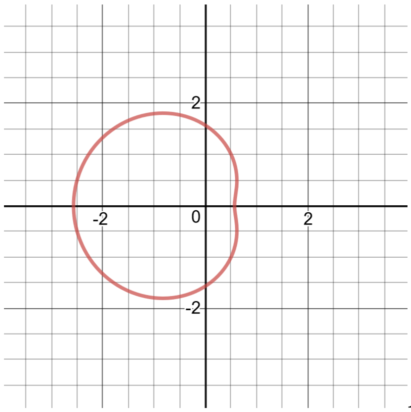
List of Common Maclaurin Series

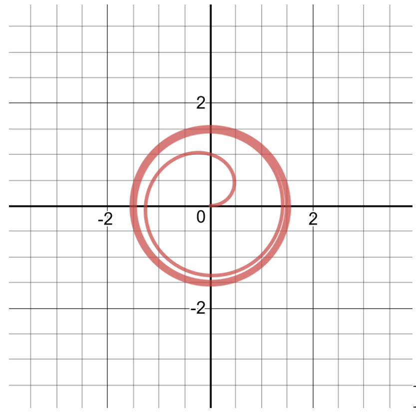
- $\dfrac{1}{1-x} = \sum_{n=0}^{\infty} x^n$, I.O.C. is $(-1, 1)$
- $\ln(1-x) = -\sum_{n=1}^{\infty} \dfrac{x^n}{n}$, I.O.C. is $[-1, 1)$
- $\ln(1+x) = \sum_{n=1}^{\infty} \dfrac{(-1)^{n+1} x^n}{n}$, I.O.C. is $(-1, 1]$
- $e^x = \sum_{n=0}^{\infty} \dfrac{x^n}{n!}$, I.O.C. is $(-\infty, \infty)$
- $\cos x = \sum_{n=0}^{\infty} \dfrac{(-1)^n x^{2n}}{(2n)!}$, I.O.C. is $(-\infty, \infty)$
- $\sin x = \sum_{n=0}^{\infty} \dfrac{(-1)^n x^{2n+1}}{(2n+1)!}$, I.O.C. is $(-\infty, \infty)$
- $\arctan x = \sum_{n=0}^{\infty} \dfrac{(-1)^n x^{2n+1}}{2n+1}$, I.O.C. is $[-1, 1]$

Part I: Multiple Choice

1. Which of the following could be the polar graph of $r = \arctan\theta$ for $\theta \geq 0$?



A



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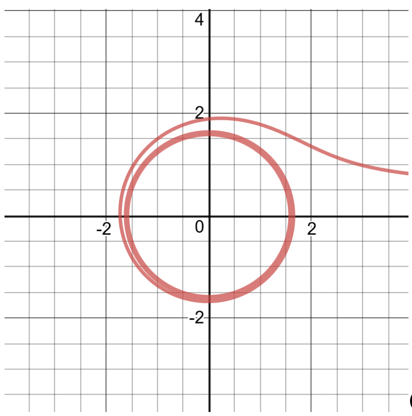

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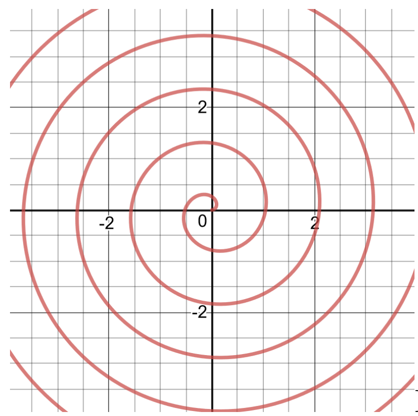


D

2. Let $f(x) = 4\arctan(x) - \pi$. What is the approximate value of $f(1.1)$ if you use the second degree Taylor polynomial for $f(x)$ centered at $a = 1$ to estimate it?

