Name: Solutions **Section #:** ______

UF-ID: ______ **TA Name:** ______

A: Sign the back of your scantron sheet.

B: On the indicated spaces on the front of your scantron, **in pencil**, write and encode:

1. Your name (last name, first initial, middle initial)
2. Your UFID Number
3. Your 4-digit Section Number

C: Under "special codes", code in the test ID number 1,1:

• 2 3 4 5 6 7 8 9 0
• 2 3 4 5 6 7 8 9 0

D. At the top right of your scantron, for "Test Form Code", encode A .

• B C D E

E: Some basic information about the exam:

1. This exam has two parts: a 10-question multiple choice section worth 50 points, and a 4-question free response section worth 55 points. The entire exam is out of 100 points.
2. You will have 100 minutes to take the exam.
3. You may write on your exam.
4. Raise your hand if you need more scratch paper or if you have a problem with the test. DO NOT LEAVE YOUR SEAT UNLESS YOU ARE FINISHED WITH THE EXAM.

F: KEEP YOUR SCANTRON COVERED AT ALL TIMES

G: When you are finished:

1. Before turning in your test, check for transcribing errors. Any mistakes you leave in are there to stay.
2. Turn in your scantron and the free-response portion of your exam to your TA, or the proctor designated as your TA. Be prepared to show your UF-ID card.
3. Solutions to the exam will be posted on Canvas after the exam is over.

The Honor Pledge: "On my honor, I have neither given nor received unauthorized aid doing this exam."

Signature: ______

Part I: Multiple Choice There are **10** questions on this portion of the exam. Fill in the answers to these questions on the provided scantron sheet. Only answers on the scantron will be graded. Each problem is worth 5 points, for a total of 50 points on this portion of the exam.

1. Evaluate $\int x^2 \ln(5x)\,dx$.

$u = \ln(5x) \quad dv = x^2\,dx$

$du = \frac{1}{x}\,dx \quad v = \frac{x^3}{3}$

(A) $2x\ln(5x) - 2x + C$

(B) $\dfrac{x^3\ln(5x)}{3} - \dfrac{x^3}{9} + C$

(C) $\dfrac{x^3\ln(5x)}{3} + \dfrac{x^3}{9} + C$

(D) $\dfrac{x^3\ln(5x)}{3} - \dfrac{x^3}{45} + C$

(E) $\dfrac{x^3\ln(5x)}{3} + \dfrac{x^3}{45} + C$

$$\int x^2 \ln(5x)\,dx = \frac{x^3\ln(5x)}{3} - \int \frac{x^2}{3}\,dx$$

$$= \frac{x^3\ln(5x)}{3} - \frac{x^3}{9} + C$$

2. The partial fraction decomposition of $\dfrac{3x^2+3x+1}{x(x+1)^2}$ can be written in the form $\dfrac{A}{x} + \dfrac{B}{x+1} + \dfrac{C}{(x+1)^2}$. Find $A + B + C$.

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

$$\frac{3x^2+3x+1}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

cover up: $A = \frac{0+0+1}{1^2} = 1$

$C = \frac{3-3+1}{-1} = -1$

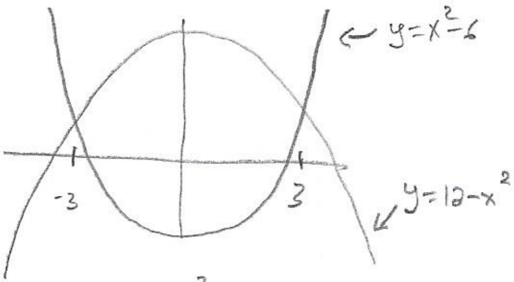
$3x^2+3x+1 = (x+1)^2 + Bx(x+1) - x$

$x = 1 \Rightarrow 7 = 4 + 2B - 1 \Rightarrow 2B = 4$

$\Rightarrow B = 2$ $\quad A+B+C = 1+2-1 = 2$

3. What is the area of the region bounded by the curves $y = x^2 - 6$ and $y = 12 - x^2$?

(*A*) 24 (*B*) 36 (*C*) 54 (*D*) 72 (*E*) 108



$x^2-6 = 12-x^2$

$2x^2 = 18 \Rightarrow x^2 = 9$

$x = \pm 3$

$$\text{Area} = \int_{-3}^{3} \left[(12-x^2)-(x^2-6)\right]dx = \int_{-3}^{3} (18-2x^2)\,dx$$

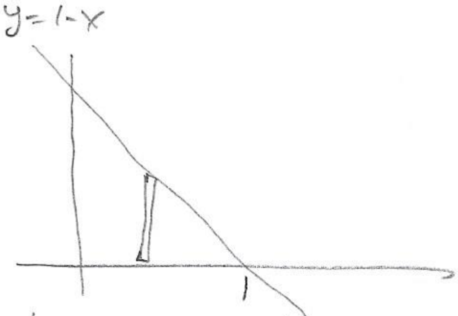
$$= \left[18x - \frac{2x^3}{3}\right]_{-3}^{3}$$

$$= (54-18)-(-54+18)$$

$$= 36+36 = 72$$

4. Find the volume of the solid whose base is the triangle bounded by $y = 1 - x$, $x = 0$, and $y = 0$ and whose cross sections perpendicular to the x-axis are squares.

