Name: Solutions **Section #:** ____________

UF-ID: ______________________ **TA Name:** ____________

A: Sign the back of your scantron sheet.

B: On the indicated spaces on the front of your scantron, **in pencil**, write and encode:

1. Your name (last name, first initial, middle initial)
2. Your UFID Number
3. Your 4-digit Section Number

C: Under "special codes", code in the test ID number 4,1:

1 2 3 • 5 6 7 8 9 0
• 2 3 4 5 6 7 8 9 0

D. At the top right of your scantron, for "Test Form Code", encode A .

• B C D E

E: Some basic information about the exam:

1. This exam has 22 multiple choice section worth 110 points. The entire exam is out of 100 points.
2. You will have 120 minutes to take the exam.
3. You may write on your exam.
4. Raise your hand if you need more scratch paper or if you have a problem with the test. DO NOT LEAVE YOUR SEAT UNLESS YOU ARE FINISHED WITH THE EXAM.

F: KEEP YOUR SCANTRON COVERED AT ALL TIMES

G: When you are finished:

1. Before turning in your test, check for transcribing errors. Any mistakes you leave in are there to stay.
2. Turn in your scantron and the free-response portion of your exam to your TA, or the proctor designated as your TA. Be prepared to show your UF-ID card.
3. Solutions to the exam will be posted on Canvas after the exam is over.

The Honor Pledge: "On my honor, I have neither given nor received unauthorized aid doing this exam."

Signature: ______________________

There are **22** questions on this exam. Fill in the answers to these questions on the provided scantron sheet. Only answers on the scantron will be graded. Each problem is worth 5 points, for a total of 110 points on the exam. Your grade will be taken out of 100 points.

1. The length of the curve $y = 2\sec x$ from $x = 0$ to $x = \frac{\pi}{4}$ is given by $\int_0^{\pi/4} \sqrt{1 + 4\sec^2 x \tan^2 x}\, dx$.

(A) True

(B) False

$$\frac{dy}{dx} = 2\sec x \tan x$$

$$L = \int_a^b \sqrt{1 + (dy/dx)^2}\, dx = \int_0^{\pi/4} \sqrt{1 + (2\sec x \tan x)^2}\, dx$$

$$= \int_0^{\pi/4} \sqrt{1 + 4\sec^2 x \tan^2 x}\, dx$$

2. Evaluate $\int_0^1 x^2 e^x \, dx$.

(A) $5e - 2$

(B) $2 - e$

(C) e

(D) $e - 2$

(E) $2 - 5e$

u	dv
x^2	$+\ e^x$
$2x$	$-\ e^x$
2	$+\ e^x$
0	e^x

$$\int_0^1 x^2 e^x \, dx = \left[x^2 e^x - 2x e^x + 2e^x\right]_0^1$$

$$= \left[e - 2e + 2e\right] - \left[0 - 0 + 2\right]$$

$$= e - 2$$

3. Let X be a continuous random variable with probability density function $f(x)$ where $f(x) = 0$ for $x < 0$ and $f(x) = e^{-x}$ for $x \geq 0$. What is the median of X?

(A) 1 (B) $\ln 2$ (C) e^2 (D) $\sqrt{e}$ (E) $\ln(1/2)$

$$\int_0^m e^{-x}\,dx = -e^{-x}\Big|_0^m$$

$$= -e^{-m} + 1 = 1/2$$

$$e^{-m} = 1/2$$

$$-m = \ln(1/2) = -\ln 2 \Rightarrow m = \ln 2$$

4. What is the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{n^2(x+2)^n}{2^{3n}}$?

(A) 8 (B) 2 (C) 0 (D) 1 (E) ∞

$$\lim_{n\to\infty} \left| \frac{(n+1)^2 (x+2)^{n+1}}{2^{3n+3}} \cdot \frac{2^{3n}}{n^2 (x+2)^n} \right| = \lim_{n\to\infty} \frac{(n+1)^2}{n^2} \cdot \frac{1}{2^3} \cdot |x+2|$$

$$= \frac{|x+2|}{8} < 1$$

$$|x+2| < 8$$

ROC

5. Which of the following integrals gives the area of the region enclosed by one petal of the polar curve $r = 4\cos(3\theta)$?

