REVIEW

1. Find the integral.
$$\int \frac{dx}{(x^2+4)^2}$$
(hint: trig substitution)

A. $\frac{1}{8}\arctan\left(\frac{x}{4}\right) + \frac{x}{(x^2+4)} + c$

B. $\frac{1}{16}\arctan\left(\frac{x}{2}\right) + \frac{x}{8(x^2+4)} + c$

C. $\frac{1}{8}\arctan\left(\frac{x}{2}\right) + \frac{x}{4(x^2+4)} + c$

D. $\frac{1}{16}\arctan\left(\frac{x}{4}\right) - \frac{x}{8(x^2+4)} + c$

see attachment

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2. Evaluate the integral.
$$\int \frac{\sec x}{\tan^3 x}\,dx$$
(trig integration)

see attachment

A. $\frac{1}{2}(-\csc x\cot x + \ln|\csc x - \cot x|) + c$

B. $\frac{1}{2}(-\csc x\cot x - \ln|\csc x - \cot x|) + c$

C. $\frac{1}{2}(\ln|\csc x - \cot x|) + c$

D. $\frac{1}{2}(\csc x\cot x + \ln|\csc x - \cot x|) + c$

E. $\frac{1}{2}(\csc x\cot x - \ln|\csc x - \cot x|) + c$

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3. Evaluate the integral.

$$\int \frac{3x}{(x^2+4x+6)}\,dx$$

(partial fraction Case III)

A. $\frac{3}{2}\ln(x^2+4x+6) - \frac{6}{\sqrt{2}}\arctan\left(\frac{x+2}{\sqrt{2}}\right) + c$

see attachment.

B. $\frac{1}{2}\ln(x^2+4x+6) + \frac{6}{\sqrt{2}}\arctan\left(\frac{x+2}{\sqrt{2}}\right) + c$

C. $\frac{3}{2}\ln(x^2+4x+6) + \frac{6}{\sqrt{2}}\arctan\left(\frac{x+2}{\sqrt{2}}\right) + c$

D. $\frac{1}{2}\ln(x^2+4x+6) - \frac{6}{\sqrt{2}}\arctan\left(\frac{x+2}{\sqrt{2}}\right) + c$

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4. Evaluate the integral.

$$\int \frac{x^2-x-1}{(x-1)(x+2)^2}\,dx$$

see attachment

(partial fraction Case II)

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5. Indicate a good method for evaluating the integral. see attachment.

$$\int \frac{8x}{\sqrt{12-6x-x^2}}\,dx$$

A. substitution ($u = 12-6x-x^2$, $du = (-6-2x)\,dx$)

B. integration by parts ($u = 12-6x-x^2$, $v' = 8x$)

C. a trigonometric method

D. trigonometric substitution ($x+3 = \sqrt{21}\sin\theta$)

3A

6. Evaluate the integrals:

(a) $\int \frac{1}{x\sqrt{x+9}}\,dx$. (rationalizing sub and PF). see attachment!

(b) $\int \frac{e^{\sqrt[4]{x}}}{\sqrt[4]{x}}\,dx$. (rationalizing sub and IBP). $u=\sqrt[4]{x}$

$u^4 = x$

$4u^3 = dx$

$= \int \frac{e^u \, 4u^{\cancel{3}\,2}\,du}{\cancel{u}} = 4\int e^u u^2\,du$

IBP

$u^2 \quad + \quad e^u$

$2u \quad - \quad e^u$

$2 \quad + \quad e^u$

$\int 0 = \quad e^u$

$= 4(u^2e^u - 2ue^u + 2e^u) + C$

$= 4e^{\sqrt[4]{x}}\left((\sqrt[4]{x})^2 - 2\sqrt[4]{x} + 2\right) + C$

$x^{2/4} = x^{1/2}$

$= 4e^{\sqrt[4]{x}}\left(\sqrt{x} - 2\sqrt[4]{x} + 2\right) + C$

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7. Sequence:

(a) $\left\{\frac{1}{2^n}\right\}$ is bounded and convergent. (T/F) $\to 0$

(b) $\left\{\sin\left(\frac{n\pi}{2}\right)\right\}$ is bounded and ~~convergent~~. (T/F) $(-1)^n$

(c) $\left\{\cos\left(\frac{(2n+1)\pi}{2}\right)\right\}$ is bounded and ~~divergent~~. (T/F) $= \{0, 0, \dots, 0, \dots\}$ $\to 0$

