UF | UNIVERSITY *of* FLORIDA

Department of Mathematics

MAC 2313
Exam 1A
Fall 2019

A. Sign your bubble sheet on the back at the bottom in ink.

B. In pencil, write and encode in the spaces indicated:

1) Name (last name, first initial, middle initial)

2) UF ID number

3) **SKIP** Section number

C. Under "special codes" code in the test ID numbers 1, 1.

● 2 3 4 5 6 7 8 9 0
● 2 3 4 5 6 7 8 9 0

D. At the top right of your answer sheet, for "Test Form Code", encode A.

● B C D E

E. **1)** This test consists of 14 multiple choice questions, ranging from four points to five points in value, plus 2 free response questions worth 20 points total. The test is counted out of 80 points, and there are 8 bonus points available.

2) The time allowed is 90 minutes.

3) You may write on the test.

4) Raise your hand if you need more scratch paper or if you have a problem with your test. DO NOT LEAVE YOUR SEAT UNLESS YOU ARE FINISHED WITH THE TEST.

F. KEEP YOUR BUBBLE SHEET COVERED AT ALL TIMES.

G. When you are finished:

1) Before turning in your test **check carefully for transcribing errors**. Any mistakes you leave in are there to stay.

2) You must turn in your scantron and tearoff sheets to your discussion leader or exam proctor. Be prepared to show your picture I.D. with a legible signature.

3) The answers will be posted in Canvas within one day after the exam. Your discussion leader will return your tearoff sheet with your exam score in discussion. Your score will also be posted in Canvas within one week of the exam.

3. Find parametric equations of the line of intersection of the two planes given by $x + y + z = 1$ and $x - 2y + 3z = 1$.

a. $x = -4 + 3t, y = 2 - t, z = 3 + 2t$

b. $x = 1 + 3t, y = -t, z = -2t$

c. $x = 1 + 5t, y = -2t, z = -3t$

d. $x = -4 + 5t, y = 2 + 2t, z = 3 - 3t$

e. $x = 5t, y = -1 - 2t, z = -3t$

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4. Consider a cycloid curve $\vec{r}(t) = \langle t - \sin t, 1 - \cos t \rangle$. Which of the following is/are correct?

P. The vector $\langle -1, 1 \rangle$ is orthogonal to a tangent vector of the curve $\vec{r}(t)$ at $t = \frac{\pi}{2}$.

Q. The vector $\left\langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\rangle$ is a unit tangent vector to the curve $\vec{r}(t)$ at $t = \frac{\pi}{2}$.

R. The curve $\vec{r}(t)$ has a cusp at $t = \pi$.

a. P only

b. R only

c. Q and R

d. P and Q

e. P and R

NOTE: Be sure to bubble the answers to questions 1−14 on your scantron.

Questions 1 − 12 are worth 5 points each.

1. Let $\vec{a} = \langle 1, 1, -3\rangle$ and $\vec{b} = \langle -1, -3, 1\rangle$. Find the unit vector in the opposite direction of $\vec{a} - \vec{b}$.

a. $\left\langle \frac{1}{3}, \frac{2}{3}, -\frac{2}{3} \right\rangle$

b. $\left\langle -\frac{1}{3}, -\frac{2}{3}, \frac{2}{3} \right\rangle$

c. $\left\langle \frac{1}{3}, -\frac{2}{3}, \frac{2}{3} \right\rangle$

d. $\left\langle -\frac{1}{3}, \frac{2}{3}, -\frac{2}{3} \right\rangle$

e. $\left\langle -\frac{1}{3}, \frac{2}{3}, \frac{2}{3} \right\rangle$

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2. Two lines $\vec{r_1}(t) = \langle t, 1-2t, 4+2t\rangle$ and $\vec{r_2}(t) = \langle 2, -3t, 4+4t\rangle$ intersects at the point $(2, -3, 8)$. Find the angle between $\vec{r_1}(t)$ and $\vec{r_2}(t)$.

a. $\cos^{-1}\left(\frac{2}{15}\right)$

b. $\cos^{-1}\left(\frac{\sqrt{29}}{15}\right)$

c. $\cos^{-1}\left(\frac{8}{\sqrt{17}\sqrt{5}}\right)$

d. $\cos^{-1}\left(\frac{10}{\sqrt{77}}\right)$

e. $\cos^{-1}\left(\frac{14}{15}\right)$

7. Find the vector projection of $\vec{u} = \langle 2, -3, 1\rangle$ onto $\vec{v} = \langle 1, -1, 1\rangle$.

a. $\left\langle \frac{6}{7}, -\frac{9}{7}, \frac{3}{7} \right\rangle$

b. $\left\langle -\frac{6}{7}, \frac{9}{7}, -\frac{3}{7} \right\rangle$

c. $\langle 6, -6, 6\rangle$

d. $\langle 2, -2, 2\rangle$

e. $\langle 2, -2, -2\rangle$

8. Let $\vec{r}(t) = \langle \sin(2t), t^2 + 2, e^t\rangle$. Find symmetric equations of the tangent line to the curve $\vec{r}(t)$ at the point $(0, 2, 1)$.