(*A*) $\frac{1}{6}$ (*B*) $\frac{1}{4}$ (*C*) $\frac{1}{3}$ (*D*) $\frac{1}{2}$ (*E*) 1



$$V = \int_a^b A(x)\,dx = \int_0^1 (1-x)^2\,dx$$

$$= \int_0^1 (1-2x+x^2)\,dx$$

$$= \left[x - x^2 + \frac{x^3}{3}\right]_0^1$$

$$= 1/3$$

5. Evaluate $\int_0^{\pi/2} \sin^2 x \cos^3 x\, dx$.

(A) 0 (B) $\frac{\pi^3}{3} - \frac{\pi^5}{5}$ (C) $\frac{\pi^5}{5} - \frac{\pi^3}{3}$ (D) $-\frac{2}{15}$ (E) $\frac{2}{15}$

$$\int_0^{\pi/2} \sin^2 x \cos^3 x\, dx = \int_0^{\pi/2} \sin^2 x \cos^2 x \cos x\, dx$$

$$= \int_0^{\pi/2} \sin^2 x (1 - \sin^2 x) \cos x\, dx$$

$u = \sin x \Rightarrow du = \cos x\, dx$

$x = 0,\ u = 0$

$x = \pi/2,\ u = 1$

$$= \int_0^1 u^2(1-u^2)\, du = \int_0^1 (u^2 - u^4)\, du = \left[\frac{u^3}{3} - \frac{u^5}{5}\right]_0^1 = \frac{1}{3} - \frac{1}{5} = \frac{2}{15}$$

6. Which of the following trigonometric substitutions should be made to evaluate $\int \frac{1}{\sqrt{-x^2 - 4x + 5}}\, dx$?

(A) $x + 2 = 3\sin\theta$

(B) $x - 2 = 3\sec\theta$

(C) $x - 2 = 3\tan\theta$

(D) $x + 2 = 3\sec\theta$

(E) $-x^2 - 4x + 5 = 3\tan\theta$

$$-x^2 - 4x + 5 = -(x^2 + 4x - 5)$$

$$= -(x^2 + 4x + 4 - 4 - 5)$$

$$= -((x+2)^2 - 9)$$

$$= 9 - (x+2)^2$$

$$\int \frac{1}{\sqrt{-x^2 - 4x + 5}}\, dx = \int \frac{1}{\sqrt{9 - (x+2)^2}}\, dx$$

$a = 3 \quad u = x + 2 \quad \Rightarrow \quad x + 2 = 3\sin\theta$

7. How many of the followings integrals are improper?

✓ I. $\displaystyle\int_0^{\infty} \frac{\sin x}{x^2+1}\,dx$ Infinite limit of integration

X II. $\displaystyle\int_{-2}^{2} \frac{x}{x^2-9}\,dx$ Integrand is continuous on $[-2,2]$

✓ III. $\displaystyle\int_{-\infty}^{1} \frac{e^x}{x^2+1}\,dx$ Infinite limit of integration

✓ IV. $\displaystyle\int_{\pi}^{-\pi} \cot x\,dx$ Integrand is not defined when $x=0$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

8. Let R be the region bounded by the curves $y = x^2 + 1$ and $y = 5x - 3$. Which of the following integrals gives the volume of the solid formed when R is rotated about the x-axis using the Washer Method?

