(x) = \frac{x^5}{5} - 3x^4 + 6$, then which of the following statements is/are true?

P. The graph of $f(x)$ has horizontal tangent lines at $x = 0$ and $x = 12$.

Q. The graph of $f(x)$ has inflection points at $x = 0$ and $x = 9$.

R. $f(x)$ is both decreasing and concave down on the interval $(0, 9)$ only.

a. R only b. P only c. P, Q and R

d. P and R only e. P and Q only

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11. The demand and cost functions for a certain product are $p(x) = 90 - 0.5x$ and $C(x) = 20x + 1800$, $0 \leq x \leq 100$. Find the production level x that will maximize **average profit** $\overline{P}(x) = \frac{P(x)}{x}$.

a. 36 b. 72 c. 64 d. 40 e. 60

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12. Write the equation of the tangent line to $f(x) = e^{x^2-4x}$ at $x = 0$.

a. $y = 0$ b. $y = -4x+1$ c. $y = x$ d. $y = -4x$ e. $y = x+1$

13. Find the expression equivalent to

$$\ln \frac{3e^x}{(x-2)^3}$$

For which x-values is it defined (in interval notation)?

a. $\ln 3 + \ln x - 3\ln(x-2)$ domain: $(0,2) \cup (2,\infty)$

b. $\ln 3 + \ln x - (x-2)^3$ domain: $(0,\infty)$

c. $\ln 3 + x - 3\ln(x-2)$ domain: $(2,\infty)$

d. $3\ln x - \ln(x-2)$ domain: $(2,\infty)$

e. $\ln 3 + x - 3\ln(x-2)$ domain: $(0,2) \cup (2,\infty)$

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14. If \$2000 is invested in a bank account at 4% annual interest compounded continuously, how long will it take for the amount in the account to increase to \$5000? Assume that there are no deposits to or withdrawals from the account during that time.

a. $\dfrac{\ln \frac{5}{2}}{0.04}$ years

b. $\dfrac{\ln 5}{0.02}$ years

c. $\dfrac{0.08}{\ln 5}$ years

d. $0.04 \ln \dfrac{5}{2}$ years

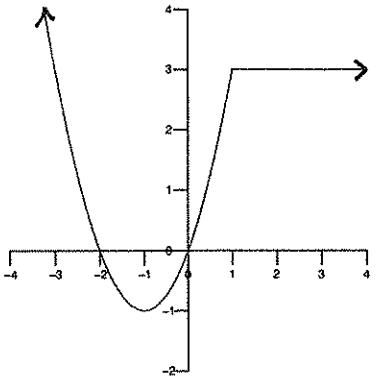
e. $\dfrac{0.04}{\ln \frac{5}{2}}$ years

Be sure to work the bonus problems on the next page!

Bonus!! (1 point each)

Use the given graph of $f'(x)$ to determine whether the following statements are true or false for the function $f(x)$.

Be sure to bubble the correct answers on your scantron.



15. $f'(x) = 3$ for all $x \geq 1$.

a. True b. False

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16. $f(x)$ is decreasing on the interval $(-2, 0)$.

a. True b. False

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17. $f(x)$ has a relative minimum at $x = -2$.

a. True b. False

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18. Since $f'(x)$ is decreasing on the interval $(-\infty, -1)$, $f(x)$ is concave up on $(-\infty, -1)$.

a. True b. False

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19. $f(x)$ has an inflection point at $x = -1$.

a. True b. False

MAC 2233 Test 3B Part II Fall 2007

Sect#_____ Name_____

UF ID_____ Signature_____

SHOW ALL WORK TO RECEIVE FULL CREDIT!!

1. The population of bacteria (in hundreds) in a certain culture after t hours of observation in a laboratory is given by the formula

$P(t) = \frac{30t^2}{t^2+3}+10, \quad 0 \le t \le 6. \quad P'(t) = \frac{180t}{(t^2+3)^2}$ and $P''(t) = \frac{540(1-t^2)}{(t^2+3)^3}$.

a) What is the population of bacteria at the beginning of the observation?

