Multiple Choice:

version A: B C A C D, D A D B D, E B

version B: C C E D C, A B D A E, E B

version C: C D C A C, B A C E A, D E

version D: D C D D B, A C D D E, C E

Free Response:

version A:

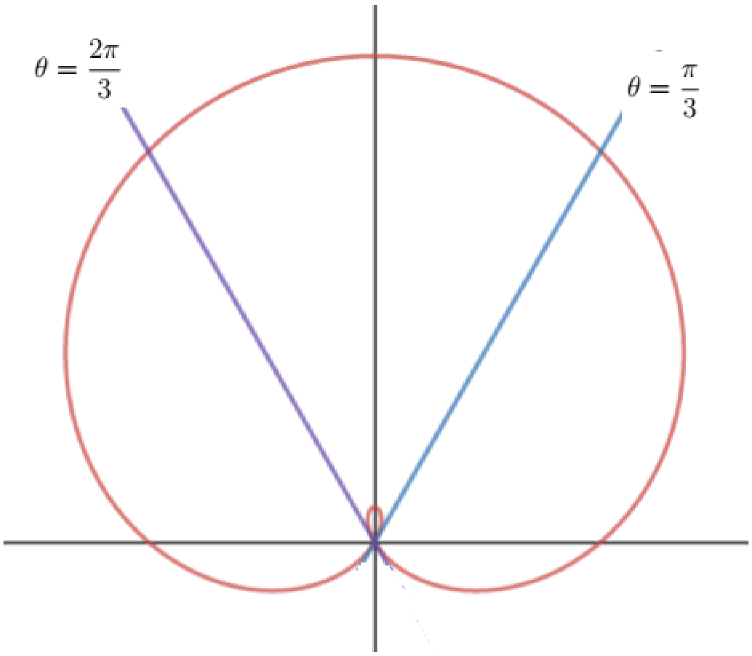
1. HTL at $t = -\frac{1}{2}$, $y = \frac{3}{4}$.

VTL at $t = 1$, $x = 18$;
VTL at $t = 4$, $x = -9$.

2. (a) $f(x) = \sqrt{3+x} = 2 + \frac{1}{4}(x-1) + \sum_{n=2}^{\infty} \frac{(-1)^{n+1} 1 \cdot 3 \cdots (2n-3)}{2^{3n-1} \cdot n!}(x-1)^n$

(b) Ratio test. $R = 4$

(c) $\sqrt{4.1} \sim 2 + \frac{1}{40} - \frac{1}{6400}$



3. (a)

(b)

$$A = \int_{2\pi/3}^{\pi} r^2 \, d\theta + \int_{\pi}^{3\pi/2} r^2 \, d\theta - \int_{\pi/3}^{\pi/2} r^2 \, d\theta$$

OR,

$$A = \frac{1}{2} \int_0^{2\pi} r^2 \, d\theta - \int_{\pi/3}^{2\pi/3} r^2 \, d\theta$$

version B:

1. HTL at $t = -\frac{3}{2}, \quad y = \frac{-3}{4}$.

   VTL at $t = 1, \quad x = 12$;
   VTL at $t = 2, \quad x = 11$.

2. (a) $f(x) = \sqrt{2+x} = 2 + \frac{1}{4}(x-2) + \sum_{n=2}^{\infty} \frac{(-1)^{n+1} 1 \cdot 3 \cdots (2n-3)}{2^{3n-1} \cdot n!}(x-2)^n$

   (b) Ratio test. $R = 4$

   (c) $\sqrt{4.1} \sim 2 + \frac{1}{40} - \frac{1}{6400}$

3. See version (A)

version C:

1. HTL at $t = -1, \quad y = 2$.

VTL at $t = 2, \quad x = 25$;
VTL at $t = 4, \quad x = 21$.

