REVIEW

1. Find the integral.

$$\int \frac{dx}{(x^2+4)^2}$$

(hint: trig substitution)

A. $\frac{1}{8}\arctan\left(\frac{x}{4}\right) + \frac{x}{(x^2+4)} + c$

B. $\frac{1}{16}\arctan\left(\frac{x}{2}\right) + \frac{x}{8(x^2+4)} + c$

C. $\frac{1}{8}\arctan\left(\frac{x}{2}\right) + \frac{x}{4(x^2+4)} + c$

D. $\frac{1}{16}\arctan\left(\frac{x}{4}\right) - \frac{x}{8(x^2+4)} + c$

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2. Evaluate the integral.

$$\int \frac{\sec x}{\tan^3 x}\, dx$$

(trig integration)

A. $\frac{1}{2}(-\csc x \cot x + \ln|\csc x - \cot x|) + c$

B. $\frac{1}{2}(-\csc x \cot x - \ln|\csc x - \cot x|) + c$

C. $\frac{1}{2}(\ln|\csc x - \cot x|) + c$

D. $\frac{1}{2}(\csc x \cot x + \ln|\csc x - \cot x|) + c$

E. $\frac{1}{2}(\csc x \cot x - \ln|\csc x - \cot x|) + c$

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3. Evaluate the integral.

$$\int \frac{3x}{(x^2+4x+6)}\,dx$$

(partial fraction Case III)

A. $\frac{3}{2}\ln(x^2+4x+6) - \frac{6}{\sqrt{2}}\arctan\left|\frac{x+2}{\sqrt{2}}\right| + c$

B. $\frac{1}{2}\ln(x^2+4x+6) + \frac{6}{\sqrt{2}}\arctan\left|\frac{x+2}{\sqrt{2}}\right| + c$

C. $\frac{3}{2}\ln(x^2+4x+6) + \frac{6}{\sqrt{2}}\arctan\left|\frac{x+2}{\sqrt{2}}\right| + c$

D. $\frac{1}{2}\ln(x^2+4x+6) - \frac{6}{\sqrt{2}}\arctan\left|\frac{x+2}{\sqrt{2}}\right| + c$

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4. Evaluate the integral.

$$\int \frac{x^2-x-1}{(x-1)(x+2)^2}\,dx$$

(partial fraction Case II)

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5. Indicate a good method for evaluating the integral.

$$\int \frac{8x}{\sqrt{12-6x-x^2}}\,dx$$

A. substitution $(u = 12-6x-x^2,\ du = (-6-2x)\ dx)$

B. integration by parts $(u = 12-6x-x^2,\ v' = 8x)$

C. a trigonometric method

D. trigonometric substitution $(x+3 = \sqrt{21}\sin\theta)$

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6. Evaluate the integrals:

   (a) $\displaystyle\int \frac{1}{x\sqrt{x+9}}\,dx.$ (rationalizing sub and ________).

   (b) $\displaystyle\int \frac{e^{\sqrt[4]{x}}}{\sqrt[4]{x}}\,dx.$ (rationalizing sub and ________).

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7. Sequence:

   (a) $\left\{\dfrac{1}{2^n}\right\}$ is bounded and convergent. (T/F)

   (b) $\left\{\sin\left(\dfrac{n\pi}{2}\right)\right\}$ is bounded and convergent. (T/F)

   (c) $\left\{\cos\left(\dfrac{(2n+1)\pi}{2}\right)\right\}$ is bounded and divergent. (T/F)

   (d) $\left\{\tan\left(\dfrac{\pi}{n}\right)\right\}_{n=3}$ is bounded and convergent. (T/F)

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8. Determine if the series converges (by what test?)

(a) $\displaystyle\sum_{n=5}^{\infty} \frac{e^{1/(n^2)}}{n^3}$

(b) $\displaystyle\sum_{n=5}^{\infty} \frac{(-1)^n + 4}{n^3}$

(c) $\displaystyle\sum_{n=5}^{\infty} \tan\left(\frac{1}{n}\right)$

(d) $\displaystyle\sum_{n=5}^{\infty} \frac{(-1)^n n}{n+1}, \quad \sum_{n=5}^{\infty} \cos(n\pi)$

(e) $\displaystyle\sum_{n=5}^{\infty} \frac{(-1)^n}{\ln n},$

(f) $\displaystyle\sum_{n=1}^{\infty} (-2)^n, \quad \sum_{n=1}^{\infty} \left(\frac{-1}{2}\right)^n,$

(g) $\displaystyle\sum_{n=5}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n+1)}{n!},$

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9. (a) Let s_n be the $n-$partial sum of the series $\sum a_n$.
Suppose that $s_n = \dfrac{1}{\ln n}$ for all $n \geq 5$. Determine the sum of $\sum a_n$, if possible.