(a) -0.22 (b) 0.18 (c) 0.19 (d) 0.22 (e) 0.99

3. Use a known Maclaurin series to find the sum of the following series.

$$3 - \frac{\pi^2}{2! \cdot 3} + \frac{\pi^4}{4! \cdot 3^3} - \frac{\pi^6}{6! \cdot 3^5} + \frac{\pi^8}{8! \cdot 3^7} + \ldots$$

(a) $\frac{3}{2}$ (b) $-\frac{3}{2}$ (c) $\frac{1}{6}$ (d) $-\frac{1}{6}$ (e) $\frac{3\pi}{2}$

4. How many of the graphs of the following parametric equations are circles?

- $x(t) = 2 + 3\sin(t), y(t) = -1 + 3\cos(t)$
- $x(t) = \cos(2t), y(t) = \sin(2t)$
- $x(t) = -1 + 1\cos(t), y(t) = 2 + 3\sin(t)$
- $x(t) = 9 + \cos(t), y(t) = 6 + \cos(t)$

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

5. Consider the Maclaurin series for the function

$$g(x) = \int \ln(1 + x^2)\, dx\ , g(0) = 0$$

Which of the following statements is true about the interval of convergence for the Maclaurin series for $g(x)$?

(a) The interval of convergence does not contain either of its endpoints.
(b) The interval of convergence contains the left endpoint, but not the right.
(c) The interval of convergence contains the right endpoint, but not the left.
(d) The interval of convergence contains both of its endpoints.
(e) The interval of convergence is $(-\infty, \infty)$

6. The Taylor series for a function $f(x)$ centered at -3 is given by

$$f(x) = 2 - 2(x + 3) - 2(x + 3)^3 + 6(x + 3)^4 + \cdots$$

Which statement below is FALSE?

(a) $f(-3) = 2$
(b) $f'(-3) = -2$
(c) $f''(-3) = 0$
(d) $f'''(-3) = -2$
(e) The series must converge at $x = -3$

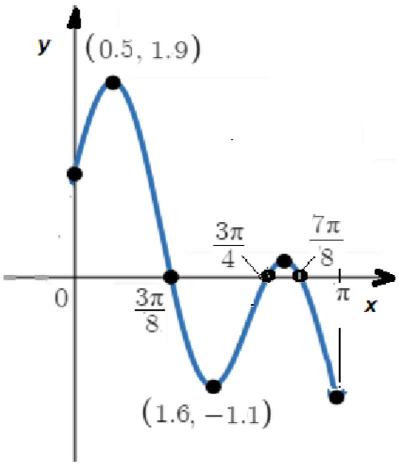
7. Which of the following is the rectangular form of the polar equation $r = \sqrt[3]{\cos(\theta) + \sin(\theta)}$?
   (a) $(x^2 + y^2)^2 = x + y$
   (b) $x^4 + y^4 = x + y$
   (c) $x^2 + y^2 = \sqrt[3]{x + y}$
   (d) $(x + y)^{5/3} = 1$
   (e) None of the above

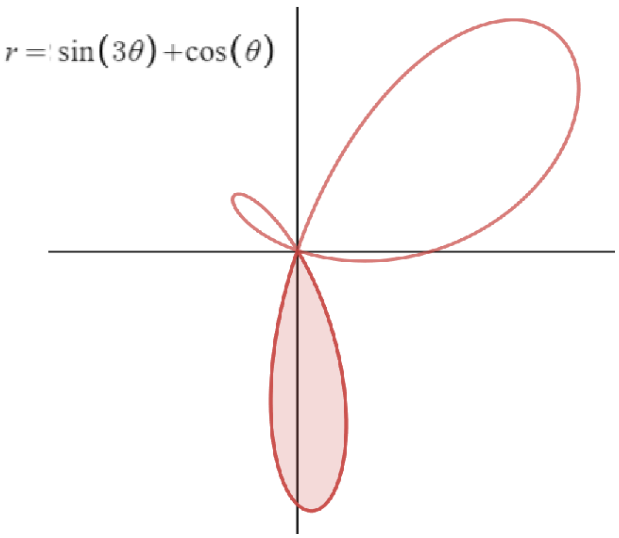
8. Using power series, evaluate the limit $\lim_{x \to 0} \dfrac{x^2 e^x}{\cos(x) - 1}$.
   (a) $\dfrac{1}{2}$ (b) $-\dfrac{1}{2}$ (c) 2 (d) -2 (e) $-\dfrac{1}{6}$

