

MAC 2233 TEST 2A
FALL 2007

- A. Sign your scantron sheet in the white area on the back in ink.
- B. Write and code in the spaces indicated:
- 1) Name (last name, first initial, middle initial)
 - 2) UF ID number
 - 3) Discussion section number
- C. Under "special codes", code in the test ID number 2, 1.
- | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | • | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 0 |
| • | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 0 |
- D. At the top right of your answer sheet, for "Test Form Code" encode A .
- | | | | | |
|---|---|---|---|---|
| • | B | C | D | E |
|---|---|---|---|---|
- E. This test consists of eleven five-point multiple choice questions, five one-point bonus questions, and two pages (both sides) of partial credit questions worth 25 points.
- The time allowed is 90 minutes.
- F. WHEN YOU ARE FINISHED:
- 1) Before turning in your test check for transcribing errors. Any mistakes you leave in are there to stay.
 - 2) You must turn in the tear off sheets and your scantron to your discussion leader. Be prepared to show your picture I.D. with a legible signature.
 - 3) The answers will be posted on the MAC2233 homepage after the exam.

1. Write the equation of the tangent line to $f(x) = \left(1 - \frac{2}{x}\right)^4$ at $x = 1$.

a. $y = 8x - 7$

b. $y = -8x + 9$

c. $y = -4x + 5$

d. $y = -8x + 7$

e. $y = -4x - 3$

2. Find $f'(x)$ if $f(x) = \frac{3x - x^2}{\sqrt{x}}$. $f'(x) =$ _____.

a. $\frac{2\sqrt{x} - 3}{x}$

b. $3\sqrt{x} - x^{\frac{3}{2}}$

c. $\frac{3 - 2x}{x^{\frac{3}{2}}}$

d. $\frac{3x^2 - 3x}{2}$

e. $\frac{3 - 3x}{2\sqrt{x}}$

3. A frustrated student threw his calculus book upwards off of the roof of a building 160 feet high with an initial velocity of 48 feet per second. With what velocity did the book hit the ground below? Use the formula $h(t) = -16t^2 + v_0t + h_0$.

a. -112 ft/sec

b. -12 ft/sec

c. 0 ft/sec

d. -80 ft/sec

e. -96 ft/sec

4. Find each value of x at which the graph of $f(x) = x(3x + 6)^3$ has a horizontal tangent line.

a. $x = -2$ only

b. $x = -2$ and $x = -\frac{1}{2}$

c. $x = 0$ only

d. $x = -2$ and $x = 0$

e. $x = -2$ and $x = -1$

5. Data from data entry classes show that an average student will learn to type according to the model $N(t) = \frac{80t + 120}{t + 4}$ where $N(t)$ is the number of words per minute a student can type accurately after t weeks in a class. Find the average rate of change in the number of words typed per minute in the first four weeks of the course ($t = 0$ to $t = 4$), and the rate at which the number of words typed per minute is increasing after six weeks in the course.

Average rate of change
from $t = 0$ to $t = 4$

Rate of change
after six weeks

Learning is increasing weekly at the rate of

- | | | |
|----|---------------------------------|---------------------------------|
| a. | $\frac{25}{4}$ words per minute | 2 words per minute |
| b. | $\frac{85}{2}$ words per minute | $\frac{15}{4}$ words per minute |
| c. | $\frac{25}{2}$ words per minute | 2 words per minute |
| d. | $\frac{25}{2}$ words per minute | 6 words per minute |
| e. | $\frac{25}{4}$ words per minute | 6 words per minute |

6. For what value of k is the tangent line to $f(x) = kx^2 + 4\sqrt{x}$ perpendicular to $y = \frac{1}{2}x + 6$ at $x = 4$? Hint: what is the slope of the tangent line when $x = 4$?

- | | | | | | | | | | |
|----|----------------|----|----------------|----|----------------|----|-----------------|----|---------------|
| a. | $-\frac{1}{8}$ | b. | $-\frac{3}{8}$ | c. | $\frac{3}{16}$ | d. | $-\frac{1}{16}$ | e. | $\frac{5}{8}$ |
|----|----------------|----|----------------|----|----------------|----|-----------------|----|---------------|