$4\cos(3\theta) = 0 \Rightarrow 3\theta = -\pi/2, \pi/2, \ldots \Rightarrow \theta = -\pi/6, \pi/6, \ldots$

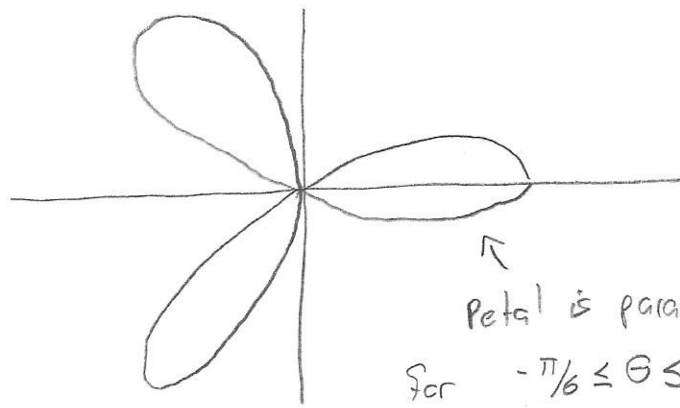
(A) $16\int_{-\pi/3}^{\pi/3} \cos(3\theta)\, d\theta$

(B) $8\int_{-\pi/6}^{\pi/6} \cos(3\theta)\, d\theta$

(C) $8\int_{-\pi/3}^{\pi/3} \cos^2(3\theta)\, d\theta$

(D) $16\int_{-\pi/6}^{\pi/6} \cos^2(3\theta)\, d\theta$

(E) $8\int_{-\pi/6}^{\pi/6} \cos^2(3\theta)\, d\theta$



Petal is parametrized for $-\pi/6 \le \theta \le \pi/6$

$$\text{Area} = \frac{1}{2}\int_{\alpha}^{\beta} [r(\theta)]^2\, d\theta = \frac{1}{2}\int_{-\pi/6}^{\pi/6} [4\cos(3\theta)]^2\, d\theta$$

$$= \int_{-\pi/6}^{\pi/6} 8\cos^2(3\theta)\, d\theta$$

6. Find the length of the parametric curve with equations $x(t) = 3t^2 - 5$ and $y(t) = 4t^2 + 1$ from $t = \sqrt{2}$ to $t = \sqrt{5}$.

(A) 10 (B) 20 (C) 5 (D) 15 (E) 25

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_{\sqrt{2}}^{\sqrt{5}} \sqrt{(6t)^2 + (8t)^2}\, dt$$

$$= \int_{\sqrt{2}}^{\sqrt{5}} \sqrt{100t^2}\, dt = \int_{\sqrt{2}}^{\sqrt{5}} 10t\, dt = 5t^2 \Big|_{\sqrt{2}}^{\sqrt{5}}$$

$$= 5\cdot 5 - 5\cdot 2 = 15$$

7. Evaluate $\int \sec^5 x \tan^3 x \, dx$.

(A) $\dfrac{\sec^7 x}{7} - \dfrac{\sec^5 x}{5} + C$

(B) $\dfrac{\sec^5 x}{5} - \dfrac{\sec^3 x}{3} + C$

(C) $\dfrac{\sec^6 x}{6} - \dfrac{\sec^4 x}{4} + C$

(D) $\dfrac{\sec^4 x}{4} - \dfrac{\sec^6 x}{6} + C$

(E) $\dfrac{\sec^5 x}{5} - \dfrac{\sec^7 x}{7} + C$

$$\int \sec^5 x \tan^3 x \, dx = \int \sec^4 x \tan^2 x \sec x \tan x \, dx$$

$$= \int \sec^4 x (\sec^2 x - 1) \sec x \tan x \, dx \qquad u = \sec x, \quad du = \sec x \tan x \, dx$$

$$= \int u^4 (u^2 - 1) \, du$$

$$= \int (u^6 - u^4) \, du$$

$$= \frac{u^7}{7} - \frac{u^5}{5} + C = \frac{\sec^7 x}{7} - \frac{\sec^5 x}{5} + C$$