9. Compute the 15th derivative $f^{(15)}(0)$ of the function $f(x) = e^{-x^5}$.
   (a) $\dfrac{15!}{6}$ (b) $-\dfrac{15!}{6}$ (c) 1 (d) -1 (e) 0

10. Which of the following is a power series representation of the function $f(x) = \dfrac{1}{(7 + x)^3}$?
   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{3n} x^{3n}}{7^{3n+3}}$
   (b) $\displaystyle\sum_{n=2}^{\infty} \frac{n(n-1)x^{n-2}}{2 \cdot 7^{n+1}}$
   (c) $\displaystyle\sum_{n=2}^{\infty} \frac{n(n-1)x^{n-2}}{7^{n+1}}$
   (d) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n n(n-1)x^{n-2}}{2 \cdot 7^{n+1}}$
   (e) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n n(n-1)x^{n-2}}{7^{n+1}}$

11. The flat and polar curves of $r = \sin(3\theta) + \cos(\theta)$ are provided below. If the area of the shaded region below is given by the integral $\int_a^b \frac{1}{2}(\sin(3\theta) + \cos(\theta))^2 \, d\theta$, what are the correct values for the bounds a and b?





(a) $a = \frac{11\pi}{8}$ and $b = \frac{13\pi}{8}$

(b) $a = \frac{7\pi}{8}$ and $b = \pi$

(c) $a = \frac{3\pi}{4}$ and $b = \frac{7\pi}{8}$

(d) $a = 0.5$ and $b = 1.6$

(e) $a = \frac{3\pi}{8}$ and $b = \frac{3\pi}{4}$

12. What is the arc length of $x(\theta) = (1+\theta)^2, y(\theta) = (1+\theta)^3$ from $\theta = 0$ to $\theta = 1$?

(a) $\frac{1}{27}(13^{\frac{3}{2}} - 40^{\frac{3}{2}})$ (b) $\frac{1}{27}(40^{\frac{3}{2}} - 13^{\frac{3}{2}})$ (c) $\frac{1}{27}(36^{\frac{3}{2}} - 9^{\frac{3}{2}})$ (d) $\frac{1}{27}(9^{\frac{3}{2}} - 36^{\frac{3}{2}})$ (e) $\frac{1}{27}$

Name: ______________________________

Part II: Free Response

FR Scores	
1	/4
2	/7
3	/5
FR Total	/16

1. Find all values of t at which the curve $x(t) = 2t^3 - 15t^2 + 24t + 7, y(t) = t^2 + t + 1$ has horizontal tangent lines, then give equations for these lines. Then, find all of the t values for which this curve has vertical tangent lines, and give equations for those as well.