2. (a) $f(x) = \sqrt{1+x} = 2 + \frac{1}{4}(x-3) + \sum_{n=2}^{\infty} \frac{(-1)^{n+1} 1 \cdot 3 \cdots (2n-3)}{2^{3n-1} \cdot n!}(x-3)^n$

(b) Ratio test. $R = 4$

(c) $\sqrt{4.1} \sim 2 + \frac{1}{40} - \frac{1}{6400}$

3. See version (A)

version D:

1. HTL at $t = -2$, $y = -2$.

   VTL at $t = 1$, $x = 13$;
   VTL at $t = 3$, $x = 5$.

2. (a) $f(x) = \sqrt{5+x} = 2 + \frac{1}{4}(x+1) + \sum_{n=2}^{\infty} \frac{(-1)^{n+1} 1 \cdot 3 \cdots (2n-3)}{2^{3n-1} \cdot n!}(x+1)^n$

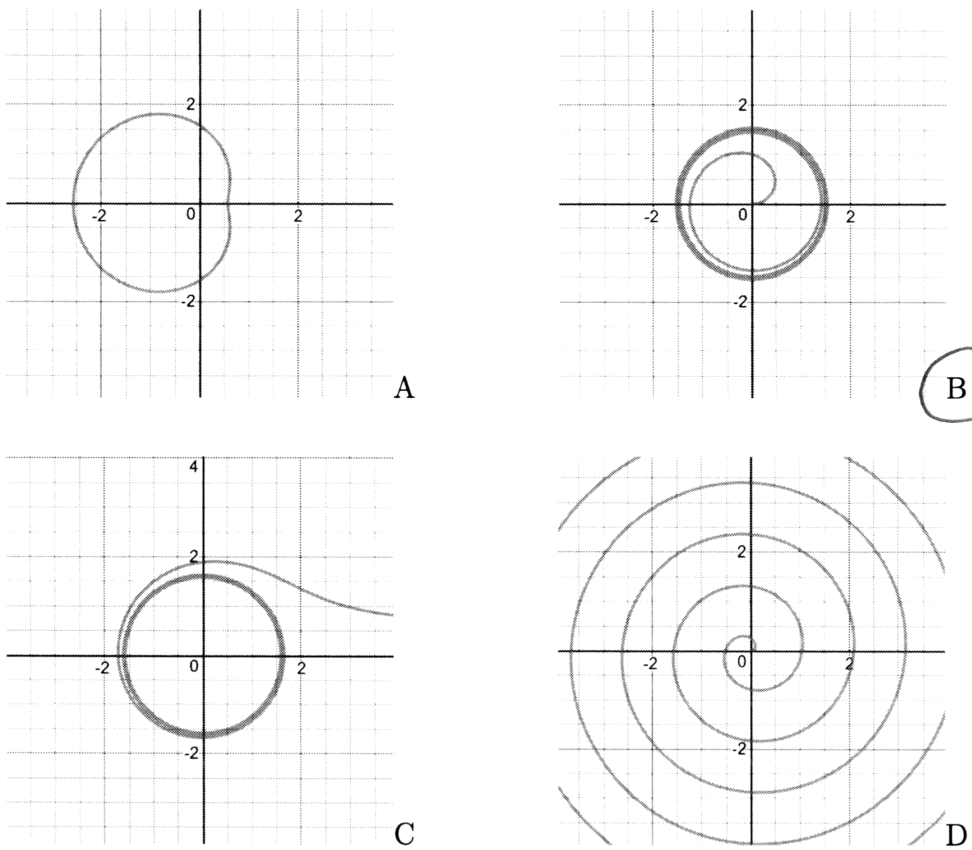
   (b) Ratio test. $R = 4$

   (c) $\sqrt{4.1} \sim 2 + \frac{1}{40} - \frac{1}{6400}$

3. See version (A)

Part I: Multiple Choice There are **12** questions on this portion of the exam. Fill in the answers to these questions on the provided scantron sheet. Only answers on the scantron will be graded. Each problem is worth 2 points, for a total of 24 points on this portion of the exam.

