UF | UNIVERSITY of FLORIDA

Department of Mathematics

MAC 2313
Exam 3B
Fall 2019

A. Sign your bubble sheet on the back at the bottom in ink.

B. In pencil, write and encode in the spaces indicated:

1) Name (last name, first initial, middle initial)

2) UF ID number

3) **SKIP** Section number

C. Under "special codes" code in the test ID numbers 3, 2.

1 2 ● 4 5 6 7 8 9 0

1 ● 3 4 5 6 7 8 9 0

D. At the top right of your answer sheet, for "Test Form Code", encode B.

A ● C D E

E. **1)** This test consists of 14 multiple choice questions worth 68 points and 2 free response questions worth 20 points. The test is counted out of 80 points, and there are 8 bonus points available.

2) The time allowed is 90 minutes.

3) You may write on the test.

4) Raise your hand if you need more scratch paper or if you have a problem with your test. DO NOT LEAVE YOUR SEAT UNLESS YOU ARE FINISHED WITH THE TEST.

F. KEEP YOUR BUBBLE SHEET COVERED AT ALL TIMES.

G. When you are finished:

1) Before turning in your test **check carefully for transcribing errors**. Any mistakes you leave in are there to stay.

2) You must turn in your scantron and tearoff sheets to your discussion leader or exam proctor. Be prepared to show your picture I.D. with a legible signature.

3) The answers will be posted in Canvas within one day after the exam. Your discussion leader will return your tearoff sheet with your exam score in discussion. Your score will also be posted in Canvas within one week of the exam.

3. Let E be the solid bounded by the cones $z = 2\sqrt{x^2+y^2}$ and $z = 6 - \sqrt{x^2+y^2}$. Which of the following integrals represents the volume of the solid E?

$$I_1 = \int_1^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (6 - 3\sqrt{x^2+y^2})\, dy\, dx$$

$$I_2 = \int_0^{2\pi} \int_0^2 r(6-r)\, dr\, d\theta$$

$$I_3 = \int_0^{2\pi} \int_0^2 \int_{2r}^{6-r} r\, dz\, dr\, d\theta$$

a. I_1 only

b. I_2 only

c. I_3 only

d. I_2 and I_3

e. None of them

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4. Identify the surfaces in spherical coordinates.

$$S_1: \ \rho\cos\phi = 1, \qquad S_2: \ \rho\sin\phi = 2, \qquad S_3: \ \theta = \frac{\pi}{6}$$

a. S_1 : cylinder, S_2 : horizontal plane, S_3 : cone

b. S_1 : horizontal plane, S_2 : cylinder, S_3 : cone

c. S_1 : horizontal plane, S_2 : cylinder, S_3 : vertical plane

d. S_1 : cone, S_2 : sphere, S_3 : vertical plane

e. S_1 : sphere, S_2 : horizontal plane, S_3 : cylinder

NOTE: Be sure to bubble the answers to questions 1–14 on your scantron.

Questions 1 – 12 are worth 5 points each.

1. Find the volume under the surface $z = e^x$ and above the triangular region D in the xy-plane with vertices $(0,0)$, $(2,2)$, and $(2,0)$.

a. $e^2 - 3$

b. $e^2 - 2$

c. $e^2 - 1$

d. e^2

e. $e^2 + 1$

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2. Change $(-1,1,2)$ from rectangular to cylindrical coordinates (r,θ,z).

a. $(\sqrt{2}, 3\pi/4, 2)$

b. $(2, 3\pi/4, 2)$

c. $(\sqrt{2}, 5\pi/4, 2)$

d. $(2, 7\pi/4, 2)$

e. $(\sqrt{2}, 7\pi/4, 2)$

7. Let E be the solid given in spherical coordinates by $0 \leq \rho \leq \sqrt{18}$, $0 \leq \theta \leq \pi/2$, and $0 \leq \phi \leq \pi/4$. Describe the solid E in rectangular coordinates.