(b) Let s_n be the $n-$partial sum of the series $\sum a_n$.
Suppose that $s_n = \cos\left(\dfrac{1}{n}\right)$ for all $n \geq 5$. Determine the sum of $\sum a_n$, if possible.

(c) If $|a_n| \leq \dfrac{1}{n}$ for all n, then $\sum a_n$ converges conditionally. (T/F)

(d) If $|a_n| \geq \dfrac{1}{n}$ for all n, then $\sum a_n$ converges conditionally. (T/F)

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10. Consider the series $\sum_{1}^{\infty} \frac{(-1)^n n}{(3n)!}$. According to the alternating series error estimation theorem, what is the least number of terms needed to approximate the sum of the series with an error no more than $\frac{1}{100000}$?
(alternating series error estimation)

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11. Evaluate the sum of the series $\sum_{3}^{\infty} \left[\cos\left(\frac{\pi}{n+1}\right) - \cos\left(\frac{\pi}{n}\right)\right]$.
(telescoping series)

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12. Find the interval of convergence. $\sum_{3}^{\infty} \frac{n(x-4)^n}{2^n}$

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13. (a) Find the series representation of $\frac{1}{(x-2)^2}$.

(b) Find the series representation of $\frac{1}{(1+x)(1-2x)}$ and the radius of convergence.

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14. Find the function with the following Maclaurin series.

$$1 - \frac{6^3x^3}{3!} + \frac{6^5x^5}{5!} - \frac{6^7x^7}{7!} + \cdots$$

A. $-\sin(6x)$

B. $-6x + \sin(6x) + 1$

C. $-6x + \sin(6x)$

D. $\sin(6x) + 1$

E. $+6x + \sin(6x) - 1$

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15. Suppose that $F(x)$ is a power series center at 0 with radius of convergence $R = 4$. Which of the following is <u>false</u>?

A. $F'(x)$ is a power series with radius of convergence $R = 4$.

B. $F(4x)$ is a power series with radius of convergence $R = 16$.

C. $\int F(4x)\,dx$ is a power series with radius of convergence $R = 1$.

D. $x^3F(x)$ is a power series with radius of convergence $R = 4$.

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16. What is the derivative $f^{(4)}(-2)$ for a function with Taylor series

$$T(x) = 3(x+2) + (x+2)^2 - 4(x+2)^3 + 2(x+2)^4 - (x+2)^7 + \cdots \ ?$$

A. 16 B. 8 C. 2 D. 1/12 E. 48

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17. Find the sum of the series $\displaystyle\sum_{n=2}^{\infty}\left[\frac{(-1)^{n+1}}{7^n} + \frac{(-1)^{n+1}}{n}\right]$.

A. $\ln 2 - \dfrac{7}{6}$ B. $\ln 2 - \dfrac{57}{56}$ C. $\ln 2 + \dfrac{5}{6}$

D. $\ln 2 - \dfrac{1}{56}$ E. $\ln 2 + \dfrac{1}{56}$

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18. Which of the following descriptions is <u>incorrect</u>?

A. $\theta = \dfrac{\pi}{4}$ is equivalent to the line $y = x, x \geq 0$

B. $r = \dfrac{\pi}{4}$ is equivalent to the circle centered at the poll with radius $\dfrac{\pi}{4}$

C. $r\cos\theta = 2$ is equivalent to the vertical line $x = 2$

D. $r^2 = 2$ describes a circle of radius 2, center at the origin

E. $r = 4\sin\theta$ describes a circle of radius 2, center at $(0, 2)$

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19. Find the area of the region lies inside both curves.

(a) $r_1^2 = 4\cos(2\theta),\ r_2 = 4$

(b) $r_1 = 1 + \sin\theta,\ r_2 = \sqrt{2}\cos\theta,$ (Let $0 < \theta_1 < \pi/2$ be the angle where the two curves intersect).

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20. Find the equation of the tangent line at the given value of parameter.

$$x = \sin\theta,\ y = \cos\theta,\ \theta = \frac{\pi}{3}$$

A. $y = \sqrt{3}x + \sqrt{2}$ B. $y = -\sqrt{3}x + 3\sqrt{2}$ C. $y = x - \sqrt{2}$

D. $y = -\sqrt{3}x + 2$ E. $y = -\dfrac{3\sqrt{2}}{2}x + 3$

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21. Find all points on the curve $x = \sec t, y = \tan t$ at which horizontal and vertical tangents exist.

