Name: ______________________ **Section #:** ____________

UFID: ______________________ **TA Name:** ____________

A: Sign the back of your scantron sheet **in ink**.

B: On the indicated spaces on the front of your scantron, **in pencil**, write and encode:

1. Your name (last name, first initial, middle initial)
2. Your UFID Number
3. Your 4-digit Section Number

C: Under "special codes", code in the test ID number 4,1:

1 2 3 • 5 6 7 8 9 0
• 2 3 4 5 6 7 8 9 0

D. At the top right of your scantron, for "Test Form Code", encode A .

• B C D E

E: Some basic information about the exam:

1. This exam has 22 multiple choice questions worth 5 points each. The entire exam has 110 points available for you to earn, but will only be graded out of 100 points.
2. You will have 2 hours to take the exam.
3. You may write on your exam.
4. Raise your hand if you need more scratch paper or if you have a problem with the test. DO NOT LEAVE YOUR SEAT UNLESS YOU ARE FINISHED WITH THE EXAM.

F: KEEP YOUR SCANTRON COVERED AT ALL TIMES

G: When you are finished:

1. Before turning in your test, check for transcribing errors. Any mistakes you leave in are there to stay.
2. Turn in your scantron to your TA, or the proctor designated as your TA. Be prepared to show your UF-ID card.
3. Solutions to the exam will be posted on Canvas after the exam is over.

The Honor Pledge: "On my honor, I have neither given nor received unauthorized aid doing this exam."

Signature: ______________________________________

Multiple Choice: There are **22** questions on this portion of the exam. Fill in the answers to these questions on the provided scantron sheet. Only answers on the scantron will be graded. Each problem is worth 5 points, for a total of 110 points on the exam.

1. Evaluate $\int_0^{\infty} e^{-x}\sin(3x)dx$.

(A) 0 (B) $-3/10$ (C) $3/10$ (D) $-1/3$ (E) The integral diverges.

2. Evaluate $\int \cos^2(x)\sin^3(x)dx$?

(A) $\dfrac{\cos^3(x)}{3} - \dfrac{\cos^5(x)}{5} + C$

(B) $\dfrac{\cos^5(x)}{5} - \dfrac{\cos^3(x)}{3} + C$

(C) $\dfrac{\sin^5(x)}{5} - \dfrac{\sin^3(x)}{3} + C$

(D) $\dfrac{\sin^3(x)}{3} - \dfrac{\sin^5(x)}{5} + C$

(E) $\dfrac{-\cos^3(x)\sin^4(x)}{12} + C$

3. Which of the following is the best strategy for computing $\int \frac{8x}{\sqrt{7-6x-x^2}}dx$?

(A) Complete the square, then use the trig sub $4(x+3) = \sec(\theta)$.

(B) Complete the square, then use the trig sub $x+3 = 4\sin(\theta)$.

(C) Use the substitution $u = 12 - 6x - x^2$.

(D) Use Integration By Parts with $u = 12 - 6x - x^2$ and $dv = 8xdx$.

(E) Use the Quotient Rule for integrals.

4. What is the sum of the coefficients of the partial fraction decomposition of $\frac{1}{x^4 - 9x^2}$?

(A) $-1/9$ (B) $1/9$ (C) $-2/27$ (D) $-4/27$ (E) 0

5. Set up, but do not evaluate, an integral to find the volume of a solid generated by revolving the region bounded by $y = x^2 + 1$ and $y = 3 - x^2$ around the x-axis.

(A) $\pi \int_{-1}^{1} 10 - 4x^2 dx$

(B) $\pi \int_{-1}^{1} 8 - 8x^2 dx$

(C) $\pi \int_{-1}^{1} 8x^2 - 8 dx$

(D) $\pi \int_{-1}^{1} 4x^2 - 10 dx$

(E) $\pi \int_{-1}^{1} 2x^4 - 4x^2 - 8 dx$

6. A semicircular metal plate with a diameter of 6 is suspended vertically in a fluid so that its diameter is at the surface of the water. In terms of ρ and g, find the hydrostatic force on the plate.