(d) $\left\{\tan\left(\frac{\pi}{n}\right)\right\}_{n=3}$ is bounded and convergent. (T/F) $\to 0$

$\lim_{n\to\infty} \tan\left(\frac{\pi}{n}\right) = \tan\left(\frac{\pi}{\infty}\right) = \tan(0) = 0$

Recall: $\tan\left(\frac{\square}{n}\right) \sim \frac{\square}{n}$

$\sin\left(\frac{\square}{n}\right) \sim \frac{\square}{n}$

$\lim_{\heartsuit \to 0} \frac{\sin(\heartsuit)}{\heartsuit} = 1$

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8. Determine if the series converges (by what test?)

(a) $\sum_{n=5}^{\infty} \frac{e^{1/(n^2)}}{n^3}$ DCT or INT conv.

(b) $\sum_{n=5}^{\infty} \frac{(-1)^n + 4}{n^3}$ DCT conv.

(c) $\sum_{n=5}^{\infty} \underbrace{\tan\left(\frac{1}{n}\right)}_{a_n}$ div $b_n = \frac{1}{n}$ $\sum \frac{1}{n}$ div. LCT $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\tan(\frac{1}{n})}{\frac{1}{n}} = 1 < \infty$, $\neq 0$

div. (d) $\sum_{n=5}^{\infty} \frac{(-1)^n n}{n+1}$, $\sum_{n=5}^{\infty} \cos(n\pi) = \sum (-1)^n$ TFD

TFD $\lim_{n\to\infty} |a_n| = \lim_{n\to\infty} \frac{n}{n+1} = 1 \neq 0$

$\rightarrow \lim_{n\to\infty} a_n \neq 0 \Rightarrow$ TFD

(e) $\sum_{n=5}^{\infty} \frac{(-1)^n}{\ln n}$, conv. (AST)

Geo (f) $\sum_{n=1}^{\infty} (-2)^n$, $\sum_{n=1}^{\infty} \left(\frac{-1}{2}\right)^n$,

div. $|r| = 2 > 1$ conv. $|r| = \frac{1}{2} < 1$

(g) $\sum_{n=5}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n+1)}{n!}$, Ratio $\lim_{n\to\infty} \frac{\cancel{1 \cdot 3 \cdot 5 \cdots (2n+1)}(2n+3)}{\cancel{1 \cdot 3 \cdot 5 \cdots (2n+1)}} \cdot \frac{\cancel{n!}}{\cancel{(n+1)!}\,(n+1)}$ $= 2(n+1)+1$

$= \lim_{n\to\infty} \frac{2n+3}{n+1} = 2 > 1$ div.

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9. (a) Let s_n be the n-partial sum of the series $\sum a_n$. $= \lim_{n\to\infty} s_n = \lim_{n\to\infty} \frac{1}{\ln n} = \boxed{0} =$ sum

Suppose that $s_n = \frac{1}{\ln n}$ for all $n \geq 5$. Determine the sum of $\sum a_n$, if possible.

(b) Let s_n be the n-partial sum of the series $\sum a_n$. $= \lim_{n\to\infty} s_n = \lim_{n\to\infty} \cos(\frac{1}{n}) = \cos(0) = \boxed{1}$ = sum

Suppose that $s_n = \cos\left(\frac{1}{n}\right)$ for all $n \geq 5$. Determine the sum of $\sum a_n$, if possible.

(c) If $|a_n| \leq \frac{1}{n}$ for all n, then $\sum a_n$ converges conditionally. (T/F) → not ~~...~~ enough info

(d) If $|a_n| \geq \frac{1}{n}$ for all n, then $\sum a_n$ converges conditionally. (T/F) ↳ div.

