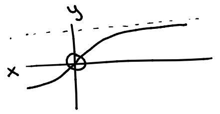
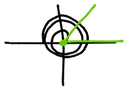
MAC2312 Exam 3 Review Answers

Wednesday, December 2, 2020 3:46 PM

① $r = \arctan\theta \quad \theta \geq 0$





$\boxed{b}$

② $f(x) = 4\arctan(x) - \pi$

$f(1.1) \qquad a = 1$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)(x-a)^n}{n!}$$

n	$f^{(n)}(a)$	$f^{(n)}(1)$
0	$4\arctan(x) - \pi$	$4\frac{\pi}{4} - \pi = 0$
1	$4\frac{1}{(x^2+1)}$	$\frac{4}{2} = 2$
2	$-4\frac{2x}{(x^2+1)^2}$	$\frac{-\cancel{4}\cdot 2}{\cancel{4}} = -2$

$$\frac{f^{(0)}(1)(x-a)^0}{0!}$$

$x = 1.1$

$$0 + \frac{2(\overset{.1}{x-1})^1}{1!} - \frac{2(x-1)^2}{2!}$$

$2(0.1) - (0.01) = \boxed{0.19} \quad \boxed{c}$

③ $\overset{0}{3} - \overset{1}{\frac{\pi^2}{2!\cdot 3}} + \overset{2}{\frac{\pi^4}{4!\cdot 3^3}} \cdots$

$$\sum_{n=0} \frac{3\,(-1)^n\,\pi^{2n}}{(2n)!\;3^{2n}} = 3\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}\Big(\underset{\underset{x}{\uparrow}}{\frac{\pi}{3}}\Big)^{2n}$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$3\cos\frac{\pi}{3} = \boxed{\frac{3}{2}} \quad \boxed{a}$

④ circle

$b\sin(ax)$ & $b\cos(ax)$

✓ • $3\sin t$ & $3\cos t$

✓ • $\cos 2t$ & $\sin 2t$

✗ • $\cos t$ & $3\sin t$

✗ • $\cos t$ & $\cos t$

$\boxed{2} \quad \boxed{c}$

⑤ $g(x) = \int \boxed{\ln(1+x^2)}\,dx \qquad [-1,1]$

$$\ln(1+x^2) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{2n}}{n} \qquad (-1,1]$$

$\sum \frac{(-1)^{n+1}}{n} \cancel{(1)^{2n}}$ ✓ alternating series

$\sum \frac{(-1)^{n+1} \cancel{(-1)^{2n}}}{n}$ ✓

$\boxed{d}$

⑥ $f(x) = \overset{0}{2} - \overset{1}{2(x+3)} - \overset{3}{2(x+3)^3} + \overset{4}{6(x+3)^4}$

$\uparrow$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)(x-a)^n}{n!} \qquad \frac{f(a)}{0!} = 2$$

$f(-3) = 2$ ✓

$\frac{f'(a)}{} = -2$

$\sum_{n=0} \frac{\quad}{n!}$ $\qquad \frac{\quad}{0!}$ - -

$f(-3) = 2$ ✓ $\qquad \frac{f'(a)}{1!} = -2$

$f'(-3) = -2$ ✓ $\qquad \frac{f''(a)}{2!} = 0$

$f''(-3) = 0$ ✓

$\boxed{d}$ $f'''(-3) = -2$ ✗ $\qquad \frac{f'''(a)}{3!} = -2$

conv. @ $x = -3$ ✓ $\qquad f'''(a) = -12$

⑦ $r = \sqrt[3]{\cos\theta + \sin\theta}$

$(r^3 = \cos\theta + \sin\theta)\, r$ $\qquad y = r\sin\theta$

$\qquad x = r\cos\theta$

$(r^2)^2 \; r^4 = r\cos\theta + r\sin\theta$ $\qquad r^2 = x^2 + y^2$

$\boxed{(x^2+y^2)^2 = x + y}$ $\qquad \boxed{a}$

⑧ $\lim_{x \to 0} \frac{x^2 e^x}{\cos x - 1}$

$\qquad n = 0 \quad 1 \quad 2 \quad 8$

$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!}$

$\frac{x^2 e^x}{(\cancel{1} - \frac{x^2}{2!} + \frac{x^4}{4!}) \cancel{-1}} = \frac{\cancel{x^2} e^x}{-\frac{\cancel{x^2}}{2!} + \frac{\cancel{x^4} x^2}{4!} \cdots} = \frac{e^x}{-\frac{1}{2!} + \frac{x^2}{4!} \cdots}$

