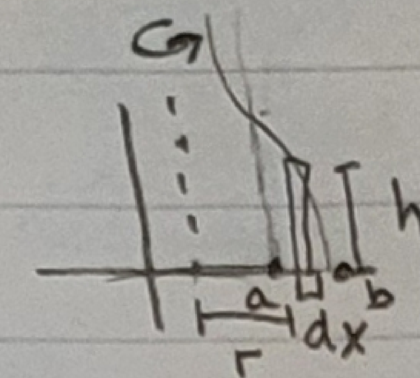
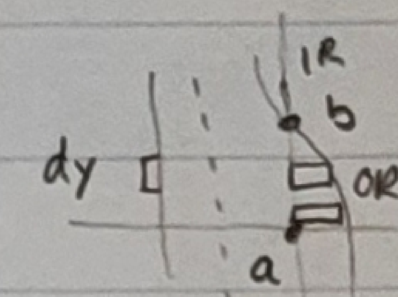
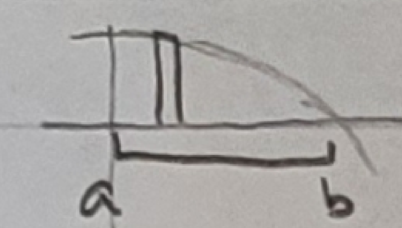
shell: $2\pi \int (\text{radius})(\text{height})\, dx$