8. Evaluate $\displaystyle\sum_{n=0}^{\infty} \frac{2}{n^2 + 3n + 2}$.

(A) $\dfrac{1}{2}$ (B) 4 (C) 2 (D) 1 (E) 3

$$\frac{2}{n^2+3n+2} = \frac{2}{(n+1)(n+2)} = \frac{A}{n+1} + \frac{B}{n+2}$$

Cover up: $A = \dfrac{2}{-1+2} = 2$

$B = \dfrac{2}{-2+1} = -2$

$$S_N = \sum_{n=0}^{N} \frac{2}{n^2+3n+2} = \sum_{n=0}^{N} \left(\frac{2}{n+1} - \frac{2}{n+2} \right)$$

$$= \left(\frac{2}{1} - \cancel{\frac{2}{2}} \right) + \left(\cancel{\frac{2}{2}} - \cancel{\frac{2}{3}} \right) + \left(\frac{2}{3} - \frac{2}{4} \right) + \dots + \left(\cancel{\frac{2}{N}} - \cancel{\frac{2}{N+1}} \right) + \left(\cancel{\frac{2}{N+1}} - \frac{2}{N+2} \right)$$

$$= 2 - \frac{2}{N+2}$$

$$\sum_{n=0}^{\infty} \frac{2}{n^2+3n+2} = \lim_{N \to \infty} S_N = \lim_{N \to \infty} \left(2 - \frac{2}{N+2} \right) = 2$$

9. Which of the following is the power series representation for $\int x\ln(1+x^3)\,dx$ centered at $a=0$?

(A) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^{n+2}}{(n+2)n}$
(B) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^{3n+2}}{(3n+2)n}$
(C) $\sum_{n=1}^{\infty} \frac{x^{3n+2}}{(3n+2)n}$
(D) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(3n+1)x^{3n}}{n}$
(E) $\sum_{n=1}^{\infty} \frac{x^{n+2}}{(n+2)n}$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^n}{n}$$

$$x\ln(1+x^3) = x\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x^3)^n}{n} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^{3n+1}}{n}$$

$$\int x\ln(1+x^3)\,dx = \int \sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^{3n+1}}{n}\,dx = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^{3n+2}}{(3n+2)\cdot n}$$

10. Find the slope of the line tangent to the polar curve $r=\cos(2\theta)$ when $\theta=\frac{\pi}{2}$.

(A) 1 (B) -1 (C) 0 (D) 2 (E) -2

$$\frac{dy}{dx} = \frac{f(\theta)\cos\theta + f'(\theta)\sin\theta}{-f(\theta)\sin\theta + f'(\theta)\cos\theta}$$

$$= \frac{\cos(2\theta)\cos\theta - 2\sin(2\theta)\sin\theta}{-\cos(2\theta)\sin\theta - 2\sin(2\theta)\cos\theta}$$

when $\theta = \pi/2$, $\frac{dy}{dx} = \frac{-1\cdot 0 - 2\cdot 0\cdot 1}{-(-1)\cdot 1 - 2\cdot 0\cdot 1} = 0$

11. Which of the polar points (r,θ) below represent the Cartesian point $(x,y) = (-1,1)$?

$$P = \left(\sqrt{2}, \frac{\pi}{4}\right) \qquad Q = \left(-\sqrt{2}, \frac{\pi}{4}\right) \qquad R = \left(-\sqrt{2}, -\frac{\pi}{4}\right) \qquad S = \left(\sqrt{2}, \frac{3\pi}{4}\right)$$