$bn = \frac{n}{(3n)!}$

10. Consider the series $\sum_{1}^{\infty} \frac{(-1)^n n}{(3n)!}$. According to the alternating series error estimation theorem, what is the least number of terms needed to approximate the sum of the series with an error no more than $\frac{1}{100000}$? (alternating series error estimation)

$|s - s_N| \le b_{N+1} = \frac{N+1}{(3(N+1))!} \le \frac{1}{10^5}$

$\rightarrow \boxed{N=2}$

11. Evaluate the sum of the series $\sum_{3}^{\infty} \left[\cos\left(\frac{\pi}{n+1}\right) - \cos\left(\frac{\pi}{n}\right)\right]$. (telescoping series)

$S_N = \cos(\pi/4) - \cos(\pi/3)$
$+ \cos(\frac{\pi}{5}) - \cos(\frac{\pi}{4})$
$+ \dots$
$+ \cos(\frac{\pi}{(n+1)}) - \cos(\frac{\pi}{n})$

$S = \lim_{N\to\infty} S_N = \lim_{N\to\infty} [\cos(\frac{\pi}{N+1}) - \cos(\frac{\pi}{3})] = 1 - \frac{1}{2} = \boxed{\frac{1}{2}}$

12. Find the interval of convergence. $\sum_{3}^{\infty} \frac{n(x-4)^n}{2^n}$

Root Test: $\lim_{n\to\infty} \left|\frac{n(x-4)^n}{2^n}\right|^{1/n} = \lim_{n\to\infty} \frac{\sqrt[n]{n}\,|x-4|}{2} = \frac{|x-4|}{2} < 1$

$\lim_{n\to\infty} \sqrt[n]{n} = 1$

$\rightarrow |x-4| < 2 = R$

$-2 < x-4 < 2$

$2 < x < 6$

ck: $x=6$ $\sum_{3}^{\infty} \frac{n 2^n}{2^n} = \sum_{3}^{\infty} n$ div. (TFD) so $\boxed{IOC = (2,6)}$

ck: $x=2$ $\sum_{3} \frac{n(-2)^n}{2^n} = \sum(-1)^n n$ div. (TFD)

see attachment

13. (a) Find the series representation of $\frac{1}{(x-2)^2}$.

(b) Find the series representation of $\frac{1}{(1+x)(1-2x)}$ and the radius of convergence.

14. Find the function with the following Maclaurin series.

$$1 - \frac{6^3x^3}{3!} + \frac{6^5x^5}{5!} - \frac{6^7x^7}{7!} + \cdots$$

A. $-\sin(6x)$
B. $-6x + \sin(6x) + 1$
C. $-6x + \sin(6x)$
D. $\sin(6x) + 1$
E. $+6x + \sin(6x) - 1$

$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$

$6x$

$\sin(6x) = \sum_{n=0}^{\infty} \frac{(-1)^n 6^{2n+1} x^{2n+1}}{(2n+1)!} = 6x - \frac{6x^3}{3!} + \frac{6^5x^5}{5!} \cdots$

$\Rightarrow \sin(6x) - 6x + 1$

15. Suppose that $F(x)$ is a power series center at 0 with radius of convergence $R = 4$. Which of the following is false?

A. $F'(x)$ is a power series with radius of convergence $R = 4$. → same R

B. $F(4x)$ is a power series with radius of convergence $R = 16$.

C. $\int F(4x)\,dx$ is a power series with radius of convergence $R = 1$. same R

D. $x^3 F(x)$ is a power series with radius of convergence $R = 4$. Same R

diff/int. power series do not change the radius of conv.

But sub might → check R

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16. What is the derivative $f^{(4)}(-2)$ for a function with Taylor series

$$T(x) = 3(x+2) + (x+2)^2 - 4(x+2)^3 + \boxed{2(x+2)^4} - (x+2)^7 + \cdots \text{ ?}$$

$c_4 = 2$

A. 16 B. 8 C. 2 D. 1/12 E. 48

$\frac{f^{(4)}(-2)}{4!} = c_2 = 2$

$\to f^{4}(-2) = 2 \cdot 4!$

$= 48$

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17. Find the sum of the series $\sum_{n=2}^{\infty}\left[\frac{(-1)^{n+1}}{7^n} + \frac{(-1)^{n+1}}{n}\right] = \sum_{n=2}^{\infty}\left(\frac{-1}{7}\right)^n \cdot (-1) + \sum_{n=2}^{\infty}\frac{(-1)^{n+1}}{n}$

$x = 1$

A. $\ln 2 - \frac{7}{6}$ B. $\ln 2 - \frac{57}{56}$ C. $\ln 2 + \frac{5}{6}$

D. $\ln 2 - \frac{1}{56}$ E. $\ln 2 + \frac{1}{56}$

geo, $r = -\frac{1}{7}$

sum $= \frac{\text{1st term}}{1 - r} = \frac{-1/49}{(1-(-1/7))} = -\frac{1}{56}$

$\ln(x+1)$, $x=1$

$\ln(2) = 1 + \sum_{n=2}^{\infty}\frac{(-1)^n}{n}$

$\ln 2 - 1$

Answer: $-\frac{1}{56} + \ln 2 - 1 = \frac{-57}{56} + \ln 2$

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18. Which of the following descriptions is incorrect?