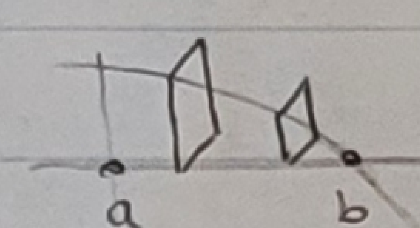
washer: $\pi \int_a^b OR^2 - IR^2\, dy$



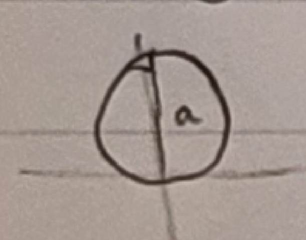
disk: $\pi \int_a^b f^2(x)\, dx$



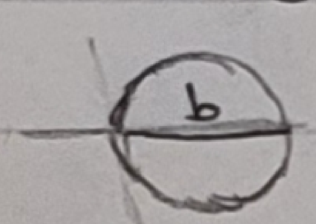
area: $\int_a^b A(x)\, dx$



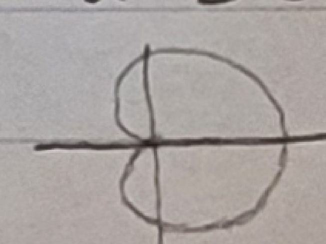
$r = a\sin\theta$



$r = b\cos\theta$

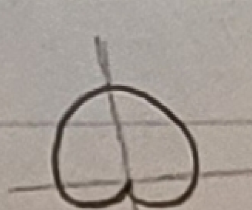


$r = a + b\cos\theta$

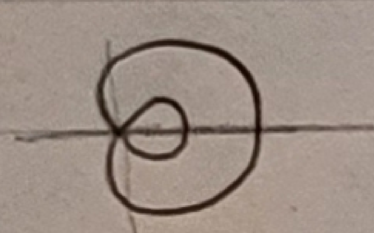


$a = b$

$r = a + b\sin\theta$

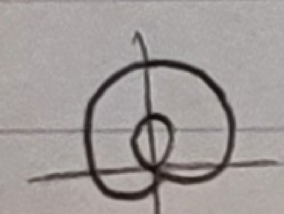


$r = a + b\cos\theta$

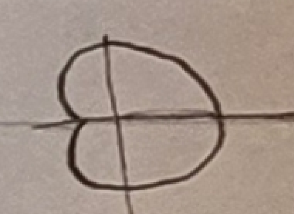


$a < b$

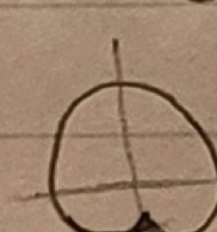
$r = a + b\sin\theta$



$r = a + b\cos\theta$



$r = a + b\sin\theta$



polar $\frac{1}{2} \int_a^b r^2\, d\theta$

Test	Series	Condition(s) of Convergence	Condition(s) of Divergence	Comment
Sec 9.2 ~~nth-Term~~ Test for Div.	$\sum_{n=1}^{\infty} a_n$		$\lim_{n\to\infty} a_n \neq 0$	This test cannot be use to show convergence. only confirms Divergence
Sec 9.2 Geometric Series	$\sum_{n=0}^{\infty} ar^n$	$\lvert r\rvert < 1$	$\lvert r\rvert \geq 1$	Sum: $S = \frac{a}{1-r}$
Sec 9.2 Telescoping Series	$\sum_{n=1}^{\infty} (b_n - b_{n+1})$	$\lim_{n\to\infty} b_n = L$		Sum: $S = b_1 - L$ $= b_1 - \lim_{n\to\infty} b_n$
Sec 9.3 p-Series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$p > 1$	$0 < p \leq 1$	
Sec 9.5 Alternating Series	$\sum_{n=1}^{\infty} (-1)^{n-1} a_n$	$0 < a_{n+1} \leq a_n$ and $\lim_{n\to\infty} a_n = 0$		Remainder: $\lvert R_N\rvert \leq a_{N+1}$
Integral Sec 9.3 (f is continuous, positive, and decreasing)	$\sum_{n=1}^{\infty} a_n$, $a_n = f(n) \geq 0$	$\int_1^{\infty} f(x)\,dx$ converges	$\int_1^{\infty} f(x)\,dx$ diverges	Remainder: $0 < R_N < \int_N^{\infty} f(x)\,dx$
Sec 9.6 Root	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} < 1$	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} > 1$ or $= \infty$	Test is inconclusive if $\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} = 1$.
Sec 9.6 Ratio	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert < 1$	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert > 1$ or $= \infty$	Test is inconclusive if $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = 1$.
Sec 9.4 Direct Comparison ($a_n, b_n > 0$)	$\sum_{n=1}^{\infty} a_n$	$0 < a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n$ converges	$0 < b_n \leq a_n$ and $\sum_{n=1}^{\infty} b_n$ diverges	
Sec 9.4 Limit Comparison ($a_n, b_n > 0$)	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ and $\sum_{n=1}^{\infty} b_n$ converges	$\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ and $\sum_{n=1}^{\infty} b_n$ diverges	

MAC2312 Final Review Work

Wednesday, December 9, 2020 12:09 PM

① $\sum \frac{n^d}{1+\sqrt{n}}$ $\qquad d = -\frac{1}{2} \longrightarrow \sum \frac{1}{n^{1/2}(1+n^{1/2})}$

$\frac{n^d}{n^{1/2}}$ $\qquad \frac{1}{2} - d > 1$

$d < -\frac{1}{2}$ $\qquad \boxed{d}$

② $\int_0^2 x^2 e^{x/2}\,dx$

u	dv
x^2 +	$e^{x/2}$
$2x$ −	$2e^{x/2}$
2 +	$4e^{x/2}$
0	$8e^{x/2}$

$2x^2 e^{x/2} - 8x e^{x/2} + 16 e^{x/2} \Big|_0^2 = 8e^1 - \cancel{16e} + \cancel{16e} - (16)$

$8e - 16$ $\qquad \boxed{b}$

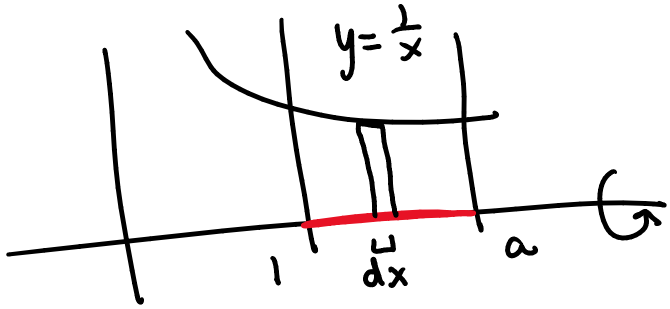
③ $x = \sin(\pi t)$ $\qquad y = t^2 - t - 3$ $\qquad \frac{dy}{dt} = 0$ —

$\frac{dx}{dt} = \pi \cos \pi t$ $\qquad \frac{dy}{dt} = 2t - 1$ $\qquad \frac{dx}{dt} = 0$ |

