List of Common Maclaurin Series

- $\dfrac{1}{1-x} = \sum_{n=0}^{\infty} x^n$, I.O.C. is $(-1,1)$
- $\ln(1-x) = -\sum_{n=1}^{\infty} \dfrac{x^n}{n}$, I.O.C. is $[-1,1)$
- $\ln(1+x) = \sum_{n=1}^{\infty} \dfrac{(-1)^{n+1}x^n}{n}$, I.O.C. is $(-1,1]$
- $e^x = \sum_{n=0}^{\infty} \dfrac{x^n}{n!}$, I.O.C. is $(-\infty,\infty)$
- $\cos x = \sum_{n=0}^{\infty} \dfrac{(-1)^n x^{2n}}{(2n)!}$, I.O.C. is $(-\infty,\infty)$
- $\sin x = \sum_{n=0}^{\infty} \dfrac{(-1)^n x^{2n+1}}{(2n+1)!}$, I.O.C. is $(-\infty,\infty)$
- $\arctan x = \sum_{n=0}^{\infty} \dfrac{(-1)^n x^{2n+1}}{2n+1}$, I.O.C. is $[-1,1]$

1. For what values of d does the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^d}{1+\sqrt{n}}$ converge?

(a) $d \in (-1, 1)$

(b) $d \in (-\infty, -\frac{1}{2}]$

(c) $d \in [\frac{1}{2}, \infty)$

(d) $d \in (-\infty, -\frac{1}{2})$

(e) $d \in (\frac{1}{2}, \infty)$

2. Compute $\displaystyle\int_0^2 x^2 e^{x/2}\, dx$.

(a) $40e - 16$ (b) $8e - 16$ (c) $\dfrac{16e}{3}$ (d) $\dfrac{5e-1}{4}$ (e) $\dfrac{4e}{3}$

3. What sort of tangent line does the parametric curve

$$x = \sin(\pi t)\ ,\ y = t^2 - t - 3\ ,\ 0 \le t \le 3$$

have when $t = \dfrac{3}{2}$?

(a) A vertical tangent line

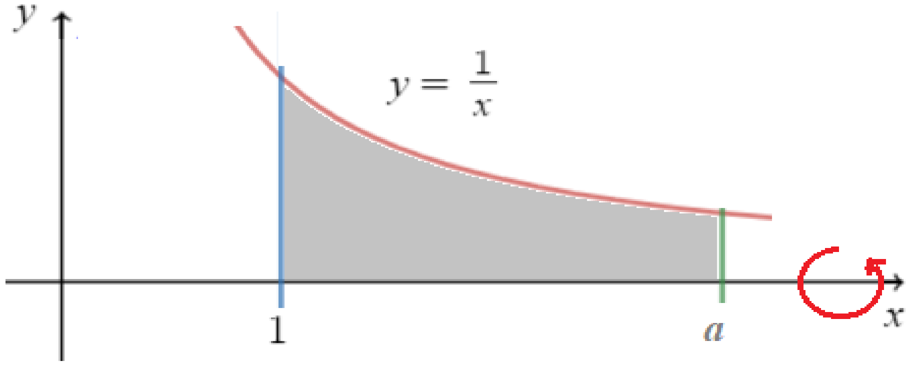
(b) A horizontal tangent line

(c) A tangent line with slope 2

(d) A tangent line with slope $\dfrac{9}{4}$

(e) There is no tangent line for this value of t

4. Consider the solid of revolution formed by revolving the area bounded by the curve $y = \dfrac{1}{x}$, the line $x = 1$, the line $y = 0$, and the line $x = a$, $(a > 1)$ about the x-axis (picture shown below). What is an integral representing the volume of this solid?



(a) $\pi \displaystyle\int_1^a \frac{dx}{x}$ (b) $2\pi \displaystyle\int_1^a \frac{dx}{x}$ (c) $\pi \displaystyle\int_1^a \frac{dx}{x^2}$ (d) $2\pi \displaystyle\int_1^a \frac{dx}{x^2}$ (e) $\pi \displaystyle\int_1^a \frac{dx}{\sqrt{x}}$