_____ bacteria

b) Show that the population is increasing over the six hours of the observation (the interval $(0, 6)$).

c) After how many hours will the population be growing at the greatest rate? Give a reason for your answer.

$t =$_____

The population at that time is _____ bacteria.

d) What is the significance of the point from (c) on the graph of $P(t)$?

2. Let $f(x) = (x+3)(x-2)^{\frac{2}{3}}$, $f'(x) = \dfrac{5x}{3(x-2)^{\frac{1}{3}}}$ and $f''(x) = \dfrac{10x-30}{9(x-2)^{\frac{4}{3}}}$.
Also $f(0) \approx 4.8$.

A. Complete the following (if none, write "none"):

1) domain of f:_____

vertical asymptote(s):_____ horizontal asymptote(s):_____

$f(x)$ has a $y-$intercept at $y =$ _____

$f(x)$ has $x-$intercept(s) at $x =$_____

2) Complete the number lines for f' and f'':

_____ f'

_____ f''

3) $f(x)$ has critical numbers at $x =$_____.

$f(x)$ has horizontal tangent lines at $x =$_____.

$f(x)$ has vertical tangent lines or cusps at $x =$_____.

4) $f(x)$ has a relative maximum at $x =$_____.

Maximum value: _____

$f(x)$ has a relative minimum at $x =$_____.

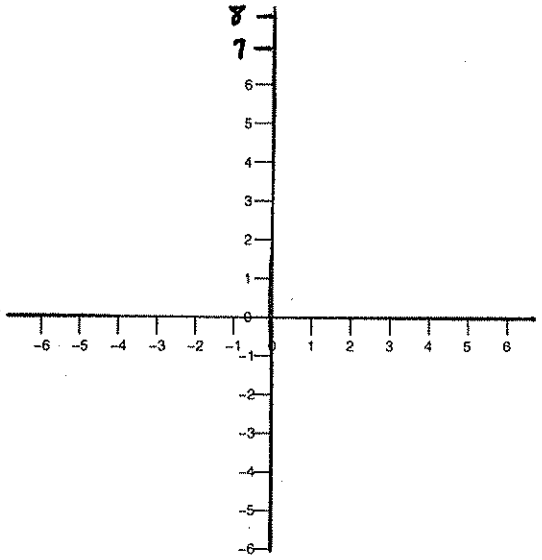
Minimum Value: _____

5) Inflection point(s): _____

Name_____ Section Number_____

2. (continued)

B. Sketch the graph of $y = f(x)$ using the information from part A. Show intercepts, relative extrema and inflection points. $f(0) \approx 4.8$.



3. Let $f(x) = \dfrac{4}{9 - 3e^x}$. Find (if none, write "none"):

a) each vertical asymptote of $f(x)$.

$x =$_____

b) $\lim_{x \to +\infty} f(x) =$_____.

c) $\lim_{x \to -\infty} f(x) =$_____.

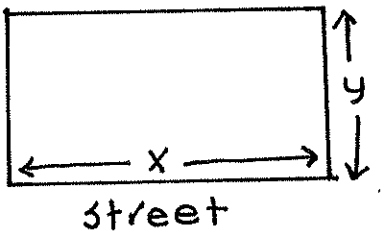
d) each horizontal asymptote of $f(x)$.

$y =$_____

4. A homeowner wishes to enclose a rectangular garden plot having an area of 1350 square feet. Three sides of the plot will be enclosed with fencing that costs \$3 per foot, and the decorative fencing for the side facing the street costs \$6 per foot. What dimensions will minimize the cost of fencing?

Function to be minimized: $C =$ _____

Constraint: _____



$x =$_____ ft.

$y =$_____ ft.

Use the Second Derivative Test to confirm your results.