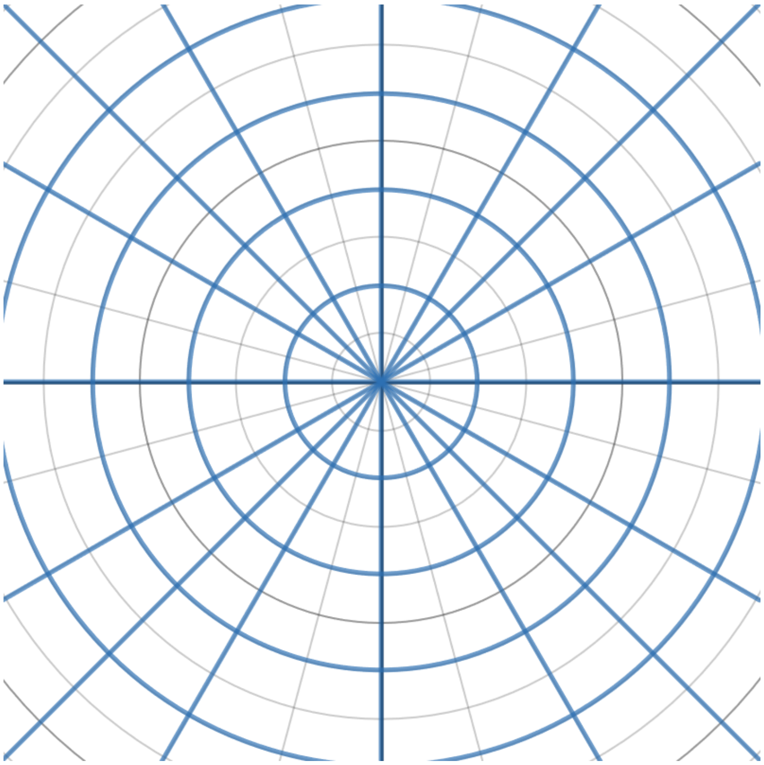
2. Let $f(x) = \sqrt{3+x}$.

   (a) Find the Taylor series for $f(x)$ centered at $a = 1$

   (b) Find the radius of convergence of this Taylor series.

   (c) Using the second degree Taylor polynomial, estimate $\sqrt{4.1}$.

3. (a) Sketch a graph of the limaçon $r = 2\sin(\theta) - \sqrt{3}$ in the provided grid.



(b) Set up, but do not evaluate, an integral to calculate the area between the inner and outer loops of this limaçon.

1. Find the Taylor series for f centered at 4 and its radius of convergence R if

$$f^{(n)}(4) = \frac{(-1)^n n!}{2^n(n+1)}.$$

A. $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n(n+1)} (x-4)^n, \ R = 2$

B. $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n(n+1)} (x-4)^n, \ R = \frac{1}{2}$

C. $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n n}{2^n} (x-4)^n, \ R = \infty$

D. $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n(n+1)} x^n, \ R = 2$

E. $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n(n+1)^2} x^n, \ R = 1$

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2. Find the Maclaurin series of $\ln(1+7x)$ and its interval of convergence I.

A. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 7^n x^n}{n}, \quad I = (-7, 7]$

B. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 7^n x^n}{n}, \quad I = \left(-\frac{1}{7}, \frac{1}{7}\right]$

C. $-\sum_{n=1}^{\infty} \frac{7^n x^n}{n}, \quad I = [-7, 7)$

D. $\sum_{n=0}^{\infty} \frac{(-1)^{n+1} 7^n x^n}{n}, \quad I = \left(-\frac{1}{7}, \frac{1}{7}\right)$

E. $-\sum_{n=0}^{\infty} \frac{7^n x^n}{n}, \quad I = \left[-\frac{1}{7}, \frac{1}{7}\right)$

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3. Find the Maclaurin series for $f(x) = x^3 e^{3-x^3}$ and use it to determine $f^{(60)}(0)$.

A. $\dfrac{-60!}{19!}$ B. $\dfrac{-60!e^3}{19!}$ C. $\dfrac{60!e^3}{20!}$ D. $\dfrac{-e^3}{19!}$ E. $\dfrac{60!}{20!}$

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4. Find a power series representation of $\displaystyle\int \frac{\ln(1-t)}{6t}\,dt$, centered at 0, and its radius of convergence R.