5. Which of the following is the correct form of the partial fraction decomposition of $\dfrac{2x^4+16}{(x^2+1)x^2}$?

(a) $2 + \dfrac{Ax+B}{x^2+1} + \dfrac{C}{x^2}$

(b) $\dfrac{Ax+B}{x^2+1} + \dfrac{C}{x} + \dfrac{D}{x^2}$

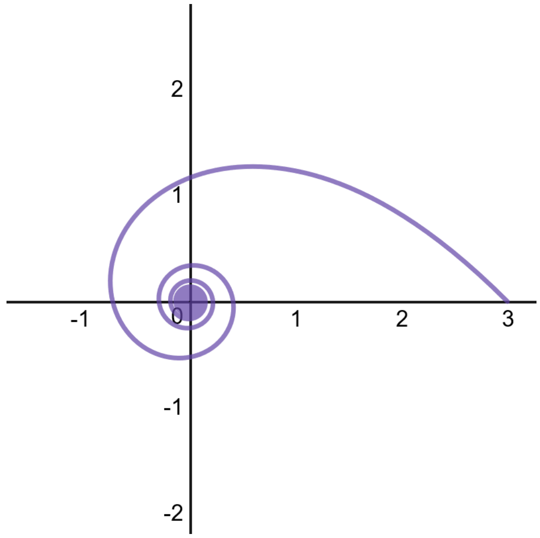
(c) $\dfrac{A}{x^2+1} + \dfrac{B}{x^2} + \dfrac{C}{x}$

(d) $\dfrac{Ax+B}{x^2+1} + \dfrac{C}{x^2}$

(e) $2 + \dfrac{Ax+B}{x^2+1} + \dfrac{C}{x} + \dfrac{D}{x^2}$

6. Evaluate $\displaystyle\int_1^e (\ln(x))^2\,dx$

(a) $e - \dfrac{e^2+1}{2}$ (b) $e - \dfrac{e^2}{2}$ (c) $\dfrac{1}{3}$ (d) $\dfrac{e^3}{3}$ (e) $e - 2$



7. The graph of a polar function over the range $0 \leq \theta < \infty$ is given above. Which of the following is the function being graphed?

   (a) $r^2 = 3\cos\theta$

   (b) $r = 3 + \theta$

   (c) $r = \dfrac{3}{1+\theta}$

   (d) $r = 3 + \sin\theta$

   (e) $r = 3\cos(8\theta)$

8. Evaluate the improper integral

$$\int_3^\infty \frac{dx}{x\ln(x)\ln(\ln(x))}.$$

   (a) $\dfrac{1}{\ln(\ln(\ln(3)))}$ (b) $\ln(\ln(\ln(3)))$ (c) $-\ln(\ln(\ln(3)))$ (d) Diverges

9. The base of a solid is the triangle in the xy-plane with vertices (0, 0), (1, 0), and (0, 1). The cross-sections of the solid perpendicular to the x-axis are squares. What is the volume of the solid?

   (a) $\dfrac{1}{3}$ (b) $\dfrac{2}{3}$ (c) 3 (d) 3π

10. Which of these integrals would compute the area of the region that is inside the circle $r = 4\cos\theta$ but outside the circle $x^2 + y^2 = 4$?

(a) $\displaystyle\int_0^{\pi/3} 16 - 16\cos^2\theta \, d\theta$

(b) $\displaystyle\int_0^{\pi/3} 16\cos^2\theta - 4 \, d\theta$

(c) $\displaystyle\pi\int_{-\pi/3}^{\pi/3} 4 - 16\cos^2\theta \, d\theta$

(d) $\displaystyle\frac{1}{2}\int_{-\pi/3}^{\pi/3} 4\cos\theta - 2 \, d\theta$

(e) $\displaystyle\int_{-\pi/3}^{\pi/3} 16\cos^2\theta - 16 \, d\theta$

11. Suppose that a function $y = f(x)$ is given with $f(x) \geq 0$ for $0 \leq x \leq 4$. If the area under the graph of $f(x)$ from $x = 0$ to $x = 4$ is revolved about the x-axis, then the volume of the solid of revolution is given by

(a) $\displaystyle 2\pi\int_0^4 x[f(x)]^2 \, dx$

(b) $\displaystyle \pi\int_0^4 x^2 f(x) \, dx$

(c) $\displaystyle 2\pi\int_0^4 \sqrt{1 + [f(x)]^2} \, dx$

(d) $\displaystyle \pi\int_0^4 [f(x)]^2 \, dx$

(e) $\displaystyle 2\pi\int_0^4 [f(x)]^2 \, dx$

12. The integral

$$\int \frac{1}{(x^2+1)^2} \, dx = A\arctan(x) + B\left(\frac{x}{x^2+1}\right) + C.$$

Find $A + B$.