1. Which of the following could be the polar graph of $r = \arctan\theta$ for $\theta \geq 0$?



2. Let $f(x) = 4\arctan(x) - \pi$. What is the approximate value of $f(1.1)$ if you use the second degree Taylor polynomial for $f(x)$ centered at $a = 1$ to estimate it?

   (a) -0.22 (b) 0.18 (c) 0.19 (d) 0.22 (e) 0.99

3. Use a known Maclaurin series to find the sum of the following series.

$$3 - \frac{\pi^2}{2!\cdot 3} + \frac{\pi^4}{4!\cdot 3^3} - \frac{\pi^6}{6!\cdot 3^5} + \frac{\pi^8}{8!\cdot 3^7} + \ldots$$

   (a) $\frac{3}{2}$ (b) $-\frac{3}{2}$ (c) $\frac{1}{6}$ (d) $-\frac{1}{6}$ (e) $\frac{3\pi}{2}$

4. How many of the graphs of the following parametric equations are circles?

- $x(t) = 2 + 3\sin(t), y(t) = -1 + 3\cos(t)$
- $x(t) = \cos(2t), y(t) = \sin(2t)$
- $x(t) = -1 + 1\cos(t), y(t) = 2 + 3\sin(t)$
- $x(t) = 9 + \cos(t), y(t) = 6 + \cos(t)$

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

5. Consider the Maclaurin series for the function

$$g(x) = \int \ln(1 + x^2)\, dx \,, g(0) = 0$$

Which of the following statements is true about the interval of convergence for the Maclaurin series for $g(x)$?

(a) The interval of convergence does not contain either of its endpoints.

(b) The interval of convergence contains the left endpoint, but not the right.

(c) The interval of convergence contains the right endpoint, but not the left.

(d) The interval of convergence contains both of its endpoints.

(e) The interval of convergence is $(-\infty, \infty)$

6. The Taylor series for a function $f(x)$ centered at -3 is given by

$$f(x) = 2 - 2(x + 3) - 2(x + 3)^3 + 6(x + 3)^4 + \cdots$$

Which statement below is FALSE?

(a) $f(-3) = 2$

(b) $f'(-3) = -2$

(c) $f''(-3) = 0$

(d) $f'''(-3) = -2$

(e) The series must converge at $x = -3$

7. Which of the following is the rectangular form of the polar equation $r = \sqrt[3]{\cos(\theta) + \sin(\theta)}$?

(a) $(x^2 + y^2)^2 = x + y$

(b) $x^4 + y^4 = x + y$

(c) $x^2 + y^2 = \sqrt[3]{x + y}$

(d) $(x + y)^{5/3} = 1$

(e) None of the above

8. Using power series, evaluate the limit $\lim_{x \to 0} \dfrac{x^2 e^x}{\cos(x) - 1}$.

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 2 (d) -2 (e) $-\frac{1}{6}$

9. Compute the 15th derivative $f^{(15)}(0)$ of the function $f(x) = e^{-x^5}$.

(a) $\frac{15!}{6}$ (b) $-\frac{15!}{6}$ (c) 1 (d) -1 (e) 0

10. Which of the following is a power series representation of the function $f(x) = \dfrac{1}{(7+x)^3}$?

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{3n} x^{3n}}{7^{3n+3}}$

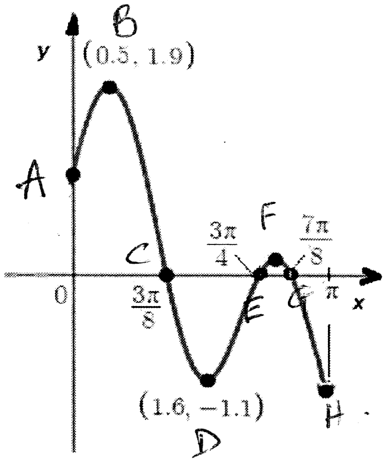
(b) $\displaystyle\sum_{n=2}^{\infty} \frac{n(n-1)x^{n-2}}{2 \cdot 7^{n+1}}$