7. Find $f'(x)$ for $f(x) = \frac{2x^2}{\sqrt{x+1}}$. $f'(x) =$ _____.

a. $8x\sqrt{x+1}$

b. $\frac{4x - 5x^2}{(x+1)^{\frac{3}{2}}}$

c. $\frac{5x^2 + 4x}{\sqrt{x+1}}$

d. $\frac{3x^2 + 4x}{(x+1)^{\frac{3}{2}}}$

e. $\frac{2x^2 + 2x + 1}{2(x+1)^{\frac{3}{2}}}$

8. The position of an object moving along a path is given by the function $s(t) = \frac{t^3}{4} - \frac{2t^2}{3} + 3t$ where $s(t)$ is the object's distance (in meters) from an observation point after t minutes. Find its acceleration after 2 minutes.

a. $\frac{16}{3}$ meter/min²

b. $\frac{10}{3}$ meter/min²

c. $\frac{5}{3}$ meter/min²

d. $-\frac{2}{3}$ meter/min²

e. $\frac{8}{3}$ meter/min²

9. Given functions $f(x)$ and $g(x)$ such that $f(3) = 2$, $f'(3) = 1$, $f'(5) = 4$, $g(3) = 5$, $g'(3) = 3$ and $g'(5) = \frac{1}{2}$. If $F(x) = (f \circ g)(x)$, find $F'(3)$.

a. 4

b. 15

c. 12

d. $\frac{5}{2}$

e. 3

10. If $xy^2 + y = x^2 - 5$, find the slope of the tangent line to the curve at the point $(-2, 1)$.

a. $\frac{10}{3}$

b. -3

c. $-\frac{4}{3}$

d. -1

e. $\frac{5}{3}$

11. The profit from a new computer is modeled by the function

$P(x) = -20,000 + 500x - 0.25x^2$ dollars where x is the number of computers sold per year (in thousands). In addition, the value of x is a function of the number of assembly line workers, n , according to the formula $x = 8n + 0.1n^2$. Use the Chain Rule to find the rate at which profit is changing with respect to n when there are 10 workers on the assembly line.

- a. Profit is decreasing by \$495 per worker.
- b. Profit is increasing by \$455 per worker.
- c. Profit is decreasing by \$22,975 per worker.
- d. Profit is increasing by \$4550 per worker.
- e. Profit is increasing by \$4950 per worker.

Be sure to answer the bonus questions on the next page!!

Sect# _____ Name _____

UF ID _____ Signature _____

SHOW ALL WORK TO RECEIVE FULL CREDIT!!

1. The demand for a certain commodity is given by $p = \frac{150}{x^2 + 4}$ where p is the price per bushel and x is the number of bushels (in millions) sold each year.

a) Economists predict a harvest of 4 million bushels for the year. According to that forecast, what is the expected selling price of a bushel of the product?

\$ _____

b) Because of flooding late in the season, economists reduce their predicted harvest from 4 to 3.8 million bushels. Use **differentials** to approximate the change in the expected price of a bushel.

Price will _____ by \$ _____
increase/decrease

c) Use parts (a) and (b) to estimate the selling price if the harvest is 3.8 million bushels.

Name _____

Section Number _____

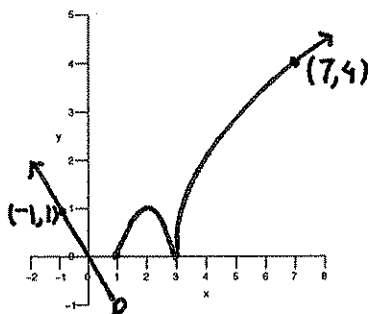
2. continued e) For the cost function $C(x) = \frac{3x^2}{100} + 10x + 100$, find

1) Average cost function $\bar{C}(x) =$ _____

2) marginal average cost when $x = 100$. Interpret your results.

Average cost is _____ by _____ per unit.
increasing/decreasing

3. Use the graph of $y = f(x)$ sketched below to answer the following:



a) Find each value of x at which $f(x)$ is **not** differentiable.

$x =$ _____

b) $f'(0) =$ _____

c) Find the average rate of change of f on $[3, 7]$.

d) Your answer to (c) is closest to which of the following: $f'(2)$, $f'(4)$ or $f'(6)$?

Give a reason for your answer.

4. Let $f(x) = \frac{1}{x-2}$.

a) Use the **limit definition** of derivative to find $f'(x)$.

$$f'(x) = \underline{\hspace{4cm}}$$

b) Write the equation of the tangent line to $y = f(x)$ at $x = 3$.

$$y = \underline{\hspace{4cm}}$$

c) Sketch the graph of $y = f(x)$ and its tangent line at $x = 3$. Show the y -intercept of the line.