(a) 0 (b) 1 (c) $\frac{3}{4}$ (d) 2

13. Use a known Maclaurin series to evaluate the sum of the series $\frac{1}{4} - \frac{1}{2 \cdot 4^2} + \frac{1}{3 \cdot 4^3} - \frac{1}{4 \cdot 4^4} + \cdots$.

(a) $(n-1)!(1-e^{-1/4})$

(b) $\ln\left(\frac{5}{4}\right)$

(c) $\frac{1}{8}$

(d) $\ln\left(\frac{1}{4}\right)$

(e) $\frac{\ln\left(\frac{1}{4}\right)}{4}$

14. Suppose that $F(x)$ is a power series with radius of convergence $R = 4$. Which below is false?

(a) $F(3x)$ is a power series with $R = \frac{4}{3}$.

(b) $F'(x)$ is a power series with $R = 4$.

(c) $\int F(x)\, dx$ is a power series with $R = 4$.

(d) $F\left(\frac{x}{4}\right)$ is a power series with $R = 16$.

(e) $xF(3x)$ is a power series with $R = 4$.

15. Which of the following best describes the graph of the parametric equation

$$x(t) = 3\sin t\ ,\ y(t) = 4\cos t$$

(a) A circle that is traced out clockwise.

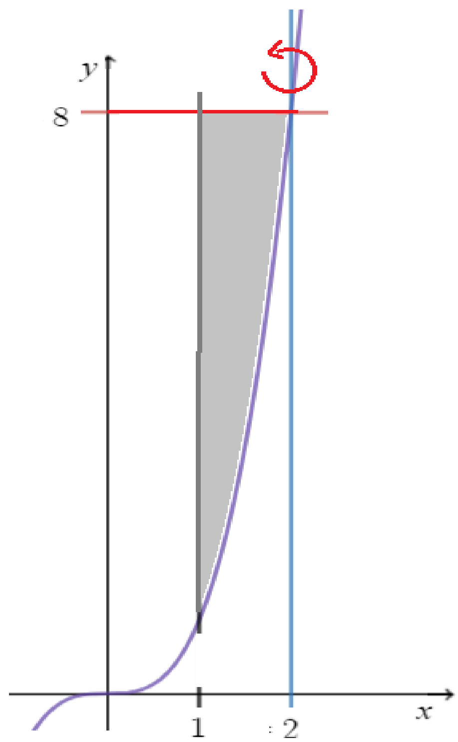
(b) An ellipse that is traced out clockwise.

(c) A circle that is traced out counterclockwise.

(d) An ellipse that is traced out counterclockwise.

(e) A hyperbola traced from left to right

16. Which of the following integrals would you get using the **shell method** to find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^3$, $y = 8$ and $x = 1$ about the line $x = 2$? (Region shown below)



(a) $2\pi \int_1^8 (2 - y)(1 - \sqrt[3]{y})\, dy$

(b) $\pi \int_1^2 \left[8^2 - (x^3)^2\right]\, dx$

(c) $2\pi \int_1^8 (\sqrt[3]{y} - 1)(8 - y)\, dy$

(d) $2\pi \int_1^2 (2 - x)(8 - x^3)\, dx$

(e) $\pi \int_1^8 (2 - y)(1 - \sqrt[3]{y})\, dy$

17. Graph the polar curves $r = 2 + 2\sin\theta$ and $r = 4\sin\theta$. At how many distinct points do the graphs intersect?

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

18. Use the second degree Taylor Polynomial, T_2, for $f(x) = \sin x$ centered at $\frac{\pi}{4}$ to approximate $\sin\left(\frac{\pi}{5}\right)$.