(A) P and R only

(B) Q and S only

(C) Q and R only

(D) P and S only

(E) R and S only

$P:\ x = \sqrt{2}\cdot\cos(\pi/4) = 1,\quad y = \sqrt{2}\cdot\sin(\pi/4) = 1$

$Q:\ x = -\sqrt{2}\cos(\pi/4) = -1,\quad y = -\sqrt{2}\cdot\sin(\pi/4) = -1$

$R:\ x = -\sqrt{2}\cdot\cos(-\pi/4) = -1,\quad y = -\sqrt{2}\cdot\sin(-\pi/4) = 1$

$S:\ x = \sqrt{2}\cdot\cos(3\pi/4) = -1,\quad y = \sqrt{3}\sin(3\pi/4) = 1$

12. What is the area of the region bounded by the curves $y = 5x - x^2$ and $y = x$?

$(A)\ \frac{26}{3}$ $\quad (B)\ \frac{20}{3}$ $\quad (C)\ \frac{32}{3}$ $\quad (D)\ \frac{22}{3}$ $\quad (E)\ \frac{16}{3}$

$5x - x^2 = x \Rightarrow x^2 - 4x = 0 \Rightarrow x(x-4) = 0$

$x = 0,\ x = 4$

$$\text{Area} = \int_0^4 (5x - x^2 - x)\,dx = \int_0^4 (4x - x^2)\,dx$$

$$= \left[2x^2 - \frac{x^3}{3}\right]_0^4$$

$$= 32 - \frac{64}{3} = \frac{32}{3}$$

13. Evaluate $\displaystyle\sum_{n=2}^{\infty} \frac{(-2)^{n+1}}{3^{n-2}}$.

(A) $-\frac{1}{48}$ (B) $\frac{3}{32}$ (C) $-\frac{8}{25}$ (D) $\frac{7}{16}$ (E) $-\frac{24}{5}$

$$\sum_{n=2}^{\infty} \frac{(-2)^{n+1}}{3^{n-2}} = \sum_{n=2}^{\infty} -18\left(-\frac{2}{3}\right)^n$$

$$= \frac{-18 \cdot (-2/3)^2}{1-(-2/3)}$$

$$= \frac{-8}{5/3} = -\frac{24}{5}$$

14. Let $f(x)$ be a function such that $f(0) = 2$, $f'(0) = -1$, $f''(0) = 3$, and $f'''(0) = -4$. What is the third degree Maclaurin polynomial for $f(x)$?

(A) $2 - x + 3x^2 - 4x^3$

(B) $2 - x + \frac{3}{2}x^2 - \frac{2}{3}x^3$

(C) $2 - x + \frac{3}{2}x^2 - \frac{4}{3}x^3$

(D) $2 + x + \frac{3}{2}x^2 + \frac{2}{3}x^3$

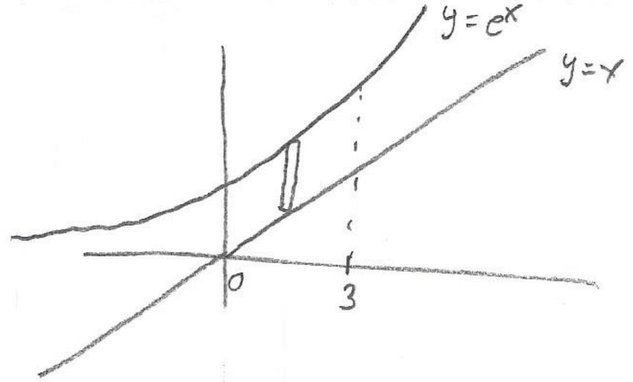
(E) $2 - \frac{1}{2}x + \frac{1}{2}x^2 - \frac{1}{6}x^3$

$$T_3(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3$$

$$= 2 - x + \frac{3}{2}x^2 - \frac{4}{6}x^3$$

$$= 2 - x + \frac{3}{2}x^2 - \frac{2}{3}x^3$$

15. Let R be the region bounded by the curves $y = e^x$, $y = x$, $x = 0$, and $x = 3$. Which of the following integrals gives the volume of the solid formed when R is rotated about the x-axis using the Washer Method?



