Name: Solutions **Section #:** ______

UF-ID: ______ **TA Name:** ______

A: Sign the back of your scantron sheet.

B: On the indicated spaces on the front of your scantron, **in pencil**, write and encode:

1. Your name (last name, first initial, middle initial)
2. Your UFID Number
3. Your 4-digit Section Number

C: Under "special codes", code in the test ID number 1,1:

• 2 3 4 5 6 7 8 9 0
• 2 3 4 5 6 7 8 9 0

D. At the top right of your scantron, for "Test Form Code", encode A .

• B C D E

E: Some basic information about the exam:

1. This exam has two parts: a 10-question multiple choice section worth 50 points, and a 4-question free response section worth 55 points. The entire exam is out of 100 points.
2. You will have 100 minutes to take the exam.
3. You may write on your exam.
4. Raise your hand if you need more scratch paper or if you have a problem with the test. DO NOT LEAVE YOUR SEAT UNLESS YOU ARE FINISHED WITH THE EXAM.

F: KEEP YOUR SCANTRON COVERED AT ALL TIMES

G: When you are finished:

1. Before turning in your test, check for transcribing errors. Any mistakes you leave in are there to stay.
2. Turn in your scantron and the free-response portion of your exam to your TA, or the proctor designated as your TA. Be prepared to show your UF-ID card.
3. Solutions to the exam will be posted on Canvas after the exam is over.

The Honor Pledge: "On my honor, I have neither given nor received unauthorized aid doing this exam."

Signature: ______

Part I: Multiple Choice There are **10** questions on this portion of the exam. Fill in the answers to these questions on the provided scantron sheet. Only answers on the scantron will be graded. Each problem is worth 5 points, for a total of 50 points on this portion of the exam.

1. Evaluate $\int \sin^3 x \cos^4 x\, dx$. $= \int \sin^2 \cos^4 x \cdot \sin x\, dx$

$= \int (1-\cos^2 x) \cos^4 x \cdot \sin x\, dx$ $\quad u = \cos x \Rightarrow du = -\sin x\, dx$, $-du = \sin x\, dx$

$= \int -(1-u^2)\, u^4\, du$

$= \int (u^6 - u^4)\, du$

$= \frac{u^7}{7} - \frac{u^5}{5} + C$

$= \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C$

(A) $-\frac{\sin^4 x \cos^5 x}{20} + C$

(B) $-\frac{\sin^7 x}{7} + \frac{\sin^5 x}{5} + C$

(C) $-\frac{\cos^7 x}{7} + \frac{\cos^5 x}{5} + C$

(D) $\frac{\sin^7 x}{7} - \frac{\sin^5 x}{5} + C$

(E) $\frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C$

2. Which of the following is the correct form of the partial fraction decomposition of $f(x) = \frac{x^2}{x(x+1)^3(x^2+2)^2}$?

(A) $\frac{A}{x} + \frac{B}{x+1} + \frac{C+D}{x^2+2}$

(B) $\frac{A}{x} + \frac{B}{(x+1)^3} + \frac{Cx+D}{(x^2+2)^2}$

(C) $\frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2} + \frac{D}{(x+1)^3} + \frac{E}{x^2+2} + \frac{F}{(x^2+2)^2}$

(D) $\frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2} + \frac{D}{(x+1)^3} + \frac{Ex+F}{x^2+2} + \frac{Gx+H}{(x^2+2)^2}$

(E) $\frac{A}{x} + \frac{B}{x+1} + \frac{Cx+D}{(x+1)^2} + \frac{Ex^2+Fx+G}{(x+1)^3} + \frac{Hx+I}{x^2+2} + \frac{Jx^3+Kx^2+Lx+M}{(x^2+2)^2}$

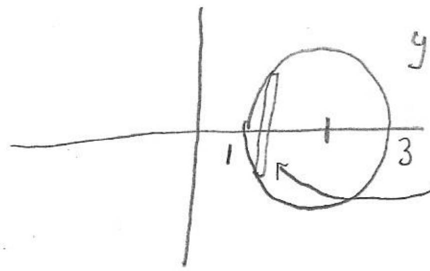
3. An innertube is formed by rotating the circle $(x-2)^2 + y^2 = 1$ about the y-axis. Which of the following integrals gives the volume of the innertube using the **Shell Method**?