a. $E = \{ (x, y, z) \,|\, 0 \leq x \leq 3,\ 0 \leq y \leq \sqrt{9 - x^2},\ \sqrt{18 - x^2 - y^2} \leq z \leq \sqrt{x^2 + y^2} \}$

b. $E = \{ (x, y, z) \,|\, 0 \leq x \leq \sqrt{18},\ 0 \leq y \leq \sqrt{18 - x^2},\ x^2 + y^2 \leq z \leq \sqrt{18 - x^2 - y^2} \}$

c. $E = \{ (x, y, z) \,|\, 0 \leq x \leq \sqrt{18},\ 0 \leq y \leq \sqrt{18 - x^2},\ \sqrt{x^2 + y^2} \leq z \leq \sqrt{18 - x^2 - y^2} \}$

d. $E = \{ (x, y, z) \,|\, 0 \leq x \leq 3,\ 0 \leq y \leq \sqrt{9 - x^2},\ \sqrt{x^2 + y^2} \leq z \leq \sqrt{18 - x^2 - y^2} \}$

e. $E = \{ (x, y, z) \,|\, -\sqrt{18} \leq x \leq \sqrt{18},\ 0 \leq y \leq \sqrt{18 - x^2},\ \sqrt{18 - x^2 - y^2} \leq z \leq x^2 + y^2 \}$

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8. Let D be the region enclosed by the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{16} = 1$. Evaluate $\displaystyle\iint_D 3\, dA$ by making an appropriate change of variables.

a. $36\,\pi$

b. $24\,\pi$

c. $18\,\pi$

d. $48\,\pi$

e. $12\,\pi$

5. Let E be the solid that lies between the surfaces $z = 2x^2 + 2y^2$ and $z = 8$. The volume of the solid E is $\int_0^{2\pi}\int_0^{a}\int_b^{c} r\,dz\,dr\,d\theta$. Find a, b, and c.

a. $a = 1,\ b = 8,\ c = 2r^2$

b. $a = 2,\ b = 8,\ c = 2r^2$

c. $a = \sqrt{8},\ b = 8,\ c = 2r^2$

d. $a = \sqrt{8},\ b = 2r^2,\ c = 8$

e. $a = 2,\ b = 2r^2,\ c = 8$

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6. Evaluate $\iint\limits_R \frac{y^2}{3x+2}\,dA$, where R is the rectangular region $\{(x,y)|0 \le x \le 1,\ 0 \le y \le 3\}$.

a. $\ln\left(\frac{5}{2}\right)$

b. $3\ln\left(\frac{5}{2}\right)$

c. $9\ln\left(\frac{5}{2}\right)$

d. $\frac{1}{9}\ln\left(\frac{11}{2}\right)$

e. $\ln\left(\frac{11}{2}\right)$

11. Write the following integral in spherical coordinates.

$$\int_0^3 \int_0^{\sqrt{9-x^2}} \int_{\sqrt{x^2+y^2}}^3 z\, dz\, dy\, dx =$$

a. $\displaystyle\int_0^{\pi} \int_0^{\pi} \int_0^3 \rho^3 \sin\phi \cos\phi \, d\rho \, d\phi \, d\theta$

b. $\displaystyle\int_0^{\pi/2} \int_0^{\pi/4} \int_0^{3\sec\phi} \rho^3 \sin\phi \cos\phi \, d\rho \, d\phi \, d\theta$

c. $\displaystyle\int_0^{\pi/2} \int_0^{\pi/4} \int_0^{3\csc\phi} \rho^3 \sin\phi \cos\phi \, d\rho \, d\phi \, d\theta$

d. $\displaystyle\int_0^{\pi/2} \int_0^{\pi/4} \int_0^{3} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

e. $\displaystyle\int_0^{\pi} \int_0^{\pi} \int_0^{3\sec\phi} \rho^3 \sin^2\phi \, d\rho \, d\phi \, d\theta$

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12. Let D be the region inside $r = \cos\theta$ and outside $r = \sin\theta$ in the first quadrant. Then $\displaystyle\iint_D \sqrt{x^2+y^2}\, dA =$

a. $\displaystyle\int_{\pi/4}^{\pi} \int_0^{\cos\theta} r^2 \, dr \, d\theta$

b. $\displaystyle\int_0^{\pi/4} \int_{\cos\theta}^{\sin\theta} r^2 \, dr \, d\theta$

c. $\displaystyle\int_0^{\pi/4} \int_0^{\cos\theta} r \, dr \, d\theta$

d. $\displaystyle\int_{\pi/4}^{\pi/2} \int_{\cos\theta}^{\sin\theta} r \, dr \, d\theta$

e. $\displaystyle\int_0^{\pi/4} \int_{\sin\theta}^{\cos\theta} r^2 \, dr \, d\theta$

9. Let E be the solid bounded by $x^2+y^2=4$, $y+z=2$, and $z=0$. Which of the following represents the volume of the solid E?