(A) $\pi \int_0^3 \left(e^{2x} - x^2\right)\, dx$

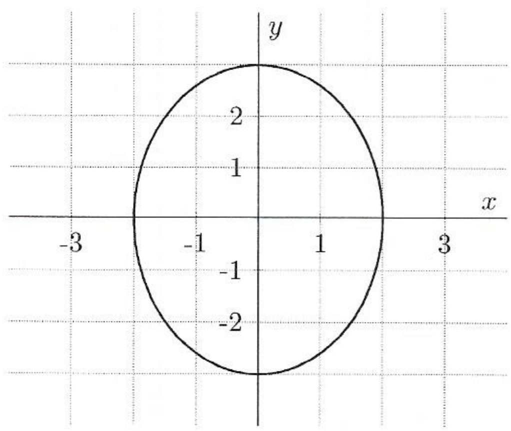
(B) $\pi \int_0^3 \left(e^x - x\right)^2\, dx$

(C) $\pi \int_0^3 x\left(e^{2x} - x^2\right)\, dx$

(D) $\pi \int_0^3 x\left(e^x - x\right)^2\, dx$

$$V = \pi \int_0^3 \left([e^x]^2 - [x]^2 \right) dx = \pi \int_0^3 \left(e^{2x} - x^2\right) dx$$

16. The graph of which of the following sets of parametric equations is shown below?



$-2 \le x \le 2$

$-3 \le y \le 3$

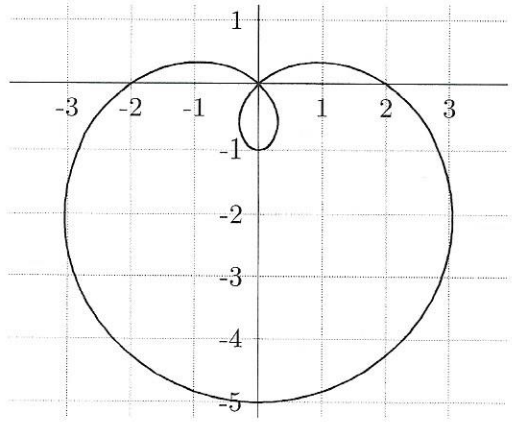
(A) $x = 3\cos t, \quad y = 2\sin t, \quad 0 \le t \le 2\pi$ $\quad -3 \le x \le 3 \quad -2 \le y \le 2$

(B) $x = 2\cos t, \quad y = 2\sin t, \quad 0 \le t \le 2\pi$ $\quad -2 \le x \le 2 \quad -2 \le y \le 2$

(C) $x = 2\cos t, \quad y = 3\sin t, \quad 0 \le t \le 2\pi$ $\quad -2 \le x \le 2 \quad -3 \le y \le 3$

(D) $x = 3\cos t, \quad y = 3\sin t, \quad 0 \le t \le 2\pi$ $\quad -3 \le x \le 3 \quad -3 \le y \le 3$

17. The graph of which polar curve is shown below?



(A) $r = 2 - 3\sin\theta$ (B) $r = 3 + 2\sin\theta$ (C) $r = 3 - 2\sin\theta$ (D) $r = 2 + 3\sin\theta$

$\theta = \pi/2$ $\theta = \pi/2$ $\theta = \pi/2$ $\theta = \pi/2$

$r = -1$ $r = 5$ $r = 1$ $r = 5$

18. Evaluate $\int_1^\infty \frac{1}{1+x^2}\,dx$

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$ (E) π

$$\int_1^\infty \frac{1}{1+x^2}\,dx = \lim_{t\to\infty} \int_1^t \frac{1}{1+x^2}\,dx = \lim_{t\to\infty} \arctan x \Big|_1^t$$

$$= \lim_{t\to\infty} \left[\arctan t - \arctan 1\right]$$

$$= \pi/2 - \pi/4$$

$$= \pi/4$$

19. Which of the following series converge?

$$\mathbf{A}: \sum_{n=1}^{\infty} \frac{1}{n^{3/2}+n+1} \qquad \mathbf{B}: \sum_{n=2}^{\infty} \frac{\sin^2 n}{n^2+n} \qquad \mathbf{C}: \sum_{n=2}^{\infty} \frac{\ln n}{n}$$