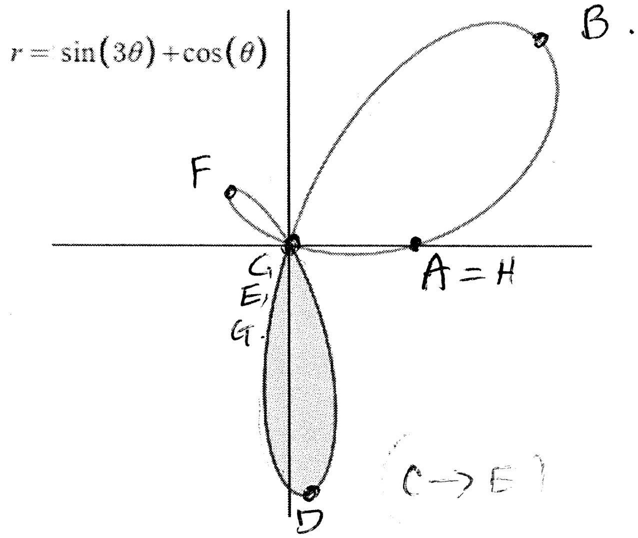
(c) $\displaystyle\sum_{n=2}^{\infty} \frac{n(n-1)x^{n-2}}{7^{n+1}}$

(d) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n n(n-1)x^{n-2}}{2 \cdot 7^{n+1}}$

(e) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n n(n-1)x^{n-2}}{7^{n+1}}$

11. The flat and polar curves of $r = \sin(3\theta) + \cos(\theta)$ are provided below. If the area of the shaded region below is given by the integral $\int_a^b \frac{1}{2}(\sin(3\theta) + \cos(\theta))^2 \, d\theta$, what are the correct values for the bounds a and b?





(a) $a = \frac{11\pi}{8}$ and $b = \frac{13\pi}{8}$

(b) $a = \frac{7\pi}{8}$ and $b = \pi$

(c) $a = \frac{3\pi}{4}$ and $b = \frac{7\pi}{8}$

(d) $a = 0.5$ and $b = 1.6$

(e) $a = \frac{3\pi}{8}$ and $b = \frac{3\pi}{4}$

12. What is the arc length of $x(\theta) = (1+\theta)^2, y(\theta) = (1+\theta)^3$ from $\theta = 0$ to $\theta = 1$?

(a) $\frac{1}{27}(13^{\frac{3}{2}} - 40^{\frac{3}{2}})$ (b) $\frac{1}{27}(40^{\frac{3}{2}} - 13^{\frac{3}{2}})$ (c) $\frac{1}{27}(36^{\frac{3}{2}} - 9^{\frac{3}{2}})$ (d) $\frac{1}{27}(9^{\frac{3}{2}} - 36^{\frac{3}{2}})$ (e) $\frac{1}{27}$

$= \int_0^1 \sqrt{[2(1+\theta)]^2 + [3(1+\theta)^2]^2} \, d\theta = \int \sqrt{4(1+\theta)^2 + 9(1+\theta)^4} \, d\theta$

$= \int (1+\theta)\sqrt{4+9(1+\theta)^2} \, d\theta = \frac{1}{18}\int u^{\frac{1}{2}} du = \frac{1}{18} \cdot \frac{2}{3} u^{\frac{3}{2}} = \frac{1}{27}(4+9(1+\theta)^2)^{\frac{3}{2}}\Big|_0^1$

$= \frac{1}{27}[40^{\frac{3}{2}} - 13^{\frac{3}{2}}]$ (u-sub), $u = 4+9(1+\theta)^2$

Name: ______________________ **Section # :** ____________

UF-ID: ______________________ **TA Name:** ____________

Part II: Free Response There are **3** questions on this portion of the exam. Show **ALL** work clearly in the space provided for each problem, unless the problem says otherwise. Your work must be complete, logical and understandable, or it will receive no credit. Please cross out or fully erase any work that you do not want graded. A total of **16** points are available on this portion of the exam.