$\pi \cos\left(\frac{3\pi}{2}\right) = 0$ $\qquad 2 \cdot \frac{3}{2} - 1 = 2$

$\boxed{a}$

④



disk $\quad \pi \int_a^b f(x)^2 \, dx$

$\boxed{c}$

$$\pi \int_1^a \left(\frac{1}{x}\right)^2 dx$$

⑤

$$\frac{2x^4+16}{(x^2+1)\,x^2}$$

x^4+x^2

$$x^4+x^2 \,\overline{\big)\, 2x^4+16}^{\;2}$$

$-(2x^4+2x^2)$

$-2x^2+16$

$\boxed{e}$

$$2+\frac{-2x^2+16}{(x^2+1)\,x^2}$$

$$\hookrightarrow \frac{Ax+B}{(x^2+1)}+\frac{C}{x}+\frac{D}{x^2}$$

⑥ $\int_1^e (\ln x)^2\,dx$

u	dv
$(\ln x)^2$	1
$\frac{2\ln x}{x}$	x

$x(\ln x)^2 - 2\int \frac{\ln x}{\cancel{x}}\cdot \cancel{x}\,dx$

$x(\ln x)^2 - 2(x\ln x - x)\Big|_1^e$

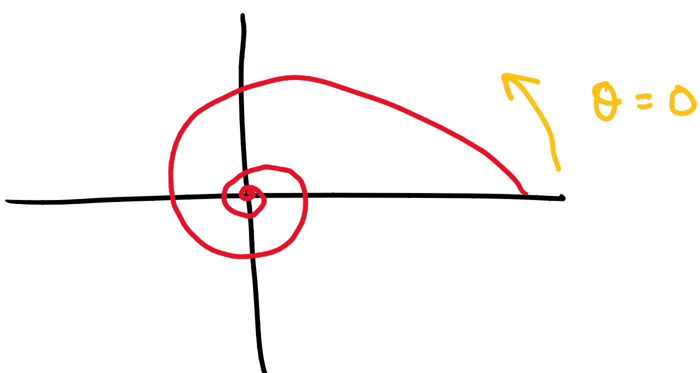
u	dv
$\ln x$	1
$\frac{1}{x}$	x

$e(1)^2 - \cancel{2(e-e)}^{\,0} - \left[1(0) - 2(1(0)-1)\right]$

$-(-2)(-1)$

$\boxed{e-2}$ $\boxed{e}$

⑦



	$r(0)=3$	$\lim_{\theta\to\infty} r = 0$
~~a)~~ $r^2=3\cos\theta$	✓	✗
~~b)~~ $r = 3+\theta$	✓	✗
$\boxed{c)}$ $r = \frac{3}{1+\theta}$	✓	✓

$\boxed{c)}\ r = \frac{3}{1+\theta}$

$\cancel{d)}\ r = \sin\theta + 3$

$\cancel{e)}\ r = 3\cos 8\theta$

⑧ $\displaystyle\int_3^\infty \frac{dx}{x\ \ln x\ \ln(\ln x)}$

$u = \ln(\ln x)$; $\infty \to \infty$; $3 \to \ln(\ln(3))$

$du = \frac{1}{\ln x \cdot x}\ dx$

$$\int_{\ln(\ln 3)}^{\infty} \frac{1}{u}\ du$$

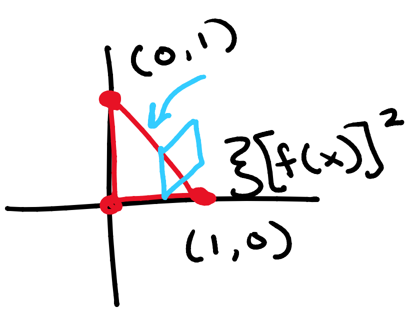
$$\ln u \Big|_{\ln(\ln 3)}^{\infty} = \lim_{u\to\infty} \ln u - \ln(\ln(\ln 3))$$