A. $\theta = \frac{\pi}{4}$ is equivalent to the line $y = x, x \geq 0$ T

$y = r\sin\theta = r\frac{\sqrt{2}}{2}$, $x = r\cos\theta = r\frac{\sqrt{2}}{2}$] $x = y$.

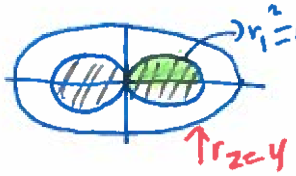
B. $r = \frac{\pi}{4}$ is equivalent to the circle centered at the poll with radius $\frac{\pi}{4}$ T

C. $r\cos\theta = 2$ is equivalent to the vertical line $x = 2$ T

D. $r^2 = 2$ describes a circle of ~~radius 2~~, center at the origin $\sqrt{2} = r$ $x^2 + y^2 = 2 \to r = \sqrt{2}$

E. $r = 4\sin\theta$ describes a circle of radius 2, center at $(0, 2)$ T.

(a)



$A = 4\int_0^{\pi/4}\frac{1}{2}r_1^2\,d\theta = 2\int_0^{\pi/4}4\cos(2\theta)\,d\theta$

$= 8\int_0^{\pi/4}\cos(2\theta)\,d\theta = \boxed{4}$

19. Find the area of the region lies inside both curves.

(a) $r_1^2 = 4\cos(2\theta)$, $r_2 = 4$

(b) $r_1 = 1 + \sin\theta$, $r_2 = \sqrt{2}\cos\theta$, (Let $0 < \theta_1 < \pi/2$ be the angle where the two curves intersect).

$V = \int_{-\frac{\pi}{2}}^{\theta_1}\frac{1}{2}r_1^2\,d\theta + \int_{\theta_1}^{\pi/2}\frac{1}{2}r_2^2\,d\theta$. (see attach.)

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20. Find the equation of the tangent line at the given value of parameter.

$$x = \sin\theta,\ y = \cos\theta,\ \theta = \frac{\pi}{3}$$

$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-\sin\theta}{\cos\theta} \to \left.\frac{dy}{dx}\right|_{\theta=\pi/3} = \frac{-\frac{\sqrt{3}}{2}}{1/2}$

$\to m = -\sqrt{3}$

A. $y = \sqrt{3}x + \sqrt{2}$ B. $y = -\sqrt{3}x + 3\sqrt{2}$ C. $y = x - \sqrt{2}$

D. $y = -\sqrt{3}x + 2$ E. $y = -\frac{3\sqrt{2}}{2}x + 3$

$y - \cos(\frac{\pi}{3}) = -\sqrt{3}\,(x - \sin(\frac{\pi}{3}))$

$\frac{dx}{dt} = \sec t\cdot\tan t = 0$ $\quad dy/dt = \sec^2 t = 0 \to$ never HTL

21. Find all points on the curve $x = \sec t, y = \tan t$ at which horizontal and vertical tangents exist.

$t = 0, \pi, 2\pi \to (x,y) = (\sec t, \tan t)$

Answer: No horizontal tangents. Vertical tangents at $(1,0), (-1,0)$

$\boxed{\text{VTL} \Rightarrow (1,0), (-1,0)}$

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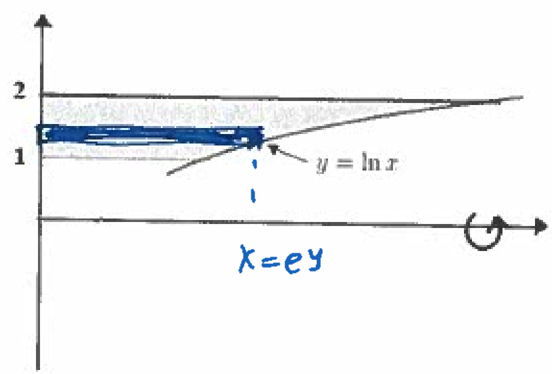
22. Using Shell method, find the volume generated by rotating about the $x-$axis the following region bounded by