(A) $\pi \int_1^3 (1-(x-2)^2)\,dx$

(B) $2\pi \int_1^3 x(1-(x-2)^2)\,dx$

(C) $4\pi \int_1^3 \sqrt{1-(x-2)^2}\,dx$

(D) $4\pi \int_1^3 x\sqrt{1-(x-2)^2}\,dx$



$y = \sqrt{1-(x-2)^2}$

height $= 2y$
$= 2\sqrt{1-(x-2)^2}$

$(x-2)^2 + y^2 = 1 \Rightarrow y^2 = 1-(x-2)^2$

$\Rightarrow y = \pm\sqrt{1-(x-2)^2}$

$$\text{Volume} = 2\pi \int_1^3 x \cdot 2\sqrt{1-(x-2)^2}\,dx$$

$$= 4\pi \int_1^3 x\sqrt{1-(x-2)^2}\,dx$$

4. Evaluate $\int e^{2x} \sin x\,dx$.

(A) $\frac{1}{5}e^{2x}(\sin x - 2\cos x) + C$

(B) $\frac{1}{5}e^{2x}(\cos x - 2\sin x) + C$

(C) $\frac{1}{5}e^{2x}(2\sin x - \cos x) + C$

(D) $\frac{1}{5}e^{2x}(2\cos x - \sin x) + C$

$u = e^{2x}$ $dv = \sin x\,dx$
$du = 2e^{2x}dx$ $v = -\cos x$ dx

$$\int e^{2x}\sin x\,dx = -e^{2x}\cos x + \int 2e^{2x}\cos x\,dx$$

$u = 2e^{2x}$ $dv = \cos x\,dx$
$du = 4e^{2x}dx$ $v = \sin x$

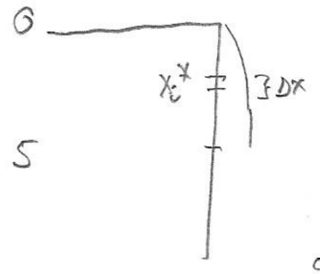
$$\int e^{2x}\sin x\,dx = -e^{2x}\cos x + \left[2e^{2x}\sin x - 4\int e^{2x}\sin x\,dx\right]$$

$$5\int e^{2x}\sin x\,dx = -e^{2x}\cos x + 2e^{2x}\sin x + c$$

$$\int e^{2x}\sin x\,dx = \frac{1}{5}e^{2x}\left[-\cos x + 2\sin x\right] + C$$

5. Suppose a chain of length 5 meters hangs down the side of a 20 meter building and has a mass density of 20 kg/m. Find the amount of work (in joules) it will take to haul the chain up the side of the building to the roof. Assume the acceleration due to gravity is g.

(A) $125g$ (B) $250g$ (C) $750g$ (D) $1250g$ (E) $1750g$



W_i = Force * Distance

= mass * g * Distance

$= 20\,\Delta x \cdot g \cdot x_i^*$

$$\text{Work} = \int_0^5 20gx\,dx = 10gx^2\Big|_0^5 = 10g\cdot 25 - 10g\cdot 0 = 250g$$