∞

$$\ln(\ln(\ln x))\Big|_3^\infty$$

$\boxed{\text{diverges}}$ $\boxed{d}$

⑨



$$V = \int_a^b A(x)\,dx$$

$$\int^1 (1-x)^2\,dx$$

$(1,0)$

$y = 1 - x$

$(y-0) = m(x-1)$

$1 - 0 = m(0-1)$

$\boxed{m = -1}$

$y = -x + b$

$0 = -1 + b$

$\boxed{b = 1}$

$$\int_0^1 (1-x)^2\,dx$$

$$-\frac{(1-x)^3}{3}\Bigg|_0^1 = 0 - \left(-\frac{1}{3}\right)$$

$$= \boxed{\frac{1}{3}}$$

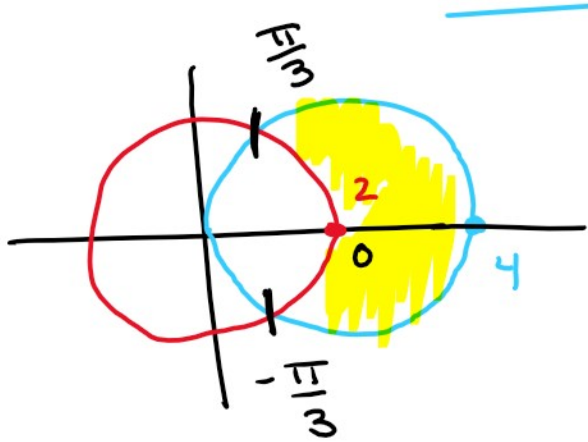
$\boxed{a}$

(10) $A = \frac{1}{2}\int_a^b r^2\,d\theta$

$r = 4\cos\theta$

$x^2 + y^2 = 4$

$r = 2$



$4\cos\theta = 2$

$\cos\theta = \frac{1}{2}$

$$2\ \frac{1}{2}\int_0^{\frac{\pi}{3}} (4\cos\theta)^2 - 2^2\,d\theta$$

$$\boxed{\int_0^{\frac{\pi}{3}} 16\cos^2\theta - 4\,d\theta}$$

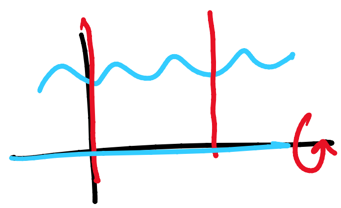
$\boxed{b}$

$$\frac{1}{\ }\int^{\frac{\pi}{3}} (4\cos\theta)^2 - 2^2\,d\theta$$

$$\frac{1}{2}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} (4\cos\theta)^2 - 2^2 \, d\theta$$

$$\frac{1}{2}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} 16\cos^2\theta - 4 \, d\theta$$

11



disk $\quad \pi \int_0^4 f(x)^2 \, dx$

$\boxed{d}$

12 $\quad \int \frac{1}{(x^2+1)^2} dx = A \arctan x + B \frac{x}{x^2+1} + C$

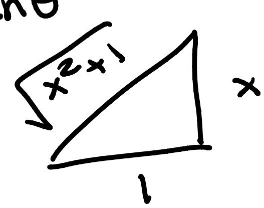
$x = \tan\theta$

$dx = \sec^2\theta \, d\theta$

$$\int \frac{\sec^2\theta \, d\theta}{(\sec^2\theta)^2} = \int \frac{1}{\sec^2\theta} d\theta = \int \cos^2\theta \, d\theta$$

$$\int \frac{1}{2}(1+\cos 2\theta) \, d\theta$$

$\frac{x}{1} = \tan\theta$



$\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta$

$\uparrow$

$\frac{1}{2}\theta + \frac{1}{4}\cdot 2\sin\theta\cos\theta$

$2\theta \cdot 4$

$$\frac{1}{2}\arctan x + \frac{1}{2}\cdot\frac{x}{\sqrt{x^2+1}}\cdot\frac{1}{\sqrt{x^2+1}}$$

$$\underbrace{\frac{1}{2}}\arctan x + \underbrace{\frac{1}{2}}\frac{x}{x^2+1}$$

$$\frac{1}{2}+\frac{1}{2} = \boxed{1} \qquad \boxed{b}$$

(13)

$$\frac{1}{1\cdot 4} - \frac{1}{2\cdot 4^2} + \frac{1}{3\cdot 4^3} - \frac{1}{4\cdot 4^4}$$

1 2 3 4

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\left(\frac{1}{4}\right)^n$$

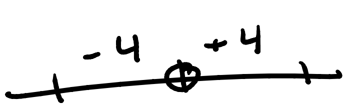
$\uparrow x$

$$\ln(1+x) \longrightarrow \ln\left(1+\frac{1}{4}\right) = \boxed{\ln\left(\frac{5}{4}\right)}$$

$\boxed{b}$

(14)

$F(x) \qquad R = 4$



$x < 4$

a) $F(3x) \qquad 3x < 4 \qquad r = \frac{4}{3}$

$x < \frac{4}{3}$

✓ b) $F'(x)$ $R = 4$

✓ c) $\int F(x)\,dx$ $R = 4$

✓ d) $F(\frac{x}{4})$ $\frac{x}{4} < 4$ $R = 16$

$x < 16$

× $\boxed{e)}$ $x F(3x)$ $\cancel{x}\ 3x < 4$

$\cancel{x} < \sqrt{\frac{4}{3}}$

(15) $x(t) = 3\sin t$ $y(t) = 4\cos t$

ellipse

$t = 0$ $x = 0$, $y = 4$

clockwise

$t = \frac{\pi}{2}$ $x = 3$, $y = 0$

$\boxed{b}$

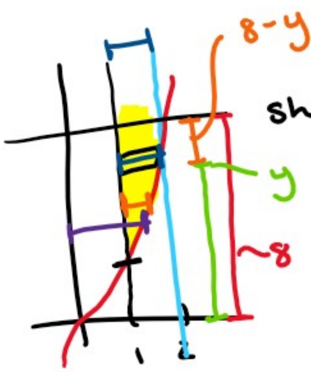
$t = 0$ $\sin t \rightarrow 0$

$\cos 2t \rightarrow 1$

$t = \frac{\pi}{2}$ $\sin t = 1$

$\cos t = -1$

(16)