$$y = \ln x, \quad y = 2, \quad y = 1.$$

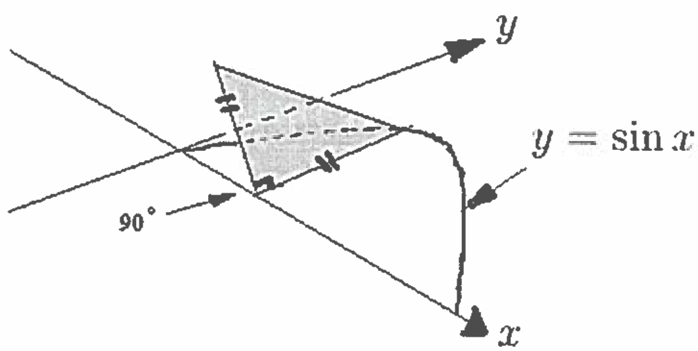
$V = \int_1^2 2\pi(e^y y) = 2\pi(e^y y - e^y)\Big|_1^2$

$= 2\pi e^2$



A. $3e^4\pi$ B. $6e^4\pi$ C. $2e^2\pi$ D. $6e^2\pi$ E. $2e^4\pi$

23. The base of a solid is bounded by the curve $y = \sin x$, and the x−axis on the interval $[0, \pi]$. The cross sections of the solid perpendicular to the x−axis are isosceles right angle triangles with the corner of the right angles lining up along the x− axis. (see diagram) Find the volume of the solid.



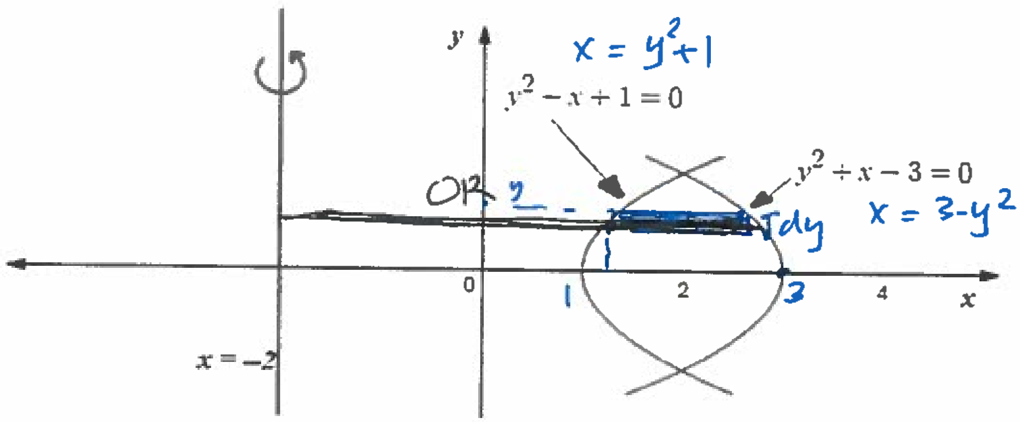
A. $\frac{301}{512}$ B. $\frac{\pi}{4}$ C. $\frac{\pi}{10}$ D. $\frac{3}{5}$ E. $\frac{\pi}{8}$

$$V = \int_0^{\pi} \sin^2 x \cdot \frac{1}{2}\, dx = \frac{1}{2} \int_0^{\pi} \frac{1 - \cos(2x)}{2}$$

$$= \frac{1}{4}\left(x - \frac{\sin(2x)}{2}\right)\Big|_0^{\pi}$$

$$= \frac{1}{4}(\pi - 0 - 0) = \frac{\pi}{4}$$

24. Using the washer method, set up an integral to find the the volume of the solid generated by revolving the shaded region bounded by $y^2 - x + 1 = 0$ and $y^2 + x - 3 = 0$ about the line $x = -2$.