a. $\displaystyle\int_0^2\int_0^{2-y}\int_0^{\sqrt{4-x^2}} dx\,dz\,dy$

b. $\displaystyle\int_{-2}^2\int_0^{y-2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} dx\,dz\,dy$

c. $\displaystyle\int_0^2\int_0^{2-y}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} dx\,dz\,dy$

d. $\displaystyle\int_{-2}^2\int_0^{2-y}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} dx\,dz\,dy$

e. $\displaystyle\int_0^2\int_0^{y-2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} dx\,dz\,dy$

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10. Reverse the order of integration: $\displaystyle\int_0^1\int_0^{2x} e^{y^2}\,dy\,dx =$

a. $\displaystyle\int_0^{2x}\int_0^1 e^{y^2}\,dx\,dy$

b. $\displaystyle\int_0^2\int_0^{y/2} e^{y^2}\,dx\,dy$

c. $\displaystyle\int_0^2\int_{y/2}^1 e^{y^2}\,dx\,dy$

d. $\displaystyle\int_0^2\int_0^{2y} e^{y^2}\,dx\,dy$

e. $\displaystyle\int_0^2\int_{2y}^1 e^{y^2}\,dx\,dy$

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Bonus Questions 13 – 14 are worth 4 points each.

13. Calculate the Jacobian, $\dfrac{\partial(x,y)}{\partial(u,v)}$, of the transformation $u = 3x - y$ and $v = x + y$.

a. $\dfrac{1}{4}$

b. $-\dfrac{1}{4}$

c. -4

d. 4

e. 1

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14. Let R be the region bounded by the lines $y = 3x - 1$, $y = 3x + 2$, $y = -1 - x$ and $y = 3 - x$. Use the change of variables $u = 3x - y$ and $v = x + y$ to rewrite the integral:

$$\iint_R \frac{3x - y}{x + y}\, dA =$$

a. $\displaystyle\int_{-1}^{3}\int_{-1}^{2} \frac{u}{4v}\, du\, dv$

b. $\displaystyle\int_{-1}^{3}\int_{-2}^{1} \frac{u}{4v}\, du\, dv$

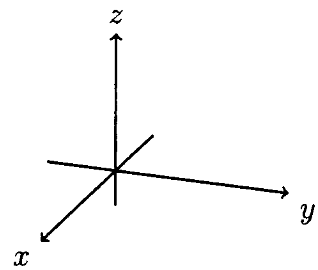
c. $\displaystyle\int_{-1}^{3}\int_{-1}^{2} \frac{u}{v}\, du\, dv$

d. $\displaystyle\int_{-1}^{3}\int_{-2}^{1} \frac{4u}{v}\, du\, dv$

e. $\displaystyle\int_{-1}^{3}\int_{-1}^{2} \frac{4u}{v}\, du\, dv$

2. (10 points)

(a) Identify and sketch the surface $z = 3 + \sqrt{9 - x^2 - y^2}$.



(b) Write the surface $z = 3 + \sqrt{9 - x^2 - y^2}$ in spherical coordinates.

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(c) Write the integral $\displaystyle\int_{-3}^{3}\int_{0}^{\sqrt{9-x^2}}\int_{3}^{3+\sqrt{9-x^2-y^2}} \sqrt{x^2+y^2}\, dz\, dy\, dx$ in spherical coordinates.

$$\int_{\square}^{\square}\int_{\square}^{\square}\int_{\square}^{\square} \square \; d\rho\, d\phi\, d\theta$$

University of Florida Honor Pledge:

On my honor, I have neither given nor received unauthorized aid doing this exam.

Signature: ______________________________

MAC 2313 Exam 3B, Part II Free Response

Name: ______________________________

TA's Name: ______________________________ Discussion Period: ______

SHOW ALL WORK TO RECEIVE FULL CREDIT

1. (10 points) (a) Write the curve $x = \sqrt{4y - y^2}$ in polar coordinates.

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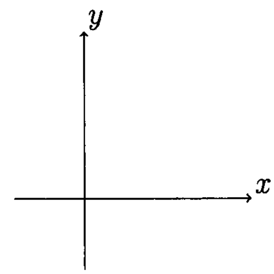
(b) Sketch the region $D = \{ (x, y) \mid 0 \leq y \leq 2,\ y \leq x \leq \sqrt{4y - y^2} \}$.



(c) Convert the integral $\displaystyle\int_0^2 \int_y^{\sqrt{4y-y^2}} \frac{1}{2}\, dx\, dy$ to polar coordinates and evaluate the integral.

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