shell: $2\pi \int (\text{radius})(\text{height})\,dy$

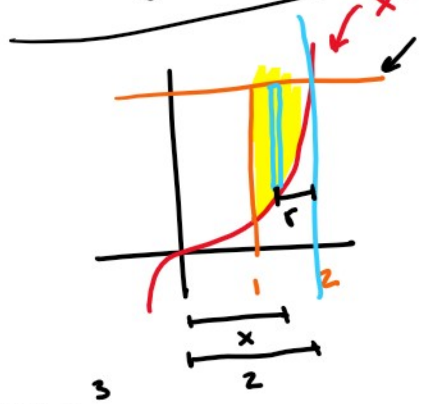
$2\pi \int_1^8 (8-y)\,(\sqrt[3]{y}-1)\,dy$

rotating about 8

$y = x^3$

$\sqrt[3]{4}$

$\sqrt[3]{y}$



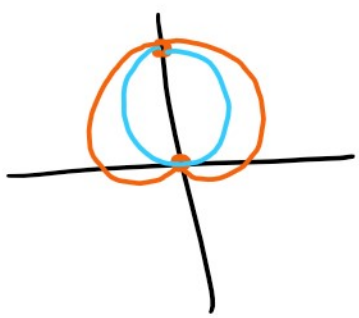
$y = x^3$

(radius)(height)

$$2\pi\int_1^2 (2-x)(8-x^3)\,dx$$

$\boxed{d}$

17 $r = 2+2\sin\theta$ $r = 4\sin\theta$



$\boxed{2}$ $\boxed{c}$

18 $f(x) = \sin x$ center @ $\frac{\pi}{4}$

$\cancel{\sin\left(x - \frac{\pi}{4} + \frac{\pi}{4}\right)}$

n	$f^{(n)}(a)$	$f^{(n)}\frac{\pi}{4}$
0	$\sin x$	$\frac{\sqrt{2}}{2}$
1	$\cos x$	$\frac{\sqrt{2}}{2}$
2	$-\sin x$	$-\frac{\sqrt{2}}{2}$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)(x-a)^n}{n!}$$

$\frac{\pi}{5}$ $\frac{4\pi}{20} - \frac{5\pi}{20}$ $\frac{\pi}{5}$

$$\frac{\frac{\sqrt{2}}{2}\cancel{\left(x-\frac{\pi}{4}\right)^0}}{} + \frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right)^1 - \frac{\frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right)^2}{}$$

$$\frac{\frac{\sqrt{2}}{2}\cancel{\left(x-\frac{\pi}{4}\right)^0}}{\cancel{0!}} + \frac{\frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right)^1}{\cancel{1!}} - \frac{\frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right)^2}{2!}$$

$$\frac{\sqrt{2}}{2} + -\frac{\pi}{20}\frac{\sqrt{2}}{2} - \frac{\frac{\sqrt{2}}{2}\left(-\frac{\pi}{20}\right)^2}{2}$$

$$\frac{\sqrt{2}}{2} - \frac{\pi\sqrt{2}}{40} - \frac{\sqrt{2}\,\pi^2}{1600} \qquad \boxed{d}$$

$\cancel{\sin(A+B) = \sin A \cos B + \cos A \sin B}$

(19)

$$\frac{dy/dt}{dx/dt} = 0$$

$$y = \frac{t^3}{3} - t$$

$$\frac{dy}{dt} = t^2 - 1$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d\left(\frac{dy}{dx}\right)}{dt}}{\frac{dx}{dt}}$$