A. $\pi \int_{-1}^{1} (5 - y^2)^2 - (3 + y^2)^2 \, dy$

B. $\pi \int_{-1}^{1} (y^2 + 5)^2 - (3 + y^2)^2 \, dy$

C. $\pi \int_{-1}^{1} (5 - y^2)^2 - (3 - y^2)^2 \, dy$

D. $\pi \int_{-1}^{1} (5 - y^2)^2 + (y^2 - 3)^2 \, dy$

E. $\pi \int_{-1}^{1} (y^2 + 5)^2 - (3 - y^2)^2 \, dy$

$V = \pi \int_{-1}^{1} [(OR)^2 - (IR)^2] \, dy$

$= \pi \int_{-1}^{1} (3 - y^2 + 2)^2 - (y^2 + 1 + 2)^2 \, dy$

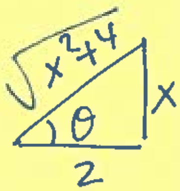
$= \pi \int_{-1}^{1} (5 - y^2)^2 - (y^2 + 3)^2 \, dy.$

Review

① $\displaystyle\int \frac{dx}{(x^2+4)^2}$ $\quad x = 2\tan\theta \rightarrow x^2+4 = 4\tan^2\theta + 4$

$\quad dx = 2\sec^2\theta\, d\theta \qquad\qquad = 4\sec^2\theta$

$$= \int \frac{2\,\cancel{\sec^2}\theta\, d\theta}{16\sec^{\cancel{4}\,2}\theta\, d\theta} = \frac{1}{8}\int \frac{d\theta}{\sec^2\theta} = \frac{1}{8}\int \cos^2\theta\, d\theta$$



$$= \frac{1}{8}\int \frac{1+\cos(2\theta)}{2}\, d\theta$$

$$\cancel{= \frac{1}{16}\int \theta}$$

$$= \frac{1}{16}\left(\theta + \frac{1}{2}\sin(2\theta)\right) + C$$

$$= \frac{1}{16}\left(\theta + \sin\theta \cdot \cos\theta\right) + C$$

$$= \frac{1}{16}\left(\arctan\left(\frac{x}{2}\right) + \frac{x}{\sqrt{x^2+4}} \cdot \frac{2}{\sqrt{x^2+4}}\right.$$

$$= \frac{1}{16}\left(\arctan\left(\frac{x}{2}\right) + \frac{2x}{x^2+4}\right)$$

$$= \frac{1}{16}\arctan\left(\frac{x}{2}\right) + \frac{1}{8}\frac{x}{x^2+4} + C.$$

B

② $\displaystyle\int \frac{\sec x}{\tan^3 x} = \int \frac{1}{\cancel{\cos x}} \cdot \frac{\cos^{\cancel{3}\,2} x}{\sin^3 x}\, dx$

$$= \int \frac{\cos^2 x}{\sin^3 x}\, dx$$

$$= \int \frac{1-\sin^2 x}{\sin^3 x}\, dx$$

$$= \int \csc^3 x - \csc x \, dx$$

$$= \frac{1}{2}\left[\ln|\csc x - \cot x| - \csc x \cot x\right] - \ln|\csc x - \cot x| + C$$

$$= -\frac{1}{2}\ln|\csc x - \cot x| - \frac{1}{2}\csc x \cot x + C.$$

13

③ $\displaystyle\int \frac{3x+6-6}{x^2+4x+6}\,dx$

$b^2-4ac = 16-24<0 \Rightarrow x^2+4x+6$ is irreducible

$\Rightarrow$ 2-step process.

① let $u = x^2+4x+6$

$du = (2x+4)\,dx$

$\frac{3}{2}\,du = 3(x+2)\,dx$

$\frac{3}{2}\,du = (3x+6)\,dx$

② D: $x^2+4x+6 = (x+2)^2+2$

$$\int \frac{3x}{x^2+4x+6}\,dx = \int \frac{3x+6}{x^2+4x+6}\,dx - \int \frac{6}{\underbrace{(x^2+4x+6)}_{(x+2)^2+2}}\,dx$$

$u = (x+2)^2+2 = x^2+4x+6$

$$\overset{①}{=} \int \frac{3}{2}\frac{du}{u} - \frac{6}{\sqrt{2}}\arctan\left(\frac{x+2}{\sqrt{2}}\right) + C$$

$$= \frac{3}{2}\ln|x^2+4x+6| - \frac{6}{\sqrt{2}}\arctan\left(\frac{x+2}{\sqrt{2}}\right) + C.$$

④ $$\int \frac{x^2 - x - 1}{(x-1)(x+2)^2}\,dx$$

$$\frac{x^2-x-1}{(x-1)(x+2)^2} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

$$A = \frac{\cancel{1}^{3} - \cancel{1} - 1}{(1+2)^2} = \frac{-1}{3^2} = -\frac{1}{9}$$

$$C = \frac{(-2)^2 - (-2) - 1}{(-2-1)\cancel{\,}} = \frac{4+2-1}{-3} = \frac{5}{-3}$$