(A) **A** and **C** only (B) **B** and **C** only (C) **A** and **B** only (D) **A**, **B**, and **C**

A: converges by LCT when comparing to $\sum 1/n^{3/2}$

B: converges by DCT when comparing to $\sum \frac{1}{n^2}$

C: Diverges by Generalized p-Series Test with $p=1$, $q=-1<1$

20. Use the Alternating Series Estimation Theorem to find the maximum possible error of using the first two nonzero terms of the Maclaurin series for $f(x) = \cos x$ to approximate $\cos(1/2)$.

(A) $\frac{1}{8}$ (B) $\frac{1}{24}$ (C) $\frac{1}{192}$ (D) $\frac{1}{128}$ (E) $\frac{1}{384}$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \cdots$$

Max error in using first two terms to approximate $\cos(1/2)$ $= |\text{third term}| = \frac{(1/2)^4}{24} = \frac{1}{16 \cdot 24} = \frac{1}{384}$

21. Find the slope of the line tangent to the parametric curve $x(t) = 3t^2 + \sqrt{t}, y(t) = 5t^3 - t - \ln t$ when $t = 1$.

(A) 2 (B) 1 (C) 5 (D) 3 (E) 4

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{15t^2 - 1 - 1/t}{6t + 1/2\sqrt{t}}$$

when $t = 1$, $\frac{dy}{dx} = \frac{15 - 1 - 1}{6 + 1/2} = \frac{13}{13/2} = 2$

22. Which of the following statements is true concerning the series $\displaystyle\sum_{n=1}^{\infty} \frac{2 \cdot 4 \cdot 6 \cdots (2n)}{1 \cdot 4 \cdot 7 \cdots (3n-2)}$?

(A) The Ratio Test Yields 1, so the test is inconclusive

(B) The Ratio Test Yields $\frac{2}{3}$, so the series converges

(C) The Ratio Test Yields $\frac{3}{2}$, so the series diverges

(D) The Ratio Test Yields 0, so the series converges

(E) The Ratio Test Yields ∞, so the series diverges

$$\lim_{n \to \infty} \left| \frac{2 \cdot 4 \cdot 6 \cdots (2n)(2n+2)}{1 \cdot 4 \cdot 7 \cdots (3n-2)(3n+1)} \cdot \frac{1 \cdot 4 \cdot 7 \cdots (3n-2)}{2 \cdot 4 \cdot 6 \cdots (2n)} \right|$$

$$= \lim_{n \to \infty} \frac{2n+2}{3n+1} = \frac{2}{3}$$

$f(x)$	Maclaurin Series	Interval of Convergence
$\frac{1}{1-x}$	$\sum_{n=0}^{\infty} x^n$	$(-1, 1)$
$\ln(1-x)$	$-\sum_{n=1}^{\infty} \frac{x^n}{n}$	$[-1, 1)$
$\ln(1+x)$	$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$	$(-1, 1]$
e^x	$\sum_{n=0}^{\infty} \frac{x^n}{n!}$	$(-\infty, \infty)$
$\cos x$	$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$	$(-\infty, \infty)$
$\sin x$	$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$	$(-\infty, \infty)$
$\arctan x$	$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$	$[-1, 1]$

Taylor Remainder Estimate:

If $\left|f^{(N+1)}(c)\right| \leq M$ for all numbers c between x and a, then the remainder $R_N(x)$ of the Taylor series satisfies the inequality

$$|R_N(x)| \leq \frac{M|x-a|^{N+1}}{(N+1)!} \quad \text{for all numbers between } x \text{ and } a.$$

Taylor Coefficients:

$$c_n = \frac{f^{(n)}(a)}{n!}$$