(A) $12\rho g$

(B) $9\rho g$

(C) $6\rho g$

(D) $36\rho g$

(E) $18\rho g$

7. Find the amount of work it takes to pump all of the water out of a full cylindrical container of radius 1 and height 2 (leave your answer in terms of ρ and g).

$(A)\ 2\pi\rho g$ $(B)\ \pi\rho g$ $(C)\ 4\pi\rho g$ $(D)\ 2\rho g$ $(E)\ \rho g$

8. Let $a_n = f(n)$, where $f(x)$ is a continuous, positive, decreasing function. If $\sum_{n=0}^{\infty}(-1)^n a_n$ is absolutely convergent, then...

(A) $\lim_{n\to\infty} a_n = 0$

(B) $\int_0^{\infty} f(x)\, dx$ converges.

(C) Both A and B.

(D) Neither A nor B.

9. Find the sum of $\sum_{n=1}^{\infty} n^{1/n} - (n+2)^{1/(n+2)}$.

$(A)\ \sqrt{2}+1$ $(B)\ \sqrt{2}-1$ $(C)\ \sqrt{2}+3$ $(D)\ \sqrt{2}-3$ (E) The series diverges.

10. For what value(s) of x is the series $\displaystyle\sum_{n=1}^{\infty} \frac{(x-3)^{3n}}{n+\sqrt{n}}$ conditionally convergent?

(A) $x = 2$

(B) $x = 4$

(C) All x in the interval $(2, 4)$.

(D) All x in the interval $[2, 4]$.

(E) There are no values of x for which the series is conditionally convergent.

11. The sequence

$$1, 1 + \frac{1}{100}, 1 + \frac{1}{100} + \frac{1}{10000}, ...$$

converges to:

(A) 100/99 (B) 1/99 (C) 199/99 (D) 2 (E) The sequence diverges.

12. How many of the following series converge?

- $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2 \ln n}$
- $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$
- $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2(\ln n)^2}$
- $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

13. Suppose that $S = \sum_{n=2}^{\infty} \frac{n^3+n+1}{n^4-n^2-n}$. Which of the following statements is true?

(A) Using $b_n = 1/n$, it can be shown that S diverges by the Direct Comparison Test.

(B) $b_n = 1/n$ won't work for the Direct Comparison Test, but it could show divergence if the Limit Comparison Test is used instead.

(C) $b_n = 1/n$, it can be shown that S converges by the Direct Comparison Test.

(D) $b_n = 1/n$ won't work for the Direct Comparison Test, but it could show convergence if the Limit Comparison Test is used instead.

(E) None of the above.

14. Suppose a power series $\sum_{n=0}^{\infty} a_n(x-1)^n$ converges for $x = 2$ and diverges for $x = -2$. Which of the following statements is necessarily true?

(A) $\sum_{n=0}^{\infty} a_n(-1)^n$ converges.

(B) $\sum_{n=0}^{\infty} a_n 3^n$ diverges.

(C) $\sum_{n=0}^{\infty} a_n 4^n$ diverges.

(D) $\sum_{n=0}^{\infty} a_n 2^n$ converges.

(E) $\sum_{n=0}^{\infty} a_n(-1)^n$ diverges.

15. Find a power series representation for the function $f(x) = \sin(x)\cos(x)$ (Hint: Double angle identity).

(A) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 4^n x^{2n+1}}{(2n+1)!}$

(B) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n-1} x^{2n}}{(2n)!}$

(C) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{4^{n+1}(2n+1)!}$

(D) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{4n+1}}{2(2n)!(2n+1)!}$

(E) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{4n+1}}{(2n)!(2n+1)!}$

16. Use the second degree Maclaurin polynomial for e^x to approximate $\sqrt[3]{e}$.

(A) 4/3

(B) 25/18

(C) $1 + \sqrt[3]{e}$

(D) $1 + \sqrt[3]{e} + \dfrac{e^{2/3}}{2}$

(E) You cannot use a Maclaurin polynomial for e^x to approximate $\sqrt[3]{e}$.