$x=0$ $$\frac{-1}{(-1)(2)^2} = \frac{-1/9}{-1} + \frac{B}{2} + \frac{\cancel{C}\ -5/3}{(2)^2}$$

$$\left(\frac{1}{4} = \frac{1}{9} + \frac{B}{2} - \frac{5}{12}\right) 36$$

$$9 = 4 + 18B - 15$$

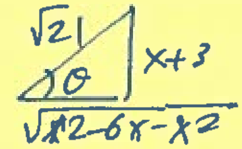
$$18B = 9 - 4 + 15 = 20$$

$$B = \frac{20}{18} = \frac{\cancel{4}^{2}\cdot 5}{2\cdot 9} = \frac{10}{9}$$

$$\int \frac{x^2-x-1}{(x-1)(x+2)^2}\,dx = -\frac{1}{9}\ln|x-1| + \frac{10}{9}\ln|x+2| + \frac{5}{3}\,\frac{1}{x+2}$$

5

$$\int \frac{8x}{\sqrt{12-6x-x^2}}\,dx = \int \frac{8x}{\sqrt{21-(x+3)^2}}$$



$12-6x-x^2$

$= 12-(6x+x^2)$

$= 12-(x^2+6x+9)+9$

$= 21-(x+3)^2$

$x+3 = \sqrt{21}\sin\theta \rightarrow dx = \sqrt{21}\cos\theta$

$$= 8\int \frac{\sqrt{21}\sin\theta - 3}{\cancel{\sqrt{21}\cos\theta}}\,\cancel{\sqrt{21}\cos\theta}\,d\theta$$

$= 8\left[-\sqrt{21}\cos\theta - 3\theta\right] + C$

$= 8\left[-\cancel{\sqrt{21}}\,\frac{\sqrt{12-6x-x^2}}{\cancel{\sqrt{21}}} - 3\tan^{-1}\left(\frac{x+3}{\sqrt{21}}\right)\right] + C$

6a a

$$\int \frac{dx}{x\sqrt{x+9}}$$

$u = \sqrt{x+9} \rightarrow \cancel{x = 9-u}$ $\quad u^2 = x+9$

$du = dx$ $\quad x = u^2-9$

$dx = 2u\,du$

$$= \cancel{\int \frac{du}{(9-u)(}}$$

$$= \int \frac{2\cancel{u}\,du}{(u^2-9)\cdot\cancel{u}} = \int \frac{2}{u^2-9}\,du$$

$$\frac{2}{u^2-9} = \frac{A}{u-3} + \frac{B}{u+3} = \frac{2}{(u-3)(u+3)}$$

$A \rightarrow 1/3$, $B \rightarrow -1/3$

$A = \frac{2}{6} = \frac{1}{3}$

$B = \frac{2}{-6} = -1/3$

$\int = \frac{1}{3}\ln|u-3| - \frac{1}{3}\ln|u+3|$

$$= \frac{1}{3}\ln\left|\frac{\sqrt{x+9}-3}{\sqrt{x+9}+3}\right| + C$$

(13)

(a) $\frac{d}{dx}\left(-\frac{1}{x-2}\right) = \frac{1}{(x-2)^2}$

$$-\frac{1}{x-2} = \frac{1}{2-x} = \frac{1}{2\left(1-\frac{x}{2}\right)}$$

$$= \frac{1}{2}\,\frac{1}{1-\frac{x}{2}} = \frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{x}{2}\right)^n$$

$$= \sum_{n=0}^{\infty}\frac{x^n}{2^{n+1}}$$

$$\frac{1}{(x-2)^2} = \frac{d}{dx}\left(-\frac{1}{x-2}\right) = \sum_{n=1}^{\infty}\frac{n x^{n-1}}{2^{n+1}}$$