6. Evaluate $\int \sec^6 x \tan^2 x\, dx$.

$= \int \sec^4 x \tan^2 x \cdot \sec^2 x\, dx$

(A) $\frac{\sec^7 x}{7} + C$

(B) $\frac{\tan^5 x}{5} + \frac{\tan^3 x}{3} + C$

(C) $\frac{\tan^7 x}{7} + \frac{2\tan^5 x}{5} + \frac{\tan^3 x}{3} + C$

(D) $\frac{\sec^5 x}{5} + \frac{\sec^3 x}{3} + C$

(E) $\frac{\sec^7 x}{7} + \frac{2\sec^5 x}{5} + \frac{\sec^3 x}{3} + C$

$= \int (\tan^2 x + 1)^2 \tan^2 x \cdot \sec^2 x\, dx$

$u = \tan x$

$\Rightarrow du = \sec^2 x\, dx$

$= \int (u^2+1)^2 \cdot u^2\, du$

$= \int u^2(u^4 + 2u^2 + 1)\, du$

$= \int (u^6 + 2u^4 + u^2)\, du$

$= \frac{u^7}{7} + \frac{2u^5}{5} + \frac{u^3}{3} + C$

$= \frac{\tan^7 x}{7} + \frac{2\tan^5 x}{5} + \frac{\tan^3 x}{3} + C$

7. Evaluate $\int \sqrt{4x^2 - 1}\,dx$.

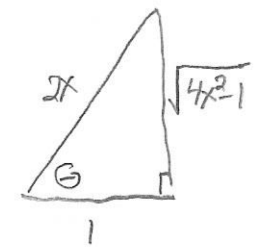
(A) $\frac{1}{2}x\sqrt{4x^2-1} + \frac{1}{4}\ln\left|2x + \sqrt{4x^2-1}\right| + C$
(B) $\frac{1}{2}x\sqrt{4x^2-1} - \frac{1}{4}\ln\left|2x + \sqrt{4x^2-1}\right| + C$
(C) $\frac{1}{4}\arcsin(2x) + \frac{1}{2}x\sqrt{4x^2-1} + C$
(D) $\frac{1}{4}\arcsin(2x) - \frac{1}{2}x\sqrt{4x^2-1} + C$

$2x = \sec\theta$

$\Rightarrow 3dx = \sec\theta\tan\theta\, d\theta$

$\Rightarrow dx = \frac{1}{2}\sec\theta\tan\theta\, d\theta$

$\sqrt{4x^2-1} = \tan\theta$



$$\int\sqrt{4x^2-1}\,dx = \int \tan\theta \cdot \frac{1}{3}\sec\theta\tan\theta\, d\theta$$
$$= \frac{1}{2}\int \sec\theta\tan^2\theta\, d\theta$$
$$= \frac{1}{2}\int \sec\theta(\sec^2\theta - 1)\, d\theta$$
$$= \frac{1}{2}\int(\sec^3\theta - \sec\theta)\,d\theta$$
$$= \frac{1}{2}\cdot\frac{1}{2}(\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|) - \frac{1}{2}\ln|\sec\theta + \tan\theta| + C$$
$$= \frac{1}{4}\sec\theta\tan\theta - \frac{1}{4}\ln|\sec\theta + \tan\theta| + C = \frac{1}{4}\cdot 2x\sqrt{4x^2-1} - \frac{1}{4}\ln\left|2x + \sqrt{4x^2-1}\right| + C$$

8. What is the sum of all of the coefficients of the partial fraction decomposition of $\frac{3x^2+4x+4}{x^3+2x^2+2x}$?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

$$\frac{3x^2+4x+4}{x^3+2x^2+2x} = \frac{3x^2+4x+4}{x(x^2+2x+2)} = \frac{A}{x} + \frac{Bx+C}{x^2+2x+2}$$

cover up $\Rightarrow A = \frac{4}{2} = 2$

irreducible

$$3x^2+4x+4 = 2(x^2+2x+2) + (Bx+C)x = 2x^2+4x+4+Bx^2+Cx$$
$$3x^2+4x+4 = (2+B)x^2 + (4+C)x + 4$$
$$\Rightarrow 3 = 2+B \Rightarrow B = 1$$
$$4 = 4+C \Rightarrow C = 0$$