$\lim_{x \to 0} \frac{e^x}{-\frac{1}{2!} + \frac{x^2}{4!} - \frac{x^4}{6!} \cdots} = \frac{1}{-\frac{1}{2}} = \boxed{-2}$ $\qquad \boxed{d}$

⑨ $f^{(15)}(0) \qquad f(x) = e^{-x^5}$

$\frac{f^{(15)}(0)\; x^{15}}{15!}$

$\qquad @\, n = 3$

$e^{♥} = \sum \frac{♥^n}{n!} \qquad \sum_{n=0}^{\infty} \frac{(-x^5)^n}{n!} \qquad -\frac{\cancel{x^{15}}}{3!} = \frac{f^{(15)}(0)\, \cancel{x^{15}}}{15!}$

$\boxed{\frac{-15!}{6}} = f^{(15)}(0)$

$\boxed{b}$

⑩ $\frac{1}{(7+x)^3} = \frac{1}{(7(1-(\frac{-x}{7})))^3}$

$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$

$\frac{1}{7+x} = \frac{1}{7(1 - -\frac{x}{7})} = \frac{1}{7} \sum \left(-\frac{x}{7}\right)^n$

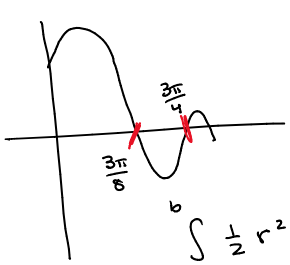
$(7+x)^{-1} = \sum \frac{(-1)^n x^n}{7^{n+1}}$

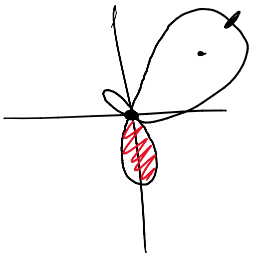
$\frac{1}{7+x^3} \qquad \frac{-1}{(7+x)^2} = \frac{\sum (-1)^n n x^{n-1}}{7^{n+1}}$

$\frac{2}{(7+x)^3} \qquad \frac{1}{(7+x)^3} = \boxed{\sum \frac{(-1)^n n(n-1) x^{n-2}}{2 \cdot 7^{n+1}}}$

$\boxed{d}$

⑪





$$\int_a^b \frac{1}{2} r^2 \, d\theta$$

$$\boxed{\begin{matrix} a = \frac{3\pi}{8} \\ b = \frac{3\pi}{4} \end{matrix}}$$

$\boxed{e}$

⑫

$$\int_0^1 \sqrt{\left(\frac{dy}{d\theta}\right)^2 + \left(\frac{dx}{d\theta}\right)^2} \, d\theta$$

$x'(\theta) = 2(1+\theta)$

$y'(\theta) = 3(1+\theta)^2$

$$\int_0^1 \sqrt{4(1+\theta)^2 + 9(1+\theta)^4} \, d\theta$$

$$\int_0^1 \underline{(1+\theta)} \sqrt{4 + 9(1+\theta)^2} \, \underline{d\theta}$$

$u = 4 + 9(1+\theta)^2$ < $u = 40$, $u = 13$

$du = 18 \underline{(1+\theta)} \, \underline{d\theta}$

$$\frac{1}{18}\int_{13}^{40} u^{1/2} \, du = \frac{\cancel{2}^{1}}{\cancel{18}_{9} \cdot 3} u^{3/2}\Big|_{13}^{40}$$

$$\boxed{\frac{1}{27}\left(40^{3/2} - 13^{3/2}\right)}$$

$\boxed{b}$

①

$x(t) = 2t^3 - 15t^2 + 24t + 7$

$y(t) = t^2 + t + 1$

HTL —— $\frac{dy}{dt} = 0$

VTL | $\frac{dx}{dt} = 0$

$y' = 2t + 1 = 0$

$t = -\frac{1}{2}$

$y(-\frac{1}{2}) = \frac{1}{4} + \frac{-1}{2} + 1 = \frac{3}{4}$

$y = \frac{3}{4}$

$x' = 6t^2 - 30t + 24$

$6(t^2 - 5t + 4)$

$(t-4)(t-1)$

$t = 1 \quad t = 4$

$x(1) = 18$

$x(4) = -9$

② $f(x) = \sqrt{3+x}$

a)

n	$f^{(n)}(a)$	$f^{(n)}(1)$
0	$(3+x)^{1/2}$	$4^{1/2} = 2$
1	$\frac{1}{2}(3+x)^{-1/2}$	$\frac{1}{2}(4)^{-1/2} = \frac{1}{4}$
2	$\frac{1}{2}(-\frac{1}{2})(3+x)^{-3/2}$	$-$
3	$\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})(3+x)^{-5/2}$	$+$
4	$\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})(3+x)^{-7/2}$	$-$
n	$\sum\limits_{n=2} \frac{1\cdot3\cdot5\cdots(2n-3)(-1)^{n+1}}{2^n(3+x)^{\frac{(2n-1)}{2}}}$	$\frac{(-1)^{n+1}\,1\cdot3\cdot5\cdots(2n-3)}{2^n(\sqrt{4})^{2n-1}}$