Answer: No horizontal tangents. Vertical tangents at $(1, 0), (-1, 0)$

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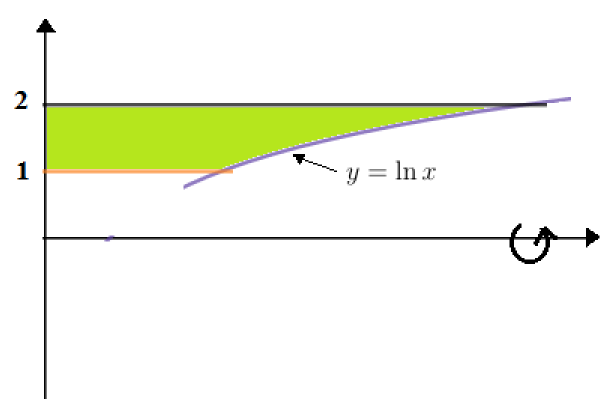
22. Using Shell method, find the volume generated by rotating about the $x-$axis the following region bounded by

$$y = \ln x, \quad y = 2, \quad y = 1.$$



A. $3e^4\pi$ B. $6e^4\pi$ C. $2e^2\pi$ D. $6e^2\pi$ E. $2e^4\pi$

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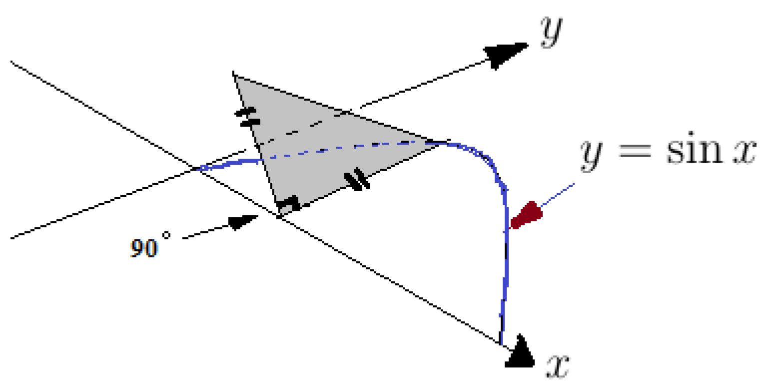
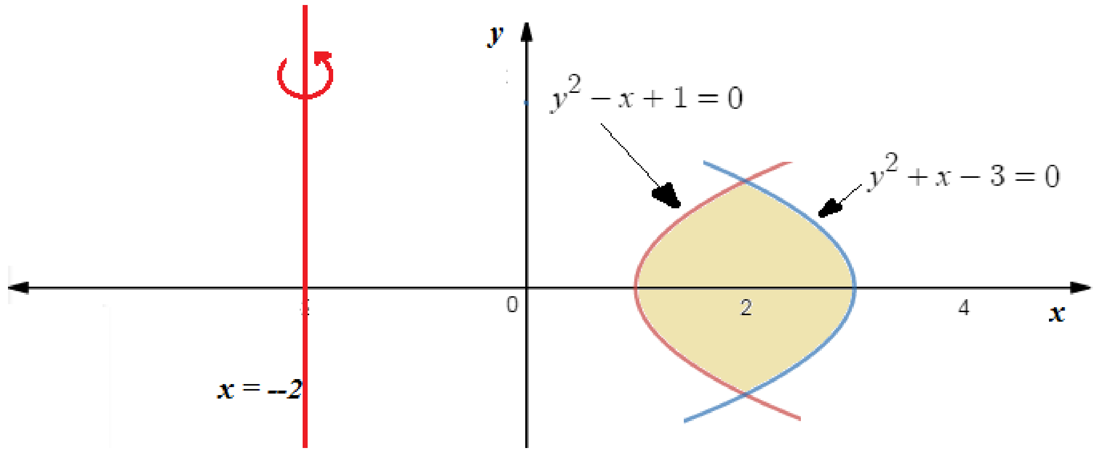
23. The base of a solid is bounded by the curve $y = \sin x$, and the $x-$axis on the interval $[0, \pi]$. The cross sections of the solid perpendicular to the $x-$axis are isosceles right angle triangles with the corner of the right angles lining up along the $x-$ axis. (see diagram) Find the volume of the solid.



A. $\frac{301}{512}$ B. $\frac{\pi}{4}$ C. $\frac{\pi}{10}$ D. $\frac{3}{5}$ E. $\frac{\pi}{8}$

24. Using the washer method, set up an integral to find the the volume of the solid generated by revolving the shaded region bounded by $y^2 - x + 1 = 0$ and $y^2 + x - 3 = 0$ about the line $x = -2$.



A. $\pi \int_{-1}^{1} (5 - y^2)^2 - (3 + y^2)^2 \; dy$

B. $\pi \int_{-1}^{1} (y^2 + 5)^2 - (3 + y^2)^2 \; dy$

C. $\pi \int_{-1}^{1} (5 - y^2)^2 - (3 - y^2)^2 \; dy$

D. $\pi \int_{-1}^{1} (5 - y^2)^2 + (y^2 - 3)^2 \; dy$

E. $\pi \int_{-1}^{1} (y^2 + 5)^2 - (3 - y^2)^2 \; dy$