$$A+B+C = 2+1+0 = 3$$

9. Evaluate $\int_0^\infty x^2 e^{-x}\,dx$.

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

$$\int x^2 e^{-x}\,dx$$

u		dv
x^2	+	e^{-x}
$2x$	-	$-e^{-x}$
2	+	e^{-x}
0	-	$-e^{-x}$

$$= -x^2e^{-x} - 2xe^{-x} - 2e^{-x} + C$$

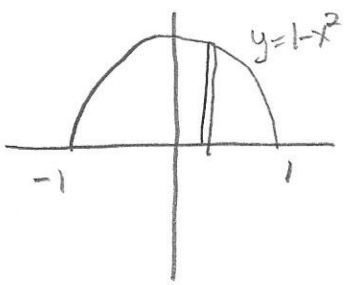
$$\int_0^\infty x^2e^{-x}dx = \lim_{t\to\infty}\int_0^t x^2e^{-x}dx$$

$$= \lim_{t\to\infty}\left[-x^2e^{-x} - 2xe^{-x} - 2e^{-x}\right]_0^t$$

$$= \lim_{t\to\infty}\left[(-t^2e^{-t} - 2te^{-t} - 2e^{-t}) - (0 - 0 - 2e^0)\right] = 2$$

10. Consider the region R bounded by the curve $y = 1 - x^2$ and the x-axis. Let A be the volume of the solid formed by revolving R about x-axis. Let B be the volume of the solid whose base is R and every cross section perpendicular to the x-axis is a square. What is the value of $\frac{A}{B}$?

(A) $\frac{1}{2\pi}$ (B) $\frac{1}{\pi}$ (C) 1 (D) π (E) 2π



$$A = \pi\int_{-1}^{1}(1-x^2)^2\,dx$$

$$B = \int_{-1}^{1}(1-x^2)^2\,dx$$

Therefore $\frac{A}{B} = \pi$

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Part II: Free Response There are **4** questions on this portion of the exam. Show **ALL** work clearly in the space provided for each problem, unless the problem says otherwise. Your work must be complete, logical, and understandable or it will receive no credit. Please cross out or fully erase any work that you do not want graded. A total of **55** points are available on this portion of the exam.

FR Scores	
1	/10
2	/15
3	/15
4	/15
FR Total	/55

1. Evaluate $\int x^5 \sin(x^3)\, dx$.

$$\int x^5 \sin(x^3)\, dx = \int x^3 \cdot x^2 \sin(x^3)\, dx \qquad u = x^3 \Rightarrow du = 3x^2 dx$$

$$\tfrac{1}{3} du = x^2 dx$$

$$= \int u \cdot \tfrac{1}{3} \sin u \, du \qquad w = \frac{u}{3} \qquad dv = \sin u \, du$$

$$dw = \tfrac{1}{3} du \qquad v = -\cos u$$

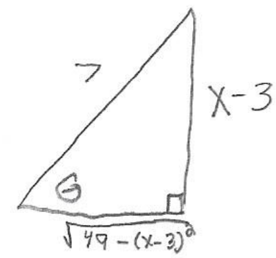
$$= -\tfrac{1}{3} u \cos u + \int \tfrac{1}{3} \cos u \, du$$

$$= -\tfrac{1}{3} u \cos u + \tfrac{1}{3} \sin u + C$$

$$= \boxed{-\tfrac{1}{3} x^3 \cos(x^3) + \tfrac{1}{3} \sin(x^3) + C}$$

2. Evaluate $\int \sqrt{40+6x-x^2}\,dx$.

$$40+6x-x^2 = -(x^2-6x-40)$$
$$= -(x^2-6x+9-9-40)$$
$$= -((x-3)^2-49)$$
$$= 49-(x-3)^2$$



$$\int \sqrt{40+6x-x^2}\,dx = \int \sqrt{49-(x-3)^2}\,dx$$

$x-3 = 7\sin\theta$

$dx = 7\cos\theta\,d\theta$

$\sqrt{49-(x-3)^2} = 7\cos\theta$

$$= \int 7\cos\theta \cdot 7\cos\theta\,d\theta$$
$$= 49\int \cos^2\theta\,d\theta$$
$$= \frac{49}{2}\int (1+\cos(2\theta))\,d\theta$$
$$= \frac{49}{2}\left[\theta + \frac{\sin(2\theta)}{2}\right] + C$$

