Exam 3 Review

Wednesday, July 21, 2021 4:18 PM

Power Series

- Radius & Interval of Convergence
- Functions
- Taylor & Maclaurin

Parametric

- w/ calculus
- Arc length

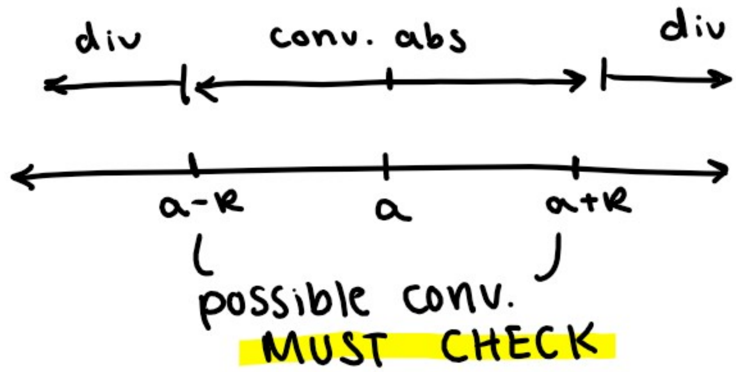
Power Series

$$\sum_{n=a}^{\infty} c_n(x-a)^n = c_0 + c_1(x-a)^1 + c_2(x-a)^2 + \cdots c_n(x-a)^n$$

always converge @ its center (a)

Radius & Interval of Convergence (ROC & IOC)

$$|x-a| < R \Rightarrow \quad -R < x-a < R$$



IOC symmetric about the center

Step 1: Find Radius of conv.

(usually w/ *Ratio*
Sometimes w/ Root
& geometric when applicable)

Step 2: Check conv. @ end points

(not w/ Ratio or Root
unless $R = 0$ or $R = \infty$)

Functions

Manipulate, substitute
differentiate or integrate

differentiating:

$$f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1(x-a)^1 + c_2(x-a)^2 \ldots$$

$$= \frac{d}{dx}(\cancel{c_0} + c_1(x-a)^1 + c_2(x-a)^2 \ldots)$$

$$f'(x) = \sum_{n=1}^{\infty} c_n \cdot n (x-a)^{n-1} = c_1 + c_2 \cdot 2 \cdot (x-a)^1$$

↳ can lose end points

integrating:

$$\int \sum_{n=0}^{\infty} c_n (x-a)^n \, dx = \sum_{n=0}^{\infty} \frac{c_n (x-a)^{n+1}}{n+1} + C$$

↳ can gain endpoints

Known Maclaurin Series (will be given)

f(x)		Series	IOC
$\frac{1}{1-x}$	=	$\sum_{n=0}^{\infty} x^n$	$(-1,1)$
$\ln(1-x)$	=	$-\sum_{n=1}^{\infty} \frac{x^n}{n}$	$[-1,1)$
$\ln(1+x)$	=	$\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{n}$	$(-1,1]$
$\arctan x$	=	$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$	$[-1,1]$
e^x	=	$\sum_{n=0}^{\infty} \frac{x^n}{n!}$	$(-\infty,\infty)$
$\cos x$	=	$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$	$(-\infty,\infty)$
$\sin x$	=	$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$	$(-\infty,\infty)$

Taylor Series

Taylor Series

centered @ a :

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n \qquad |x-a| < R$$

to find Taylor series from a function :

n	$f^{(n)}(x)$	$f^{(n)}(a)$
0	$f(x)$	$f(a)$
1	$f'(x)$	$f'(a)$
2	$f''(x)$	$f''(a)$
$\vdots$	$\vdots$	$\vdots$
n		

← try to find the pattern

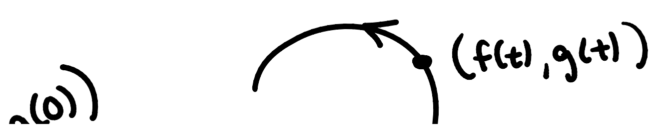
finding the n^{th} derivative :

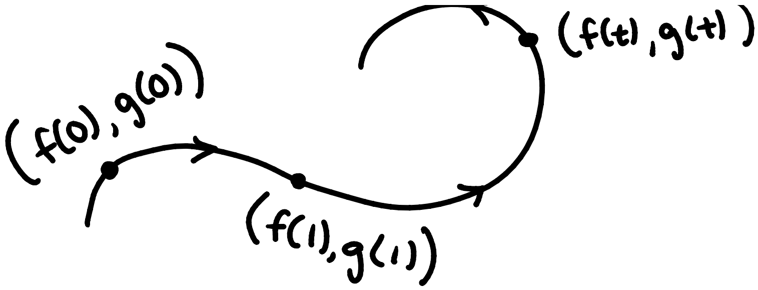
$$f^{(n)}(a) = n!\, c_n$$

Parametric

$$x = f(t) \; , \; y = g(t)$$

$(f(t), g(t))$ ← coordinantes





to graph:

t	x	y
0	f(0)	g(0)
1	f(1)	g(1)
2	f(2)	g(2)

parametric circles:

$x(t) = a\sin(bx)$ & $y(t) = a\cos(bx)$

$x(t) = a\cos(bx)$ & $y(t) = a\sin(bx)$

Calculus w/ Parametric

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

Arc Length

$$L = \int_{\alpha}^{\beta} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2} \, dt$$

regular coordinates:

$$L = \int_{a}^{b} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

exponent rules to know:

$$x^{a+b} = x^a x^b$$

$$x^{a-b} = \frac{x^a}{x^b}$$

$$x^{a \cdot b} = (x^a)^b$$

$$x^{a/b} = (x^a)^{1/b} = \sqrt[b]{x^a}$$

logarithmic rules

$$\log(ab) = \log(a) + \log(b)$$

$$\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$$

$$a\log(b) = \log(b^a)$$