(a) $\sin\left(\frac{\pi}{5}\right) \approx \frac{\sqrt{2}}{2} + \frac{\sqrt{2}\pi}{40} - \frac{\sqrt{2}\pi^2}{80}$

(b) $\sin\left(\frac{\pi}{5}\right) \approx \frac{\sqrt{2}}{2} - \frac{\sqrt{2}\pi}{40} + \frac{\sqrt{2}\pi^2}{80}$

(c) $\sin\left(\frac{\pi}{5}\right) \approx \frac{\sqrt{2}}{2} + \frac{\sqrt{2}\pi}{40} - \frac{\sqrt{2}\pi^2}{1600}$

(d) $\sin\left(\frac{\pi}{5}\right) \approx \frac{\sqrt{2}}{2} - \frac{\sqrt{2}\pi}{40} - \frac{\sqrt{2}\pi^2}{1600}$

(e) $\sin\left(\frac{\pi}{5}\right) \approx \frac{\sqrt{2}}{2} + \frac{\sqrt{2}\pi}{40} + \frac{\sqrt{2}\pi^2}{80}$

19. For what range of values of t will the graph of the parametric curve $x = \frac{t^2}{2} + t$, $y = \frac{t^3}{3} - t$ have negative slope and be concave down?

(a) $t < -1$ (b) $t > -1$ (c) $-1 < t < 1$ (d) $t < 1$ (e) $t > 1$

20. Consider the power series $S = \sum_{n=1}^{\infty} c_n(x-5)^n$. Which of the following could be an interval of convergence for S?

(a) $IOC = [-1, 7)$

(b) $IOC = [-4, -2]$

(c) $IOC = [-5, 15)$

(d) $IOC = (4, 7]$

(e) $IOC = [3, 6)$

21. Evaluate $\displaystyle\int \frac{e^{2x}}{(e^{2x}+1)(e^x+1)}\,dx$.

(a) $\ln(e^{2x}+1)+\frac{1}{2}\arctan(e^x)-\frac{1}{2}\ln|e^x+1|+C$

(b) $\frac{1}{2}\ln(e^{2x}+1)+\frac{1}{2}\arctan(e^x)-\frac{1}{2}\ln|e^x+1|+C$

(c) $-\frac{1}{2}\ln(e^{2x}+1)+\frac{1}{2}\arctan(e^x)+\frac{1}{2}\ln|e^x+1|+C$

(d) $-\ln(e^{2x}+1)+\frac{1}{2}\arctan(e^x)+\frac{1}{2}\ln|e^x+1|+C$

(e) $\frac{1}{4}\ln(e^{2x}+1)+\frac{1}{2}\arctan(e^x)-\frac{1}{2}\ln|e^x+1|+C$

22. Which of the following statements about convergence is true?

(a) If a_n and b_n are two positive sequences with $a_n \le b_n$ and $\displaystyle\sum_{n=0}^{\infty} b_n$ diverges, then $\displaystyle\sum_{n=0}^{\infty} a_n$ diverges by the Direct Comparison Test.

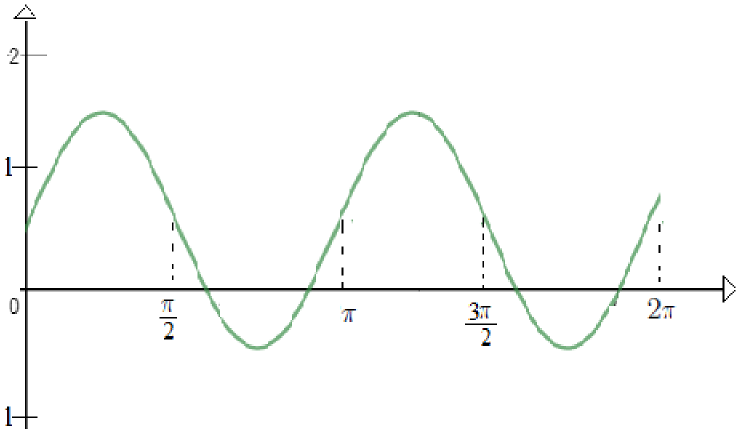
(b) If $\displaystyle\lim_{n\to\infty} \sqrt[n]{|a_n|}=1$, then $\displaystyle\sum_{n=0}^{\infty} a_n$ diverges by the Root Test.

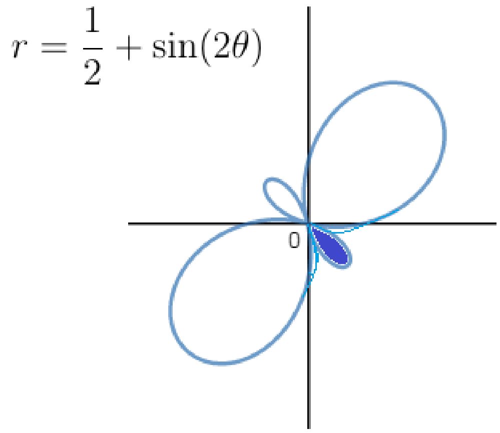
(c) If $\displaystyle\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|=1$, then $\displaystyle\sum_{n=0}^{\infty} a_n$ converges by the Ratio Test.