$\sin\theta = \frac{x-3}{7} \Rightarrow \theta = \arcsin\left(\frac{x-3}{7}\right)$

$$= \frac{49}{2}\left[\theta + \frac{2\sin\theta\cos\theta}{2}\right] + C$$

$\cos\theta = \frac{\sqrt{40+6x-x^2}}{7}$

$$= \boxed{\frac{49}{2}\left[\arcsin\left(\frac{x-3}{7}\right) + \frac{(x-3)\sqrt{40+6x-x^2}}{49}\right] + C}$$

3. Calculate the area of the region bounded by the curve $y = \dfrac{8}{x^3+4x}$ and the x-axis for $x \geq 1$.

$$\frac{8}{x^3+4x} = \frac{8}{x(x^2+4)} \to \frac{A}{x} + \frac{Bx+C}{x^2+4} \quad \text{cover up} \Rightarrow A = \frac{8}{4} = 2$$

$$8 = 2(x^2+4) + (Bx+C)x = 2x^2+8+Bx^2+Cx$$

$$0x^2+0x+8 = (2+B)x^2+Cx+8$$

$$\Rightarrow 0 = 2+B \Rightarrow B = -2$$

$$0 = C$$

$$\text{Area} = \int_1^\infty \frac{8}{x^3+4x}\,dx$$

$$= \lim_{t\to\infty} \int_1^t \frac{8}{x^3+4x}\,dx$$

$$= \lim_{t\to\infty} \int_1^t \left(\frac{2}{x} - \frac{2x}{x^2+4}\right) dx$$

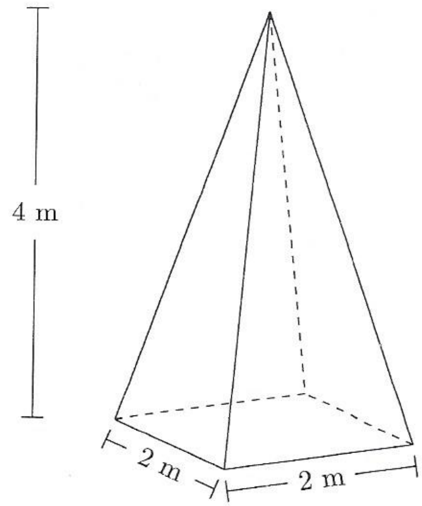
$$= \lim_{t\to\infty} \Big[2\ln|x| - \ln(x^2+4)\Big]_1^t$$

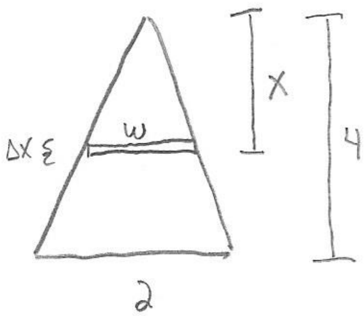
$$= \lim_{t\to\infty} \Big(\big[2\ln|t| - \ln(t^2+4)\big] - \big[2\ln 1 - \ln 5\big]\Big)$$

$$= \lim_{t\to\infty} \left[\ln\left(\frac{t^2}{t^2+4}\right) + \ln 5\right]$$

$$= \ln 1 + \ln 5 = \boxed{\ln 5}$$

4. A water tank is shaped like a square pyramid with edge of length 2 meters and height 4 meters. See the figure below. If the tank is completely full of water, then how much work is required to pump all the water out of the top of the tank? Express your answer in terms of ρ and g.





By similar triangles, $\frac{x}{4} = \frac{w}{2} \Rightarrow w = \frac{x}{2}$

Volume of slice of water $= w^2 \Delta x = \left(\frac{x}{2}\right)^2 \Delta x = \frac{x^2}{4} \Delta x$

$$\text{Work} = \int_0^4 \rho g \cdot x \cdot \frac{x^2}{4} dx = \rho g \int_0^4 \frac{x^3}{4} dx$$

$$= \rho g \left[\frac{x^4}{16} \right]_0^4$$

$$= \rho g \cdot \frac{4^4}{16}$$

$$= \boxed{16 \rho g}$$