(b) $$\frac{1}{(1+x)(1-2x)} = \frac{\cancel{A} \to 1/2}{1+x} + \frac{\cancel{B} \to 2/3}{1-2x}$$

$$A = \frac{1}{1-2\cdot(-1)} = \frac{1}{1+2} = \frac{1}{3}$$

$$B = \frac{1}{1+\frac{1}{2}} = \frac{1}{3/2} = \frac{2}{3}$$

$\downarrow R=1$ $\downarrow R=\frac{1}{2}$

$$\cancel{\sum}\frac{1}{(1+x)(1-2x)} = \frac{1}{2}\cdot\frac{1}{1+x} + \frac{2}{3}\,\frac{1}{1-2x}$$

$$= \frac{1}{2}\sum_{n=0}^{\infty}(-x)^n + \frac{2}{3}\sum_{n=0}^{\infty}(\cancel{-}2x)^n$$

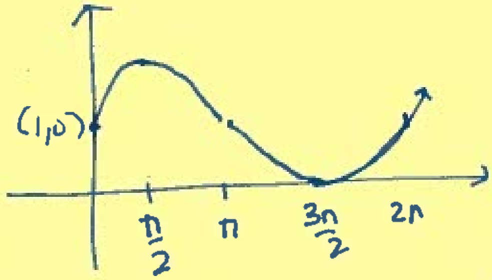
$$= \frac{1}{2}\sum_{n=0}^{\infty}\cancel{(-x)^n}(-1)^n\cdot x^n + \frac{2}{3}\sum_{n=0}^{\infty}2^n\cdot x^n$$

$$\left.\begin{array}{ll} |2x|<1 \to |x|<\frac{1}{2} \\ |x|<1 \quad\;\; |x|<1 \end{array}\right\} \to R=\frac{1}{2}$$

19) (b) $r_1 = 1+\sin\theta$ $\qquad r_2 = \sqrt{2}\cos\theta$ $\qquad 0<\theta_1<\frac{\pi}{2}$ intersection

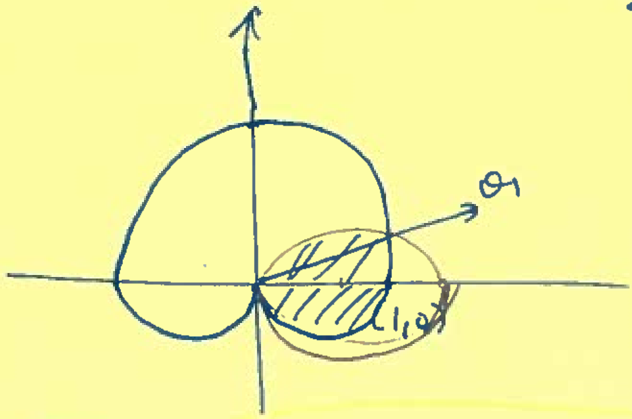
circle $\left(\frac{\sqrt{2}}{2},0\right)$

$r=\frac{\sqrt{2}}{2} \neq \frac{1}{2}$



$1-\frac{\sqrt{2}}{2}<\frac{\sqrt{2}}{2}$

$1<\sqrt{2}$



$$\text{Area} = \underbrace{\int_{\frac{3\pi}{2}}^{2\pi} \frac{1}{2} r_1^2\, d\theta + \frac{1}{2}\int_0^{\theta_1} r_1^2\, d\theta}_{} + \frac{1}{2}\int_{\theta_1}^{\pi/2} r_2^{\,2}\, d\theta$$

or combine

$$= \int_{-\pi/2}^{\theta_1} \frac{1}{2} r_1^2\, d\theta \quad + \quad \frac{1}{2}\int_{\theta_1}^{\pi/2} r_2^2\, d\theta.$$

20) $\cancel{dx}$ $\left.\frac{dy}{dx}\right|_{\frac{\pi}{3}} = \left.\frac{-\sin\theta}{\cos\theta}\right|_{\theta=\frac{\pi}{3}} = \frac{-\sin(\pi/3)}{\cos(\pi/3)} = \frac{-\frac{\sqrt{3}}{2}}{1/2}$

$x=\sin\theta$

$y=\cos\theta$ $\qquad \cos\pi$ $\qquad = -\sqrt{3}$

$y-\cos\left(\frac{\pi}{3}\right) = -\sqrt{3}\left(x-\sin\left(\frac{\pi}{3}\right)\right)$

$y-\frac{1}{2} = -\sqrt{3}\left(x-\frac{\sqrt{3}}{2}\right)$

$y=-\sqrt{3}x+\frac{3}{2}+\frac{1}{2} \Rightarrow \boxed{y=-\sqrt{3}x+2.}$