FR Scores	
1	/4
2	/7
3	/5
FR Total	/16

The Honor Pledge: "On my honor, I have neither given or received unauthorized aid doing this exam."

Signature: ______________________

1. Find all values of t at which the curve $x(t) = 2t^3 - 15t^2 + 24t + 7, y(t) = t^2 + t + 1$ has horizontal tangent lines, then give equations for these lines. Then, find all of the t values for which this curve has vertical tangent lines, and give equations for those as well.

+0.5 $\left[\frac{dy}{dt} = 2t+1\right] = 0$, $t = -\frac{1}{2}$.

+0.5 $\left[\frac{dx}{dt} = 6t^2 - 30t + 24\right] = 6(t^2 - 5t + 4)$

$= 6(t-1)(t-4) = 0$.

$t = 1, 4$.

HTL @ $t = -\frac{1}{2}$ +0.5, $y = (-\frac{1}{2})^2 + (-\frac{1}{2}) + 1$

$y = \frac{3}{4}$ +0.5

VTL @ $t = 1$ +0.5 $x = 2(1)^3 - 15(1)^2 + 24(1) + 7$

$x = 18$ +0.5

@ $t = 4$ +0.5 $x = 2(4)^3 - 15(4)^2 + 24(4) + 7$.

$x = -9$ +0.5

+0.5 +0.5 +0.5 +0.5

2. Let $f(x) = \sqrt{3+x}$.

(-3)

$= 2 + \frac{1}{4}(x-1) + \sum_{n=2}^{\infty} \frac{(-1)^{n+1}\, 1\cdot 3 \cdots (2n-3)}{2^{3n-1} \cdot n!}(x-1)^n$

+0.5 +0.5

(a) Find the Taylor series for $f(x)$ centered at $a = 1$

	n	$f^{(n)}(x)$	$f^{(n)}(1)$	c_n
	0	$(3+x)^{\frac{1}{2}}$	2	2
+	1	$\frac{1}{2}(3+x)^{-\frac{1}{2}}$	$\frac{1}{4}$	$\frac{1}{4}$
−	2	$\frac{1}{2}(-\frac{1}{2})(3+x)^{-\frac{3}{2}}$		
+	3	$\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})(3+x)^{-\frac{5}{2}}$		
−	4	$\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})(3+x)^{-\frac{7}{2}}$		
	⋮			
	n	$\frac{(-1)^{n+1}\, 1\cdot 3 \cdots (2n-3)}{2^n (3+x)^{\frac{2n-1}{2}}}$, $n \geq 2$	$\frac{(-1)^{n+1}\, 1\cdot 3 \cdots (2n-3)}{2^n \cdot 2^{2n-1}}$	$\frac{(-1)^{n+1}\, 1\cdot 3 \cdots (2n-3)}{2^{3n-1} \cdot n!}$

(b) Find the radius of convergence of this Taylor series.

ratio test

$L = \lim_{n\to\infty} \frac{1\cdot 3 \cdots (2n-3)(2n-1)\, 2^{3n-1} \cdot n!\, |x-1|^{n+1}}{1\cdot 3 \cdots (2n-3)\, 2^{3n+2} \cdot (n+1)!\, |x-1|^n}$] +1

(+2)

$= |x-1| \lim_{n\to\infty} \frac{(2n-1)}{8(n+1)} = \frac{|x-1|}{4} < 1 \Rightarrow |x-1| < 4 = R$

+0.5 +0.5

(c) Using the second degree Taylor polynomial, estimate $\sqrt{4.1}$.