(d) If $\displaystyle\lim_{n\to\infty} a_n=0$, then $\displaystyle\sum_{n=0}^{\infty} a_n$ converges by the Test for Divergence.

(e) If a_n and b_n are two positive sequences with $\displaystyle\lim_{n\to\infty}\frac{a_n}{b_n}=1$ and $\displaystyle\sum_{n=0}^{\infty} b_n$ diverges, then $\displaystyle\sum_{n=0}^{\infty} a_n$ diverges by the Limit Comparison Test.

1. Set up an integral for the area of the shaded region.





A. $\frac{1}{2}\int_{5\pi/8}^{7\pi/8} r^2 \, d\theta$

B. $\frac{1}{2}\int_{7\pi/12}^{11\pi/12} r^2 \, d\theta$

C. $\frac{1}{2}\int_{9\pi/12}^{11\pi/12} r^2 \, d\theta$

D. $\frac{1}{2}\int_{\pi}^{3\pi/2} r^2 \, d\theta$

E. $\frac{1}{2}\int_{3\pi/8}^{7\pi/8} r^2 \, d\theta$

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2. Evaluate the improper integral.

$$\int_e^\infty \frac{(\ln x)^2}{x^2} \, dx$$

A. $\frac{7}{6e}$

B. $\frac{5}{e}$

C. $\frac{7e}{3}$

D. Diverges

E. $\frac{e}{6}$

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3. Find the sum of the series.

$$\sum_{n=1}^{\infty}\left(\frac{(-1)^n\pi^{2n+1}}{2^{2n+1}(2n+1)!}+\frac{1}{2^n}\right) \qquad \textbf{(NOTE: } n \textbf{ starts at 1)}$$

A. $3-\frac{\pi}{2}$ **B.** $2-\frac{\pi}{2}$ **C.** 0 **D.** 1 **E.** $-\frac{\pi}{2}$

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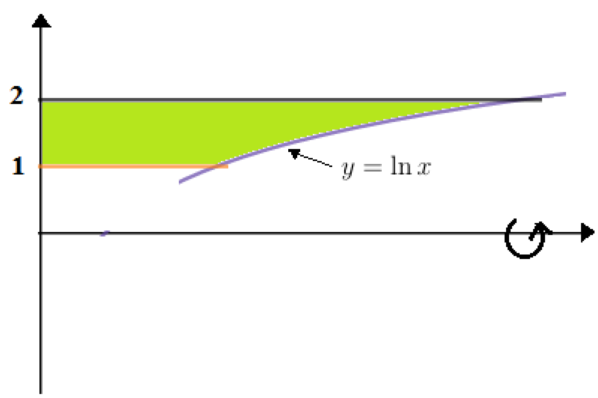
4. Using Shell method, find the volume generated by rotating about the $x-$axis the following region bounded by

$$y=\ln x, \quad y=2, \quad y=1.$$



A. $3e^4\pi$ B. $6e^4\pi$ C. $2e^2\pi$ D. $6e^2\pi$ E. $2e^4\pi$

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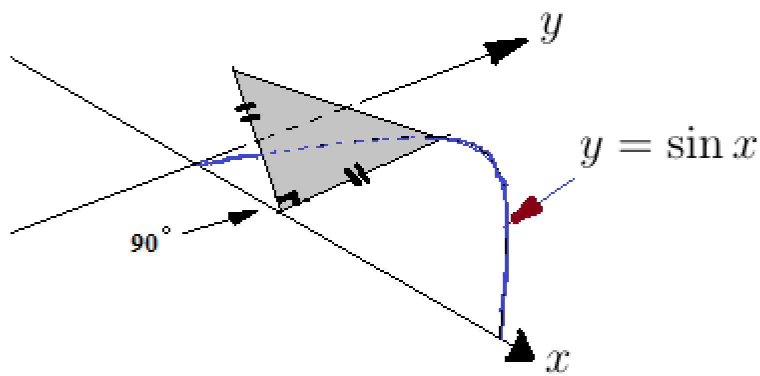
5. The base of a solid is bounded by the curve $y = \sin x$, and the $x-$axis on the interval $[0, \pi]$. The cross sections of the solid perpendicular to the $x-$axis are isosceles right angle triangles with the corner of the right angles lining up along the $x-$ axis. (see diagram) Find the volume of the solid.