$x^2+1 = 5x-3 \Rightarrow x^2-5x+4=0$

$(x-1)(x-4)=0$

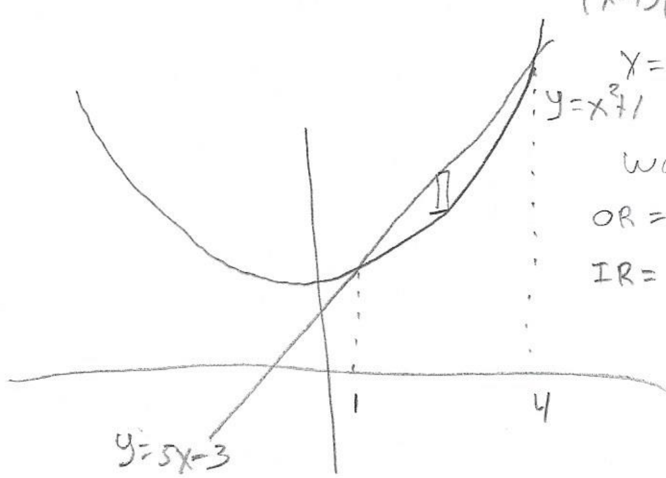
$x = 1, 4$

(A) $\pi \displaystyle\int_1^4 \left[(5x-3) - (x^2+1)\right]\,dx$

(B) $\pi \displaystyle\int_1^4 \left[(5x-3)^2 - (x^2+1)^2\right]\,dx$

(C) $\pi \displaystyle\int_1^4 x\left[(5x-3) - (x^2+1)\right]\,dx$

(D) $\pi \displaystyle\int_1^4 x\left[(5x-3)^2 - (x^2+1)^2\right]\,dx$



washer:

OR = $5x-3$

IR = x^2+1

Volume $= \pi \displaystyle\int_1^4 \left[(5x-3)^2 - (x^2+1)^2\right]\,dx$

9. Evaluate $\int_0^1 \sqrt{x^2+2x}\,dx$.

(A) $\sqrt{3}+\frac{1}{2}\ln(2+\sqrt{3})$ (B) $\sqrt{3}-\frac{1}{2}\ln(2+\sqrt{3})$ (C) $2\sqrt{3}-\ln(2+\sqrt{3})$ (D) $2\sqrt{3}+\ln(2+\sqrt{3})$

$x^2+2x = x^2+2x+1-1 = (x+1)^2-1$

$\int_0^1 \sqrt{x^2+2x}\,dx = \int_0^1 \sqrt{(x+1)^2-1}\,dx$

$x+1=\sec\theta \Rightarrow dx=\sec\theta\tan\theta\,d\theta$

$\sqrt{(x+1)^2-1} = \tan\theta$

$x=0 \Rightarrow \sec\theta=1 \Rightarrow \theta=0$

$x=1 \Rightarrow \sec\theta=2 \Rightarrow \theta=\pi/3$

$= \int_0^{\pi/3} \sec\theta\tan^2\theta\,d\theta$

$= \int_0^{\pi/3} (\sec^3\theta - \sec\theta)\,d\theta$

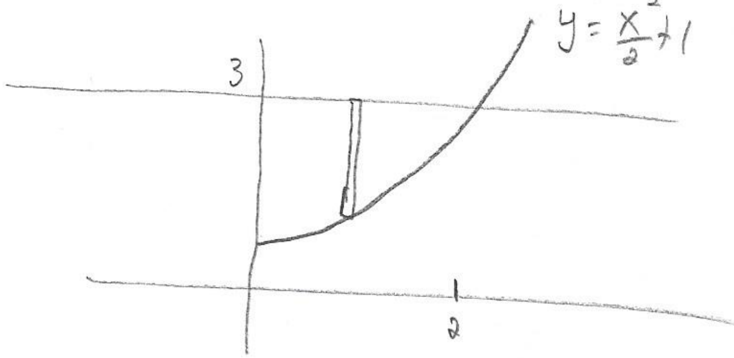
$= \left[\frac{1}{2}(\sec\theta\tan\theta + \ln|\sec\theta+\tan\theta|) - \ln|\sec\theta+\tan\theta|\right]_0^{\pi/3}$

$= \left[\frac{1}{2}(2\sqrt{3}+\ln(2+\sqrt{3})) - \ln(2+\sqrt{3})\right] - \left[\frac{1}{2}(1\cdot 0+\ln(1+0)) - \ln(1+0)\right]$

$= \sqrt{3} - \frac{1}{2}\ln(2+\sqrt{3})$

10. Let R be the region in the first quadrant bounded by the curves $y=\frac{x^2}{2}+1$, $y=3$, and $x=0$. What is the volume of the solid formed when R is rotated about the y-axis?

(A) π (B) 2π (C) $\frac{3\pi}{2}$ (D) 4π (E) 5π



$\frac{x^2}{2}+1=3 \Rightarrow x^2=4$

$x=2$

Using the Shell Method,

$V = 2\pi\int_0^2 x(3-[x^2/2+1])\,dx = 2\pi\int_0^2 x(2-x^2/2)\,dx$

$= 2\pi\int_0^2 (2x - x^3/2)\,dx$

$= 2\pi\left[x^2 - x^4/8\right]_0^2$

$= 2\pi\cdot(4-2) = 4\pi$

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Part II: Free Response There are **4** questions on this portion of the exam. Show **ALL** work clearly in the space provided for each problem, unless the problem says otherwise. Your work must be complete, logical, and understandable or it will receive no credit. Please cross out or fully erase any work that you do not want graded. A total of **55** points are available on this portion of the exam.