a. $y = z - 1, x = 1$

b. $\frac{y}{2} = z - 1, x = 2$

c. $x = z - 1, y = 2$

d. $\frac{x}{2} = z - 1, y = 0$

e. $\frac{x}{2} = z - 1, y = 2$

11. A particle moves with the position function $\vec{r}(t) = 2e^t\,\hat{\imath} + e^{-t}\,\hat{\jmath}$. Find a_T at $t = 0$, where a_T is the tangential component of acceleration.

a. $\dfrac{4}{\sqrt{5}}$

b. $\dfrac{5}{\sqrt{5}}$

c. $\dfrac{6}{\sqrt{5}}$

d. $\dfrac{3}{\sqrt{5}}$

e. $\dfrac{2}{\sqrt{5}}$

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12. Find the distance between the two parallel planes $x - 2y + 2z = 4$ and $4x - 8y + 8z = 1$.

a. $\dfrac{5}{4}$

b. $\dfrac{7}{4}$

c. $\dfrac{4}{3}$

d. $\dfrac{5}{3}$

e. $\dfrac{15}{4}$

5. Let $\vec{u}$ and $\vec{v}$ be two vectors in space with $|\vec{u}| = 2$ and $|\vec{v}| = 5$. If the angle between $\vec{u}$ and $\vec{v}$ is $\frac{\pi}{3}$, calculate $\vec{u} \cdot (\vec{u} + 2\vec{v})$.

a. 14

b. 12

c. 7

d. $2 + 10\sqrt{3}$

e. $4 + 10\sqrt{3}$

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6. The vector $\langle a, b, 1 \rangle$ is orthogonal to the plane $z = 2x + 3y + 5$. Find a.

a. $a = 2$

b. $a = 3$

c. $a = -3$

d. $a = -2$

e. Do not have enough information to find a.

9. Find an equation for the surface consisting of all points that are equidistant from the point $(0, 0, 1)$ and the plane $z = -1$, and identify the surface.

a. a paraboloid $z = x^2 + y^2$

b. a cone $z^2 = x^2 + y^2$

c. a paraboloid $z = \dfrac{x^2}{4} + \dfrac{y^2}{4}$

d. a cone $z^2 = \dfrac{x^2}{4} + \dfrac{y^2}{4}$

e. a cylinder $x^2 + y^2 = 4$

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10. Find the point of intersection of $L_1 : x = 2 + t, y = 3 - 2t, z = 1 - 3t$ and $L_2 : x = 3 + s, y = -4 + 3s, z = 2 - 7s$.

a. $(2, 3, 1)$

b. $(4, -1, -5)$

c. $(3, -4, 2)$

d. $(0, 7, 1)$

e. L_1 and L_2 are skew lines, so there is no point of intersection.

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Bonus Questions 13 – 14 are worth 4 points each.

13. Consider the surface S: $4x^2 - y^2 + z^2 = 0$. Which of the following is/are correct?

P. The surface S is a hyperboloid of one sheet.

Q. The horizontal trace in the xy-plane is a hyperbola.

R. The traces in the plane parallel to the xz-plane are ellipses.

a. P only

b. P and Q only

c. R only

d. P and R only

e. P, Q, and R

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14. If the curve C is parametrized by $\vec{r}(t) = \cos^3 t\,\hat{\imath} + \sin^3 t\,\hat{\jmath} + \hat{k}$, find its arc length for $0 \leq t \leq \frac{\pi}{2}$.

a. $\frac{3\pi^2}{8}$

b. $\frac{3\pi}{2}$

c. $\frac{9\pi}{2}$

d. $\frac{9}{2}$

e. $\frac{3}{2}$

MAC 2313 Exam 1A, Part II Free Response

Name: ______________________________

TA's Name: ______________________________ Discussion Period: ______

SHOW ALL WORK TO RECEIVE FULL CREDIT

1. (10 points) Let P be the plane containing three points $A(2,0,-1)$, $B(3,1,1)$ and $C(4,-1,2)$.

(a) Find a linear equation of the plane P.

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(b) Find the area of the triangle ABC.

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2. (10 points) Let C be a smooth curve parametrized by $\vec{r}(t) = \langle \cos(t^2), \sin(t^2), t^2 \rangle$ for $t > 0$, find

(a) a unit tangent vector of the curve C

$\hat{T}(t) =$ ____________________

(b) curvature of the curve C

$\kappa(t) =$ ____________________

University of Florida Honor Pledge:

On my honor, I have neither given nor received unauthorized aid doing this exam.

Signature: ______________________________