A. $\frac{301}{512}$ B. $\frac{\pi}{4}$ C. $\frac{\pi}{10}$ D. $\frac{3}{5}$ E. $\frac{\pi}{8}$

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6. Let $\frac{4y^2 - 7y - 12}{y(y+2)(y-3)} = \frac{A}{y} + \frac{B}{y+2} + \frac{C}{y-3}$. Evaluate $A + C$.

A. 4 B. 2.2 C. 0.5 D. 0 E. 1.8

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7. Calculate $f^{(11)}(1)$ for $f(x) = e^{3x}$ using a Taylor series that is centered at 1.

A. $27\ e^3$ B. 0 C. $11!\ 3^{11}\ e^3$

D. $3^{11}\ e^3$ E. $162\ e^3$

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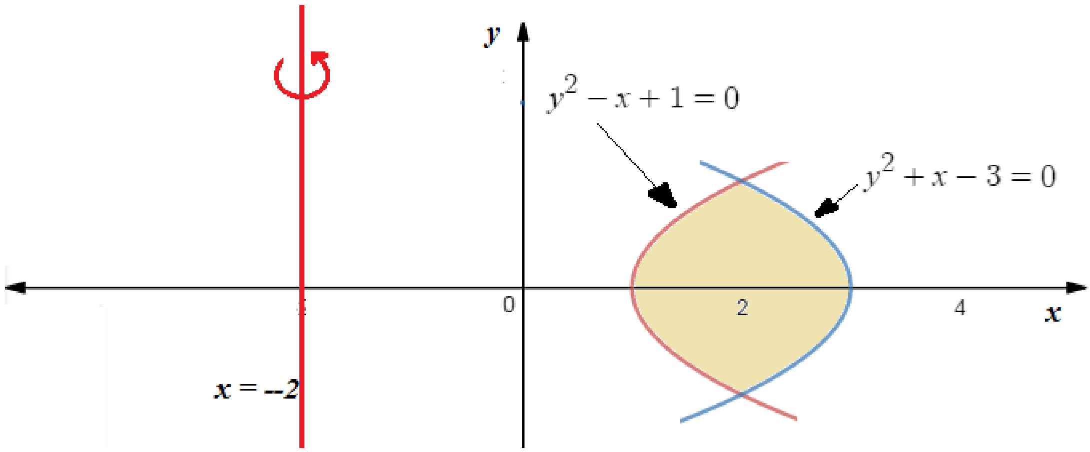
8. Using the washer method, set up an integral to find the the volume of the solid generated by revolving the shaded region bounded by $y^2 - x + 1 = 0$ and $y^2 + x - 3 = 0$ about the line $x = -2$.



A. $\pi \int_{-1}^{1} (5 - y^2)^2 - (3 + y^2)^2 \; dy$

B. $\pi \int_{-1}^{1} (y^2 + 5)^2 - (3 + y^2)^2 \; dy$

C. $\pi \int_{-1}^{1} (5 - y^2)^2 - (3 - y^2)^2 \; dy$

D. $\pi \int_{-1}^{1} (5 - y^2)^2 + (y^2 - 3)^2 \; dy$

E. $\pi \int_{-1}^{1} (y^2 + 5)^2 - (3 - y^2)^2 \; dy$

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9. Which of the following series is convergent?