17. Find the Maclaurin series for $\dfrac{x}{(1+x)^2}$.

(A) $\displaystyle\sum_{n=1}^{\infty} (n-1)(-1)^{n-1} x^n$

(B) $\displaystyle\sum_{n=1}^{\infty} n(-1)^{n-1} x^{n+1}$

(C) $\displaystyle\sum_{n=1}^{\infty} n(-1)^n x^{n+1}$

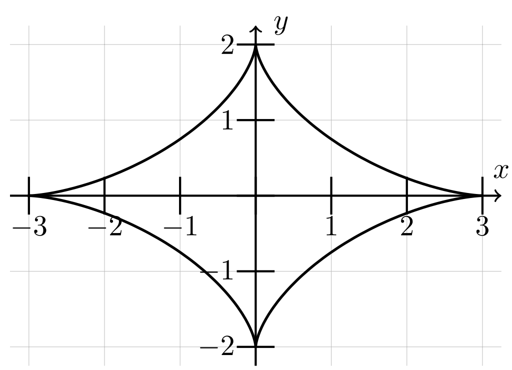
(D) $\displaystyle\sum_{n=1}^{\infty} n(-1)^n x^n$

(E) $\displaystyle\sum_{n=1}^{\infty} n(-1)^{n-1} x^n$

18. Using the Alternating Series Estimation Theorem, what is the maximum possible error of using the first 10 nonzero terms of the Maclaurin series for $\ln(1+x)$ to approximate $\ln 2$?

(A) 1/9 (B) 1/10 (C) 1/11 (D) 1/12 (E) 1/13

19. The graph of which of the following sets of parametric equations is shown below?



(A) $x(t) = 2\cos^3 t,\ y(t) = 3\sin^2 t,\ 0 \le t \le 2\pi$

(B) $x(t) = 2\cos^3 t,\ y(t) = 3\sin^3 t,\ 0 \le t \le 2\pi$

(C) $x(t) = 3\cos^3 t,\ y(t) = 2\sin^2 t,\ 0 \le t \le 2\pi$

(D) $x(t) = 3\cos^3 t,\ y(t) = 2\sin^3 t,\ 0 \le t \le 2\pi$

20. Find the length of the curve with parametric equations $x(\theta) = \cos\theta + \theta\sin\theta$ and $y(\theta) = \sin\theta - \theta\cos\theta$ for $0 \le \theta \le 2\pi$.

(A) 1 (B) π (C) 2π (D) π^2 (E) $2\pi^2$

21. Convert the polar equation $r = 4\cos\theta$ into a rectangular equation.

(A) $x^2 + y^2 = 4$

(B) $(x-2)^2 + y^2 = 4$

(C) $x^2 + (y-2)^2 = 4$

(D) $(x-2)^2 + (y-2)^2 = 4$

22. Which of the following integrals gives the area of the inner loop of $r = 1 + 2\sin\theta$?

(A) $\dfrac{1}{2}\displaystyle\int_{\pi/6}^{5\pi/6} [1 + 2\sin\theta]^2 \, d\theta$

(B) $\dfrac{1}{2}\displaystyle\int_{\pi/3}^{2\pi/3} [1 + 2\sin\theta]^2 \, d\theta$

(C) $\dfrac{1}{2}\displaystyle\int_{7\pi/6}^{11\pi/6} [1 + 2\sin\theta]^2 \, d\theta$

(D) $\dfrac{1}{2}\displaystyle\int_{4\pi/3}^{5\pi/3} [1 + 2\sin\theta]^2 \, d\theta$

Common Maclaurin Series and Formulas

April 11, 2022

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!},\ IOC = (-\infty, \infty)$$

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!},\ IOC = (-\infty, \infty)$$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!},\ IOC = (-\infty, \infty)$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n,\ IOC = (-1, 1)$$

$$\arctan(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1},\ IOC = [-1, 1]$$

$$\ln(1+x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1},\ IOC = (-1, 1]$$

Taylor Remainder Estimate:

$$|R_N(x)| \leq \frac{M|x-a|^{N+1}}{(N+1)!},$$

where $|f^{(N+1)}(c)| \leq M$ for any c in between x and a.

Taylor Coefficients:

$$a_n = \frac{f^{(n)}(a)}{n!}$$