$\uparrow$ $2^n(2^{2n-1}) = 2^{3n-1}$

$$\frac{f^{(n)}(1)(x-1)^n}{n!}$$

$$\boxed{2+\frac{1}{4}(x-1)+\sum_{n=2}^{\infty}\frac{(-1)^{n+1}\,1\cdot3\cdot5\cdots(2n-3)}{n!\,2^{3n-1}}(x-1)^n}$$

b) RoC

ratio

$n+1$ $\quad 2n+2-3$ $\quad 2(n+1)-3$

$$\lim_{n\to\infty}\left|\frac{\cancel{1\cdot3\cdot5\cdots(2n-3)}(2n-1)}{\cancel{1\cdot3\cdot5\cdots(2n-3)}}\cdot\frac{\cancel{n!}}{(n+1)\cancel{n!}}\cdot\frac{\overset{\cancel{2^{3n}}\,2^{-1}}{2^{3n-1}}}{\underset{\cancel{2^{3n}}\,2^2}{2^{3n+2}}}\;\frac{\overset{\cancel{(x-1)^n}(x-1)}{(x-1)^{n+1}}}{\cancel{(x-1)^n}}\right| = \frac{\cancel{2}\cdot\cancel{2^{-1}}}{2^2}|x-1|$$

$$|x-1|\lim_{n\to\infty}\frac{2n-1}{n+1}\cdot\frac{1}{8} = \frac{2}{8}|x-1| = \frac{1}{4}|x-1| < 1$$

$\frac{1}{4}|x-1| < 1$

$|x-1| < \boxed{4 = R}$

c) $\sqrt{4.1} = \sqrt{3+x}$

$x = 1.1$

$$2+\frac{1}{4}(x-1)+\frac{(-1)(1)(x-1)^2}{2!\;2^{3(2)-1}}$$

$$2+\frac{1}{4}(1.1-1)-\frac{(1.1-1)^2}{2\cdot 2^5}$$

$$\boxed{2+\frac{1}{4}(.1)-\frac{0.01}{2^6}}=\boxed{2+\frac{1}{40}-\frac{1}{6400}}$$

③ $r = 2\sin\theta - \sqrt{3} = 0$

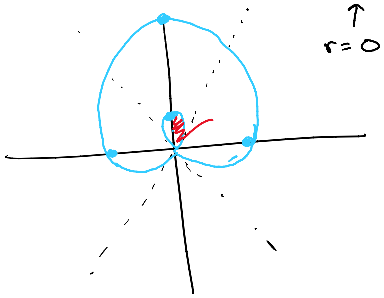
$\sin\theta = \frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{3}, \frac{2\pi}{3}$

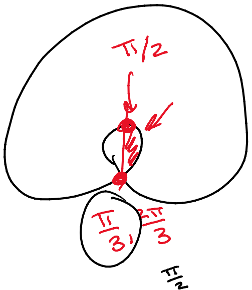
↑

$r=0$

a)



0	$-\sqrt{3}$
$\frac{\pi}{2}$	$2-\sqrt{3}$
π	$-\sqrt{3}$
$\frac{3\pi}{2}$	$-2-\sqrt{3}$
2π	$-\sqrt{3}$



b) $\frac{1}{2}\int_a^b r^2\, d\theta$

$\frac{1}{2}\int_0^{2\pi} r^2\, d\theta - \int_{\pi/3}^{\pi/2} r^2\, d\theta$

$\frac{1}{2}\int_0^{2\pi} r^2 d\theta - \frac{1}{2}\int_{\pi/3}^{2\pi/3} r^2\, d\theta$