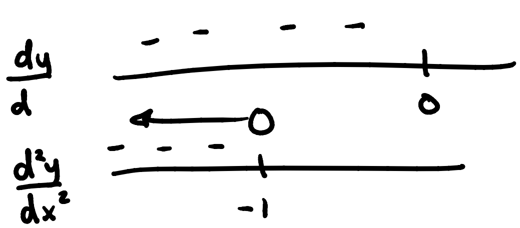
$$x = \frac{t^2}{2} + t$$

$$\frac{dx}{dt} = t + 1$$

$$\frac{dy/dt}{dx/dt} = \frac{\overset{\cancel{(t+1)}(t-1)}{t^2-1}}{\cancel{t+1}} = t - 1 = 0$$

$$t = 1$$

$$\frac{d^2y}{dx^2} = \frac{1}{t+1} \qquad t < -1$$

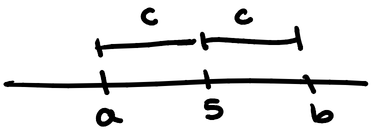


$t < -1$

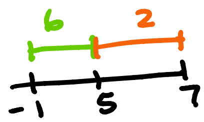
$\boxed{a}$

$$\frac{d^2y}{dx^2} = \frac{\frac{dx}{dt}\frac{d^2y}{dt^2} - \frac{d^2x}{dt^2}\frac{dy}{dt}}{\left(\frac{dx}{dt}\right)^3}$$

(20) $\sum_{n=1}^{\infty} c_n (x-5)^n$



~~a)~~ $\overset{a\ \ b}{[-1,7]}$ $c_a = 6$ $c_b = 2$



~~b)~~ $[-4,-2]$

$\boxed{c)}$ $[-5,15]$ $c_a = 10$ $c_b = 10$

~~d)~~ $(4,7]$ $c_a = 1$ $c_b = 2$

e) $[3,6)$ $c_a = 2$ $c_b = 1$

(21) $\int \frac{e^{2x}}{(e^{2x}+1)(e^x+1)} dx$ $\quad e^{2x} = e^x \cdot e^x$

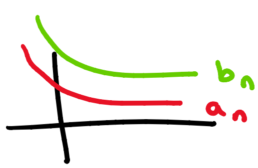
$u = e^x$

$du = e^x\,dx$

$\int \frac{u}{(u^2+1)(u+1)} du$

$= \frac{Au+B}{u^2+1} + \frac{C}{u+1}$

(22) ~~a)~~

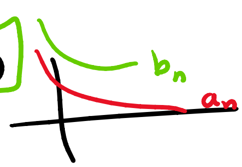


~~b)~~ inconclusive

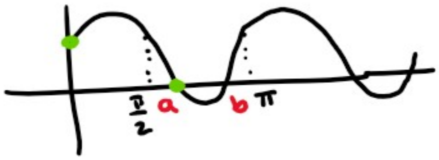
~~c)~~ inconclusive

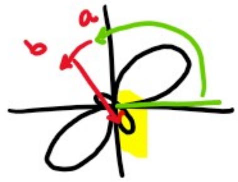
~~d)~~

$\boxed{e)}$



1





$r = \frac{1}{2} + \sin 2\theta$

$0 = \frac{1}{2} + \sin 2\theta$

$-\frac{1}{2} = \sin 2\theta$

$\frac{1}{2}\int_a^b r^2 \, d\theta$

$\downarrow$

$\frac{7\pi}{6}, \frac{11\pi}{6} = 2\theta$

$\frac{7\pi}{12}, \frac{11\pi}{12}$

$$\frac{1}{2}\int_{\frac{7\pi}{12}}^{\frac{11\pi}{12}} r^2 \, d\theta$$

$\boxed{b}$

2 $\int_e^\infty \frac{(\ln x)^2}{x^2} \, dx$

u	dv
$(\ln x)^2$	$\frac{1}{x^2}$

3 $\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!}\left(\frac{\pi}{2}\right)^{2n+1} + \left(\frac{1}{2}\right)^n$

$\uparrow$

x

$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$ $\qquad \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$

$\sum_{n=1}^{\infty} = \sum_{n=0}^{\infty} - S_0$

$S_1 + S_2 + S_3 \dots = \cancel{S_0} + S_1 + S_2 \dots - \cancel{S_0}$

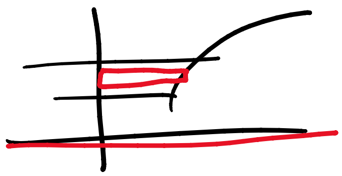
$\sum_{n=0}^{\infty}$ $\quad \sum_{n=1}$ $\quad S_0$

$\sin\frac{\pi}{2} + \frac{1}{1-\frac{1}{2}} - \left(\frac{1}{1!}\left(\frac{\pi}{2}\right) + 1\right)$

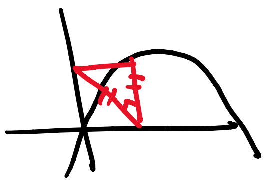
$\cancel{1} + 2 - \frac{\pi}{2} - \cancel{1} = 2 - \frac{\pi}{2}$

$\boxed{b}$

4. $2\pi \int (\text{radius})(\text{height})\, dy$



5.