A. $\sum_{n=7}^{\infty} n \tan\left(\frac{1}{n^2}\right)$

B. $\sum_{n=8}^{\infty} n^2 \sin^2\left(\frac{1}{n}\right)$

C. $\sum_{n=7}^{\infty} \sqrt{n} \tan\left(\frac{1}{n}\right)$

D. $\sum_{n=5}^{\infty} \frac{\tan(1/n)}{n}$

E. $\sum_{n=7}^{\infty} \frac{1}{\sqrt{n}} \sin\left(\frac{1}{\sqrt{n}}\right)$

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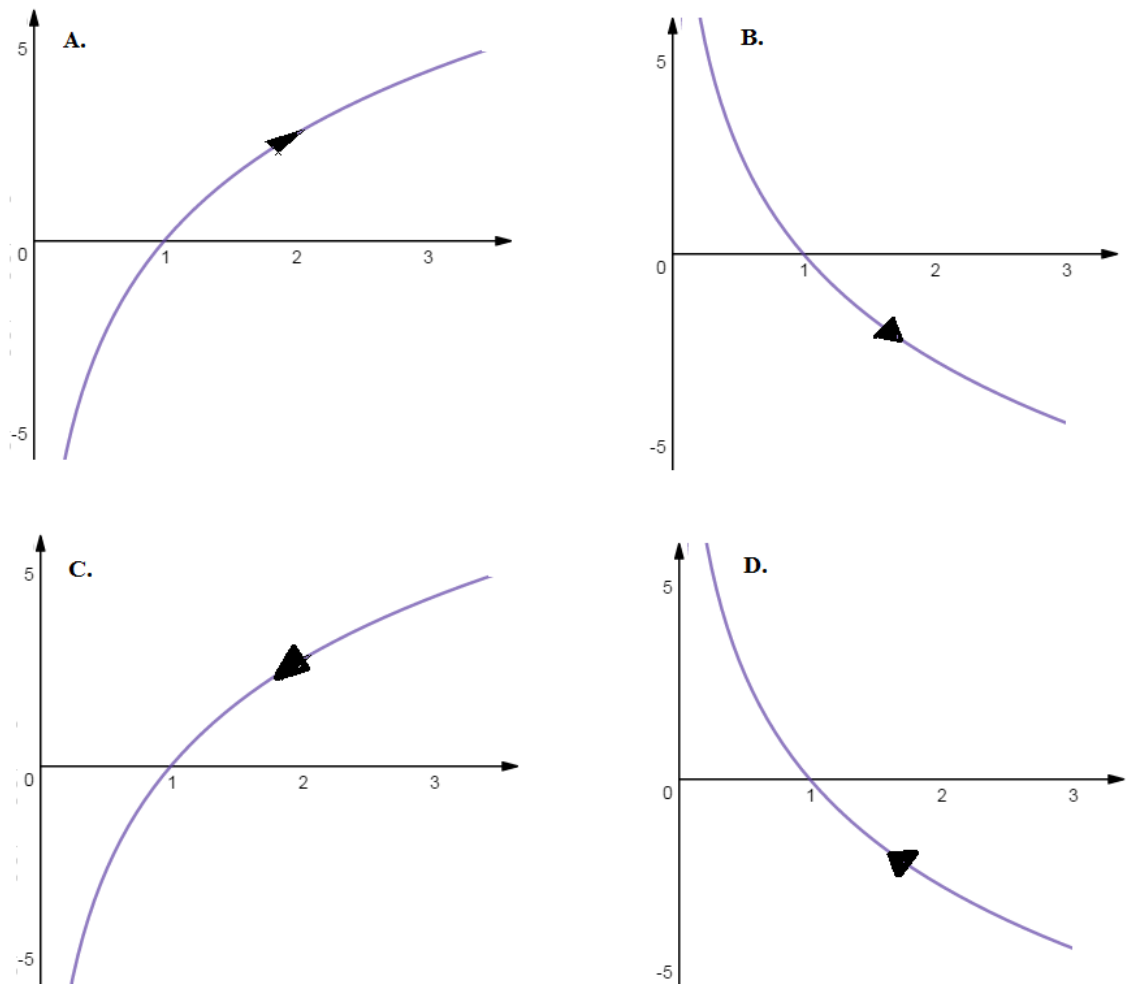
10. Consider the following.

$$x = t^4, \qquad y = \ln(t)$$

Eliminate the parameter to find a Cartesian equation of the curve. Sketch the curve and indicate the direction in which the curve is traced as the parameter increases.



11. Determine whether the series is convergent or divergent.

$$\sum_{n=7}^{\infty} \frac{1}{n\sqrt{8\ln n}}$$

A. convergent B. divergent

12. Which integral below gives the area of the region that lies inside the first curve and outside the second curve?

$$r_1 = 7 - 7\sin\theta, \qquad r_2 = 7$$

A. $\left(\int_{\pi}^{2\pi} r_1^2 \, d\theta\right) - \frac{49}{2}\pi$

B. $\left(\int_{\pi}^{2\pi} r_1^2 \, d\theta\right) - \frac{49}{4}\pi$

C. $\left(\frac{1}{2}\int_{\pi}^{2\pi} r_1^2 \, d\theta\right) - 49\pi$

D. $\left(\frac{1}{2}\int_{\pi}^{2\pi} r_1^2 \, d\theta\right) - \frac{49}{4}\pi$

E. $\left(\frac{1}{2}\int_{\pi}^{2\pi} r_1^2 \, d\theta\right) - \frac{49}{2}\pi$

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13. Find T_2, the Taylor polynomial of degree 2, centered at 5 for $f(x) = \dfrac{1}{x+1}$.