FR Scores	
1	/10
2	/15
3	/15
4	/15
FR Total	/55

1. Evaluate $\int_0^{\infty} x^3 e^{-x^4}\,dx$.

$$\int_0^{\infty} x^3 e^{-x^4} = \lim_{t\to\infty} \int_0^{t} x^3 e^{-x^4}$$

$$= \lim_{t\to\infty} -\frac{1}{4}e^{-x^4}\Big|_0^t$$

$$= \lim_{t\to\infty} \left[-\frac{1}{4e^{t^4}} + \frac{1}{4}\right]$$

$$= 0 + \frac{1}{4}$$

$$= \boxed{\frac{1}{4}}$$

2. Evaluate $\int \frac{x+3}{(x^2+1)(x-1)}\,dx$.

$$\frac{x+3}{(x^2+1)(x-1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$

cover up: $A = \frac{1+3}{1+1} = 2$

$$x+3 = 2(x^2+1) + (Bx+C)(x-1)$$

$$x+3 = 2x^2+2 + Bx^2 - Bx + Cx - C$$

$$x+3 = (B+2)x^2 + (C-B)x + (2-C)$$

$B+2=0 \Rightarrow B=-2$

$C-B=1$

$2-C=3 \Rightarrow C=-1$

$$\int \frac{x+3}{(x^2+1)(x-1)}\,dx = \int\left(\frac{2}{x-1} + \frac{-2x-1}{x^2+1}\right)dx$$

$$= \int\left(\frac{2}{x-1} - \frac{2x}{x^2+1} - \frac{1}{x^2+1}\right)dx$$

$$= \boxed{2\ln|x-1| - \ln(x^2+1) - \arctan x + C}$$

3. Evaluate $\int \sin^4 x\, dx$.

$$\int \sin^4 x\, dx = \int [\sin^2 x]^2\, dx$$

$$= \int \left[\tfrac{1}{2}(1-\cos(2x))\right]^2 dx$$

$$= \frac{1}{4}\int \left[1 - 2\cos(2x) + \cos^2(2x)\right] dx$$

$$= \frac{1}{4}\int \left[1 - 2\cos(2x) + \frac{1}{2}(1+\cos(4x))\right] dx$$

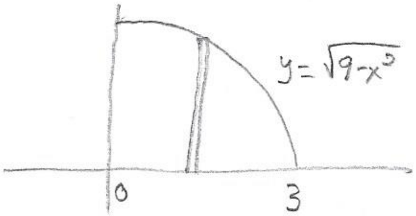
$$= \frac{1}{4}\int \left[\frac{3}{2} - 2\cos(2x) + \frac{1}{2}\cos(4x)\right] dx$$

$$= \frac{1}{4}\left[\frac{3x}{2} - \sin(2x) + \frac{\sin(4x)}{8}\right] + C$$

$$= \boxed{\frac{3x}{8} - \frac{\sin(2x)}{4} + \frac{\sin(4x)}{32} + C}$$

4. Let R be the region bounded by $y = \sqrt{9 - x^2}$, $x = 0$, and $y = 0$.

(a) Use the Disk Method to find the volume of the solid formed by rotating R about the x-axis.



$$\text{Volume} = \pi\int_0^3 \left(\sqrt{9-x^2}\right)^2 dx$$

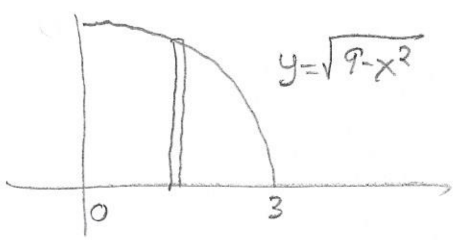
$$= \pi\int_0^3 (9-x^2)\,dx$$

$$= \pi\left[9x - \frac{x^3}{3}\right]_0^3$$

$$= \pi\left[27 - 9\right]$$

$$= \boxed{18\pi}$$

(b) Use the Shell Method to find the volume of the solid formed by rotating R about the y-axis.



$$\text{volume} = 2\pi\int_0^3 x\sqrt{9-x^2}\,dx$$

$u = 9 - x^2 \Rightarrow du = -2x\,dx$

$-\frac{1}{2}\,du = x\,dx$

$x = 0 \Rightarrow u = 9$

$x = 3 \Rightarrow u = 0$

$$= 2\pi\int_9^0 -\frac{1}{2}u^{1/2}\,du$$

$$= 2\pi \cdot -\frac{1}{3}u^{3/2}\Big|_9^0$$

$$0 + \frac{2\pi}{3}\cdot 9^{3/2} = \frac{2\pi}{3}\cdot 27 = \boxed{18\pi}$$