(+2)

$f(x) = \sqrt{3+x} \approx T_2 = 2 + \frac{1}{4}(x-1) - \frac{1}{2^5 \cdot 2}(x-1)^2$] (+1)

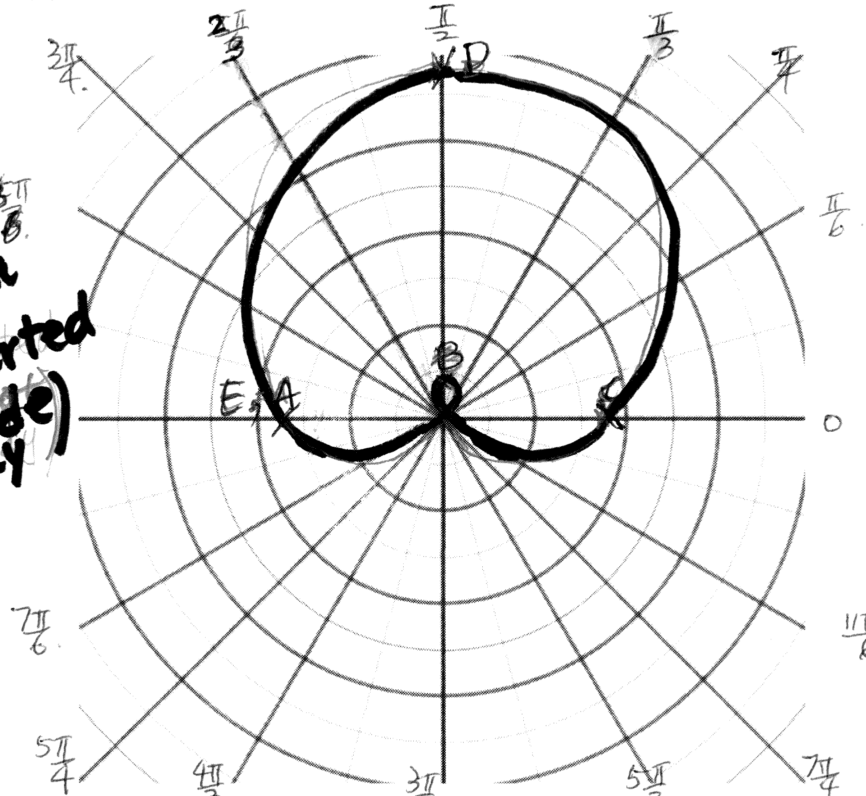
$4.1 = 3 + x \Leftrightarrow x = 1.1$] +0.5

$\sqrt{4.1} = f(1.1) \approx 2 + \frac{1}{4}(0.1) - \frac{1}{2^6}(0.1)^2$] +0.5

or $= 2 + \frac{1}{40} - \frac{1}{6400}$

$r=-\sqrt{3}\left(1-\frac{2}{\sqrt{3}}\sin\theta\right)$. $c = \frac{2}{\sqrt{3}}$, $c>1$ (inner loop).

3. (a) Sketch a graph of the limaçon $r = 2\sin(\theta) - \sqrt{3}$ in the provided grid.



+1 limacon
+1 inverted (not side way)

$r=0 \iff \sin\theta = \frac{\sqrt{3}}{2}$, $\theta = \frac{\pi}{3}, \frac{2\pi}{3}$.

$r(0) = -\sqrt{3}$ A

$r(\frac{\pi}{2}) = 2-\sqrt{3}$ B

$r(\pi) = -\sqrt{3}$ C

$r(\frac{3\pi}{2}) = -2-\sqrt{3}$. D

$r(2\pi) = -\sqrt{3}$ E

Correct shape.
correct $\theta = \frac{\pi}{3}, \frac{2\pi}{3}$,

$r(\frac{3\pi}{4}) = \frac{2\sqrt{2}}{2} - \sqrt{3} = \sqrt{2}-\sqrt{3}$

$r(\frac{5\pi}{6}) = 1-\sqrt{3}$, $r(\frac{\pi}{4}) = \sqrt{2}-\sqrt{3}$

(b) Set up, but <u>do not evaluate</u>, an integral to calculate the area between the inner and outer loops of this limaçon.

option 1: $2\left[\frac{1}{2}\int_{2\pi/3}^{\pi}(2\sin\theta-\sqrt{3})^2\,d\theta + \int_{\pi}^{3\pi/2}\frac{1}{2}(2\sin\theta-\sqrt{3})^2\,d\theta - \int_{\pi/3}^{\pi/2}\frac{1}{2}(2\sin\theta-\sqrt{3})^2\,d\theta\right]$

(or) $= \int_{2\pi/3}^{\pi} r^2\,d\theta + \int_{\pi}^{3\pi/2} r^2 - \int_{\pi/3}^{\pi/2} r^2\,d\theta$

ok if not simplified.