$A = \frac{1}{2}s^2$

$$\int_0^{\pi} \frac{1}{2}(\sin x)^2\, dx$$

~~6~~

7. $f^{(11)}(1)$ $\quad f(x) = e^{3x}$ $\quad$ centered @ 1

$$\frac{f^{(11)}(1)(x-1)^{11}}{11!}$$

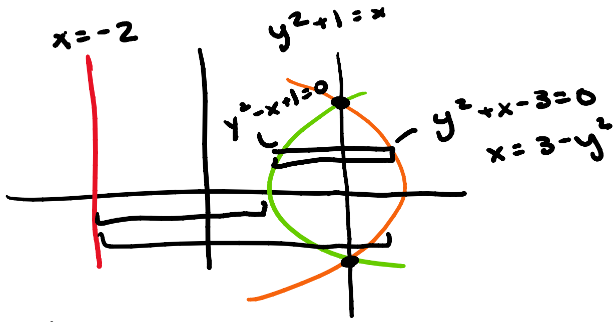
$$e^{3x} = e^{3(x-1)+3} = e^{3(x-1)}e^{3}$$

$$e^3 \sum_{n=0}^{\infty} \frac{[3(x-1)]^n}{n!} \leftarrow n = 11$$

$$e^3 \frac{3^{11}\cancel{(x-1)^{11}}}{\cancel{11!}} = \frac{f^{(11)}(1)\cancel{(x-1)^{11}}}{\cancel{11!}}$$

$$f^{(11)}(1) = 3^{11}e^3$$ $\boxed{d}$

8



$$\pi \int_a^b OR^2 - IR^2 \, dy$$

$$\pi \int_{-1}^{1} (3-y^2+2)^2 - (y^2+1+2)^2 \, dy$$

$$\pi \int_{-1}^{1} (5-y^2)^2 - (3+y^2)^2 \, dy$$

$\boxed{a}$

9

$$\lim_{\heartsuit \to 0} \frac{\sin \heartsuit}{\heartsuit} = 1$$

div

$\cancel{b)}$ $\sum n^2 \sin^2\left(\frac{1}{n}\right) \rightarrow \frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}} \; \frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}}$

$$\lim_{n\to\infty} \frac{n^2 \sin^2 \frac{1}{n}}{n^2 \cdot \frac{1}{n^2}} = 1$$

$$\frac{n^2}{n^2} = 1$$

$\cancel{a)}$ $n \cdot \frac{1}{n^2}$ $\qquad \tan\frac{1}{n^2} = \frac{\sin\frac{1}{n^2}}{\cos\frac{1}{n^2}}$

$$\lim_{n\to\infty} \cancelto{1}{\frac{\sin\frac{1}{n^2}}{\frac{1}{n^2}}} \cdot \frac{}{\cancelto{1}{\cos\frac{1}{n^2}} \cdot n} = 0$$

$\cancel{c)}$ $\frac{\sqrt{n}}{n}$

$\boxed{d)}$ $\frac{1}{n} \cdot \frac{1}{n} = \frac{1}{n^2} \leftarrow$ con

$$\frac{\frac{\tan\frac{1}{n}}{\cancel{n}}}{\frac{1}{n} \cdot \cancel{\frac{1}{n}}} = \frac{\sin\frac{1}{n}}{\frac{1}{n}} \cdot \cos\frac{1}{n} = 1$$

$$\frac{\frac{\cdots}{\cancel{n}}}{\frac{1}{n}\cdot\cancel{\frac{1}{n}}} = \frac{\cdots}{\frac{1}{n}}\cdot\cos\frac{1}{n} = 1$$

a_n $\quad$ b_n

$\sin\frac{1}{n}$ $\quad$ $\frac{1}{n}$

$$\lim_{n\to\infty}\frac{\sin\frac{1}{n}}{\frac{1}{n}} = 1$$

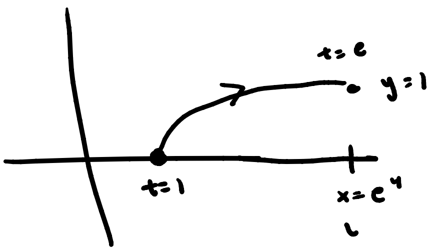
$\sin\frac{1}{n^2}$ $\quad$ $\frac{1}{n^2}$