A. $1 - x + x^2 - x^3 + x^4 - x^5$

B. $\dfrac{1}{6} - \dfrac{x-5}{6^2} + \dfrac{(x-5)^2}{6^3}$

C. $\dfrac{1}{6} - \dfrac{x-5}{6^2} + \dfrac{(x-5)^2}{2!\, 6^3}$

D. $1 - x + \dfrac{x^2}{2}$

E. $1 - (x-5) + (x-5)^2$

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14. Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \left(\frac{\cos(1/n)}{8} \right)^n.$$

A. Not enough information

B. The series is divergent

C. The series is absolutely convergent

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15. Use the ratio test to determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{n!}{3e^{n^2}}$$

A. The series is absolutely convergent

B. Not enough information

C. The series is divergent

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16. What can you say about the sequence $\{a_n\}$ and the series $\sum a_n$ if $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \frac{1}{2}$?

A. $\{a_n\}$ converges to 0 and $\sum a_n$ converges.

B. $\{a_n\}$ diverges and $\sum a_n$ diverges.

C. $\{a_n\}$ diverges and $\sum a_n$ converges.

D. $\{a_n\}$ converges to 0, not enough information about series $\sum a_n$.

E. $\{a_n\}$ converges to $\frac{1}{2}$ and $\sum a_n$ converges.

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17. Consider the series $\sum_{n=1}^{\infty}(-1)^n \arctan\left(\frac{4}{n}\right) = \sum_{n=1}^{\infty}(-1)^n b_n$.

Indicate whether the Alternating Series Test (AST) can be used; if it can be used, indicate whether the series converges or diverges.

A. The series converges by the AST.

B. The AST is inconclusive because the sequence of b_n is not decreasing.

C. The AST is inconclusive because $\lim_{n\to\infty} b_n \neq 0$.

D. The series is not alternating, therefore the AST can not be used.

E. By the AST, the series diverges because $\lim_{n\to\infty} b_n \neq 0$.

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18. Using appropriate trigonometric substitution, the integral is transformed into which below in terms of θ?

$$\int \sqrt{x^2 + 10x}\, dx$$

A. $\frac{25}{2}(\ln|\sec\theta + \tan\theta| - \sec\theta\ \tan\theta) + C$

B. $\frac{25}{2} + \frac{25}{4}\sin(2\theta) + C$

C. $\frac{25}{2}(\sec\theta\ \tan\theta + \ln|\sec\theta + \tan\theta|) + C$

D. $\frac{25}{2}(\sec\theta\ \tan\theta - \ln|\sec\theta + \tan\theta|) + C$

E. $\frac{25}{2} - \frac{25}{4}\sin(2\theta) + C$

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19. Find the radius of convergence, R, and the interval of convergence, I, of the series.

$$\sum_{n=1}^{\infty} \frac{(-1)^n (2x-1)^n}{\sqrt[4]{n}}$$

A. $R = \frac{1}{2}, \quad I = \left(-\frac{1}{2}, \frac{3}{2}\right]$

B. $R = 1, \quad I = (0, 2]$

C. $R = 0, \quad I = \{0\}$

D. $R = \frac{1}{2}, \quad I = (-1, 2]$

E. $R = \frac{1}{2}, \quad I = (0, 1]$

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20. Determine whether the telescoping series is convergent or divergent. find the sum if convergent.

$$\sum_{n=1}^{\infty} \left(e^{5/n} - e^{5/(n+1)}\right)$$

A. divergent

B. the series converges to $e^5 - 1$

C. the series converges to $1 - e^5$

D. the series converges to $e^5 + 1$

E. the series converges to e^5

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21. Evaluate $\int_0^1 x^5 \ e^{x^3} \ dx$.

A. $\frac{3}{5}$

B. $\frac{1}{3}(e-1)$

C. $\frac{1}{3}(e^3 - e)$

D. $\frac{1}{3}$

E. $e^3 - e + 1$