+1 correct coefficients
+1 squaring
+1 correct bounds

option 2. $\frac{1}{2}\int_0^{2\pi} r^2\,d\theta - 2\cdot\frac{1}{2}\int_{\pi/3}^{2\pi/3} r^2\,d\theta$.

(or) $= \frac{1}{2}\int_0^{2\pi} r^2\,d\theta - \int_{\pi/3}^{2\pi/3} r^2\,d\theta$.

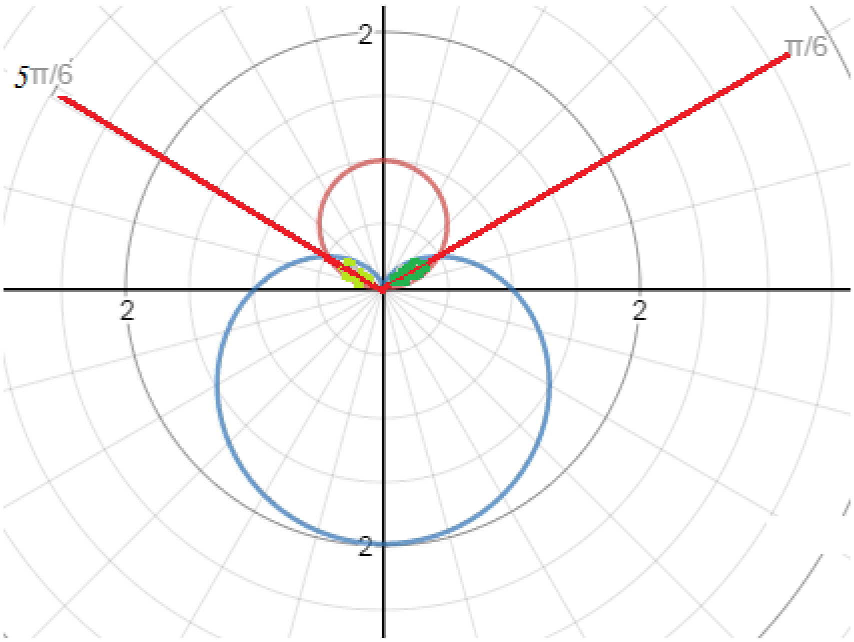
option 3 (or) $2\left[\int_{2\pi/3}^{3\pi/2}\frac{1}{2}r^2\,d\theta - \int_{\pi/2}^{2\pi/3}\frac{1}{2}r^2\,d\theta\right]$.

other answers may be possible

Version A		Version B	
1.	A	1.	C
2.	B	2.	B
3.	B	3.	D
4.	A	4.	E
5.	C	5.	A
6.	C	6.	E
7.	A	7.	D
8.	B	8.	B
9.	A	9.	D
10.	D	10.	D
11.	E	11.	D

1. $r_1 = \sin\theta, \quad r_2 = 1 - \sin\theta.$

(b) On curve r_1 : $(r,\theta) = (0,0), (1,\pi/2), (0,\pi), (-1,3\pi/2), (0,2\pi)$
On curve r_2 : $(r,\theta) = (1,0), (0,\pi/2), (1,\pi), (2,3\pi/2), (1,2\pi)$



(c)

(d) area inside BOTH curves: $A = \int_0^{\pi/6} r_1^2 \, d\theta + \int_{\pi/6}^{\pi/2} r_2^2 \, d\theta$