$2\ \frac{1}{2}\int_{\pi/3}^{\pi/2} r^2\, d\theta$

$\int_{\pi/3}^{\pi/2} r^2\, d\theta$

① $f^{(n)}(4) = \frac{(-1)^n\, n!}{2^n\,(n+1)}$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)\,(x-a)}{n!}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n \cancel{n!}}{2^n(n+1)\cancel{n!}}(x-4)^n = \sum \frac{(-1)^n}{2^n(n+1)}(x-4)^n$$

$$\left|\frac{x-4}{2}\right| < 1$$

$\boxed{a}$

$|x-4| < \boxed{2 = R}$

$(2, 6]$

$\Sigma (r)^n$

$|r| < 1$

$$\Sigma \frac{(-1)^n \left(\frac{-2}{2}\right)^n}{n+1} = \Sigma \frac{1}{n+1}$$

$(-1)^n$

② $\ln(1+7x)$

$$\ln(1+7x) = \boxed{\Sigma \frac{(-1)^{n+1}(7x)^n}{n}}$$

$(-1, 1]$

$\frac{(-1)^{n+1}(1)^n}{n}$ conv. alt.

$|7x| < 1$

$-\frac{1}{7} \le x < \frac{1}{7}$

$\frac{(-1)^{n+1}(-1)^n}{n} = \frac{1}{n}$ div

$\boxed{\left(-\frac{1}{7}, \frac{1}{7}\right]}$

$\boxed{b}$

③ $f(x) = x^3 e^{3-x^3} = x^3 e^3 e^{-x^3}$

$$\frac{f^{(60)}(0)\,x^{60}}{60!}$$

$$\Sigma \frac{(-x^3)^n}{n!} \cdot x^3 \cdot e^3$$

$$\frac{(-1)^n x^{3n+3} \cdot e^3}{n!}$$

$$\frac{(-1)\cancel{x^{60}}\,e^3}{19!} = \frac{f^{(60)}(0)\cancel{x^{60}}}{60!}$$

$3n+3 = 60$

$$\boxed{-\frac{60!\,e^3}{19!}} = f^{(60)}(0)$$

$\boxed{b}$

④ $\int \frac{\ln(1-t)}{6t}\,dt$ $[-1,1)$

$-\Sigma \frac{t^n}{n\cdot 6t} = -\int \Sigma \frac{t^{n-1}}{6n}\,dt$ $[1,1)$

$\boxed{-\Sigma \frac{t^n}{6n^2}}$

$\boxed{a}$

⑤ $\Sigma x^n = \frac{1}{1-x}$

$\Sigma n x^{n-1} = \frac{1}{(1-x)^2}$

$\cdot\, n\left(\frac{1}{11}\right)^n 11$

$\frac{1}{(1-11)^2} = \boxed{\frac{11}{100}}$

$\boxed{c}$

$\Sigma \frac{n}{11^n}$

$x = \frac{1}{11}$

$\Sigma x^n = \frac{1}{1-x}$

$\Sigma n\, x^{n-1} = \frac{1}{(1-x)^2}$

$\Sigma n x^n x^{-1} = \frac{1}{(1-x)^2}$

$\Sigma n\left(\frac{1}{11}\right)^n 11 = \frac{1}{(1-11)^2} = \frac{1}{100}$

$\Sigma \frac{n}{11^n}$ $\frac{1}{1100}$

⑥ $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n} (1)^n$ $\underline{(-1)^2 (1)^1}$