$\sin^2\frac{1}{n}$

$\sin\frac{1}{n}\cdot\sin\frac{1}{n}$ $\quad$ $\frac{1}{n}\cdot\frac{1}{n}$

(10) $t=1$

$t=e$



$\boxed{a}$

(11) div

(12) $\frac{1}{2}\int_a^b r^2\,d\theta$

(13) $$\frac{1}{x+1} = \frac{1}{(x-5)+6} = \frac{1}{6\left[1-\left(-\frac{(x-5)}{6}\right)\right]}$$

$\frac{1}{1-x}$ $\qquad$ $\frac{1}{6}\cdot\frac{1}{1-\left(-\frac{(x-5)}{6}\right)}$

$$\frac{1}{6}\sum_{n=0}^{\infty}\frac{(-1)^n(x-5)^n}{6^n}$$

$n=0$ $\quad$ $n=1$ $\quad$ $n=2$

$$\frac{1}{6} - \frac{(x-5)}{6^2} + \frac{(x-5)^2}{6^3}$$

$\boxed{b}$

(14)

$$\lim_{n\to\infty} \sqrt[n]{\left(\frac{\cos\frac{1}{n}}{8}\right)^n} = \frac{1}{8} < 1$$

$\boxed{\text{conv.}}$ $\boxed{c}$

(15)

$$\lim_{n\to\infty} \frac{\overset{(n+1)\cancel{n!}}{(n+1)!}}{\cancel{n!}} \cdot \frac{\cancel{3}\cancel{e^{n^2}}}{\cancel{3}e^{(n+1)^2}} = \lim \frac{n+1}{e^{2n+1}}$$

$$(n+1)^2 \rightarrow e^{\cancel{n^2}+2n+1}$$

$$= 0$$

conv ratio $\boxed{a}$

(16)

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{2} < 1$$

Σa_n conv. $\boxed{a}$

$\{a_n\} \leftarrow 0$

$$\lim_{n\to\infty} a_n = 0$$ Test for div

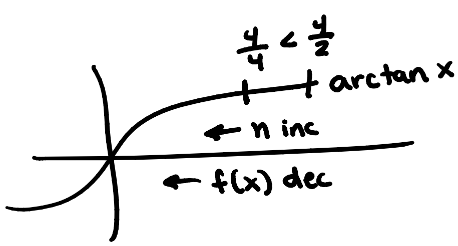
(17)

$\arctan \frac{4}{n}$

$f(n+1) < f(n)$ ✓

$$\lim_{n\to\infty} \arctan\frac{4}{n} = 0$$ ✓

$\boxed{a}$



(18) $\int \sqrt{x^2+10x}\, dx$

$\hookrightarrow x^2+10x+25 \quad -25$

$)^2 - 25$

$$\hookrightarrow \quad x^2+10x+25 \quad -25$$

$$(x+5)^2 - 25$$

$$25\left[\left(\frac{x+5}{5}\right)^2 - 1\right]$$

$$\frac{x+5}{5} = \sec\theta$$

$$dx = 5\sec\theta\tan\theta\, d\theta$$

$$\int \sqrt{25(\sec^2\theta - 1)} \cdot 5\sec\theta\tan\theta\, d\theta$$

$\boxed{d}$

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$$\lim_{n\to\infty} \frac{(2x-1)^{n+1}}{\cancel{(2x-1)^n}} \cdot \frac{n^{1/4}}{(n+1)^{1/4}}$$

$$\cancel{(2x-1)^n}(2x-1)$$

$$|2x-1| \cdot 1 < 1$$

$$-1 < 2x-1 < 1$$

$$0 < 2x < 2$$

$$0 < x < 1$$

$$(0,1]$$

$$\sum \frac{(-1)^n(-1)^n}{\sqrt[4]{n}} \qquad \frac{(-1)^n(2-1)^n}{\sqrt[4]{n}} \quad 1^n$$

div. $\qquad$ conv. AST

$\boxed{R = \frac{1}{2} \quad I = (0,1]}$ $\quad \boxed{e}$

20

$n=1 \quad e^{5/1} - \cancel{e^{5/2}}$

$n=2 \quad \cancel{e^{5/2}} - \cancel{e^{5/3}}$

$\vdots$

$n=N \quad \cancel{e^{5/N}} - e^{5/N+1}$

$$\lim_{N\to\infty} e^5 - e^{5/N+1} = e^5 - 1$$

$\boxed{b}$

21

$$\int_0^1 \overbrace{x^5}^{x^2 \quad x^3} e^{x^3}\,dx$$

$u = x^3$ — $1 - 1$, $0 - 0$

$du = 3x^2\,dx$

$$\frac{1}{3}\int_0^1 u e^u\,du$$

u	dv