$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^n$

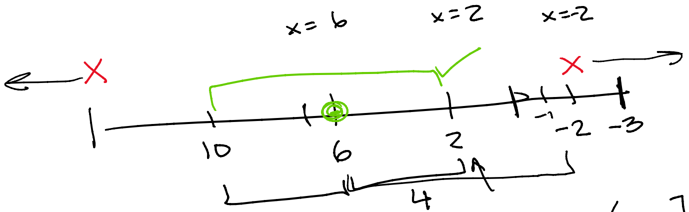
$\sum_{n=1}^{\infty} - S_1$

$\boxed{\ln 2 - 1}$

$\boxed{c}$

⑦

$\Sigma c_n \underbrace{(x-6)}^n$



$(10,2]$

P $\Sigma c_n (1)^n$ $x-6=1$ conv

$x = 7$

Q $\Sigma 4^n c_n$ $x-6=4$ not enough

$x=10$

$)^n$ $x-6=-6$ not enough

$x = 10$

R $\quad \Sigma c_n(-6)^n \quad$ $x-6=-6$ $\quad$ not enough

$x=0$

S $\quad \Sigma c_n(-9)^n \quad$ $x-6=-9$ $\quad$ $\underline{\text{div}}$

$x=-3$

$\boxed{a}$

⑧ $\int \sqrt{\frac{dx}{dt}^2 + \frac{dy}{dt}^2}$

$x'(t) = e^t - 1$

$y'(t) = \frac{4}{2} e^{t/2}$

$\uparrow$

$2e^{t/2}$

$\sqrt{(e^t-1)^2 + 4e^t}$

$e^{2t} - 2e^t + 1 + 4e^t$

$e^{2t} + 2e^t + 1$

$\sqrt{(e^t+1)^2}$

$\boxed{\int_0^2 (e^t+1)\, dt}$

$\boxed{b}$

⑨ $2 \int_a^{\pi/2} \frac{1}{2}(2-4\sin\theta)^2\, d\theta$

$\boxed{a = \frac{\pi}{6}}$ $\boxed{a}$

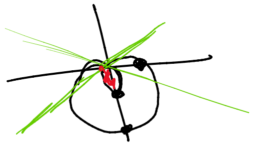
$\swarrow \frac{\pi}{2}$

$r = 2-4\sin\theta$

$r = 0 = 2-4\sin\theta$

$\frac{1}{2} = \sin\theta$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$



⑩ $x = te^t \quad y = e^{5t}$

$x = e = te^t$

$(e, e^5) \quad t=1$

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{5e^{5t}}{te^t + e^t} = \frac{5e^5}{e^1+e^1} = \frac{5e^5}{2e} = \boxed{\frac{5}{2}e^4}$$

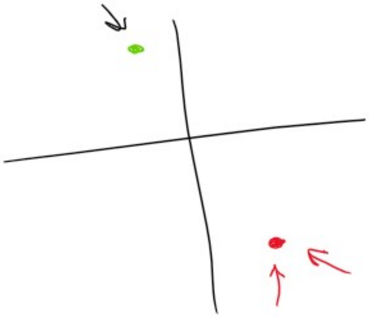
$\boxed{d}$

⑪ (r, θ) $\left(-3, \frac{2\pi}{3}\right)$

$\left(3, -\frac{\pi}{3}\right)$ ✓ $\left(-3, -\frac{4\pi}{3}\right)$ ✓

$\left(3, \frac{5\pi}{3}\right)$ ✓ $\left(-3, -\frac{10\pi}{3}\right)$ ✓

$-\frac{6\pi}{3}$ $-\frac{4\pi}{3}$



✗ $\left(3, \frac{8\pi}{3}\right)$

$\frac{6\pi}{3}$ $\frac{2\pi}{3}$

$\boxed{e}$

① $r_1 = \sin\theta$ $r_2 = 1 - \sin\theta$

a) $\sin\frac{\pi}{6} = \frac{1}{2}$ $1 - \frac{1}{2} = \frac{1}{2}$ ✓

$\sin\frac{5\pi}{6} = \frac{1}{2}$ $1 - \frac{1}{2} = \frac{1}{2}$ ✓

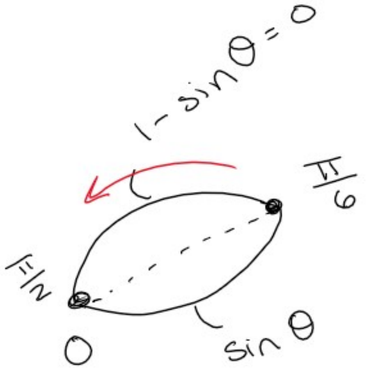
b) $r_1 = (0,0)$ $\left(1, \frac{\pi}{2}\right)$ $(0, \pi)$ $\left(-1, \frac{3\pi}{2}\right)$

$(0, 2\pi)$

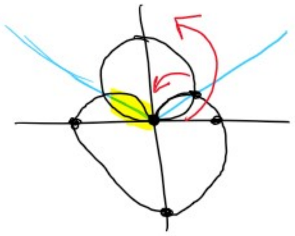
$r_2 = (1, 0)$ $\left(0, \frac{\pi}{2}\right)$ $(1, \pi)$ $\left(2, \frac{3\pi}{2}\right)$

$(1, 2\pi)$

$$\frac{1}{2}\int_a^b r^2\,d\theta$$



$$\int_0^{\frac{\pi}{6}} \sin\theta^2\,d\theta + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (1-\sin\theta)^2\,d\theta$$



$$2\left(\frac{1}{2}\int_0^{\frac{\pi}{6}} (\sin\theta)^2\,d\theta + \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (1-\sin\theta)^2\,d\theta\right)$$

$$\boxed{\int_0^{\frac{\pi}{6}} \sin^2\theta\,d\theta + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (1-\sin\theta)^2\,d\theta}$$