A. $\displaystyle -\sum_{n=1}^{\infty} \frac{t^n}{6n^2} + C,\ R = 1$

B. $\displaystyle \sum_{n=1}^{\infty} \frac{(-1)^n t^n}{6n} + C,\ R = 1$

C. $\displaystyle -\sum_{n=1}^{\infty} \frac{t^n}{6n^2} + C,\ R = \frac{1}{6}$

D. $\displaystyle \sum_{n=1}^{\infty} \frac{(-1)^{n+1} t^n}{6n} + C,\ R = 1$

E. $\displaystyle -\sum_{n=1}^{\infty} \frac{t^n}{6n^2} + C,\ R = 6$

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5. Starting with the geometric series $\displaystyle\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$, use differentiation to find the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{n}{11^n}$.

A. $\dfrac{6}{25}$ B. $\dfrac{11}{25}$ C. $\dfrac{11}{100}$ D. $\dfrac{6}{100}$ E. $\dfrac{5}{49}$

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6. Find the sum of the series $\displaystyle -\frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots = \sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n}$.

A. Diverges B. $1 - \ln 2$ C. $\ln 2 - 1$ D. $1 + \ln 2$ E. $\ln 2$

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7. Suppose that $\sum_{n=0}^{\infty} c_n(x-6)^n$, a power series with finite radius of convergence, $R > 0$, converges for $x = 2$ and diverges for $x = -2$. What can be said about the following series

P. $\sum_{n=0}^{\infty} c_n$ Q. $\sum_{n=0}^{\infty} c_n 4^n$ R. $\sum_{n=0}^{\infty} c_n(-1)^n 6^n$ S. $\sum_{n=0}^{\infty} c_n(-1)^n 9^n$

A.	P. Convergent	Q. Not enough info.	R. Not enough info.	S. Divergent
B.	P. Divergent	Q. Not enough info.	R. Not enough info.	S. Divergent
C.	P. Convergent	Q. Not enough info.	R. Divergent	S. Divergent
D.	P. Convergent	Q. Convergent	R. Not enough info.	S. Divergent
E.	P. Convergent	Q. Divergent	R. Divergent	S. Convergent

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8. Set up the integral that represents the length of the curve

$$x = e^t - t, \quad y = 4e^{t/2}, \quad 0 \le t \le 2$$

A. $\int_0^2 2(e^t - 1)\, dt$ B. $\int_0^2 (e^t + 1)\, dt$ C. $\int_0^2 2(e^t + 1)\, dt$

D. $\int_0^2 (e^t - 1)\, dt$ E. $\int_0^2 \sqrt{2}(e^t + 1)\, dt$

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9. Let $A = 2\int_{\alpha}^{\pi/2} \frac{1}{2}(2 - 4\sin\theta)^2\, d\theta$ be the area of the inner loop of $r = 2 - 4\sin\theta$. What is the value of α?

A. $\alpha = \frac{\pi}{6}$ B. $\alpha = \frac{5\pi}{6}$ C. $\alpha = \frac{\pi}{3}$ D. $\alpha = 0$ E. $\alpha = \frac{\pi}{4}$

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10. What is the slope of the tangent line to the curve

$$x = te^t, \; y = e^{5t}$$

at the point (e, e^5)?

A. $3e^2$ B. e^4 C. $4e^3$ D. $\frac{5}{2}e^4$ E. $\frac{3}{5}e^2$

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11. Which point (r, θ) below does <u>not</u> represent the same point as $\left(-3, \frac{2\pi}{3}\right)$?

A. $\left(3, -\frac{\pi}{3}\right)$ B. $\left(-3, \frac{-4\pi}{3}\right)$ C. $\left(3, \frac{5\pi}{3}\right)$

D. $\left(-3, \frac{-10\pi}{3}\right)$ E. $\left(3, \frac{8\pi}{3}\right)$

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1. Given two curves $r_1 = \sin\theta, \quad r_2 = 1 - \sin\theta.$

(a) Verify that the two curves intersect at $\theta = \frac{\pi}{6}, \frac{5\pi}{6}.$

(b) Give the r value of each of the given points

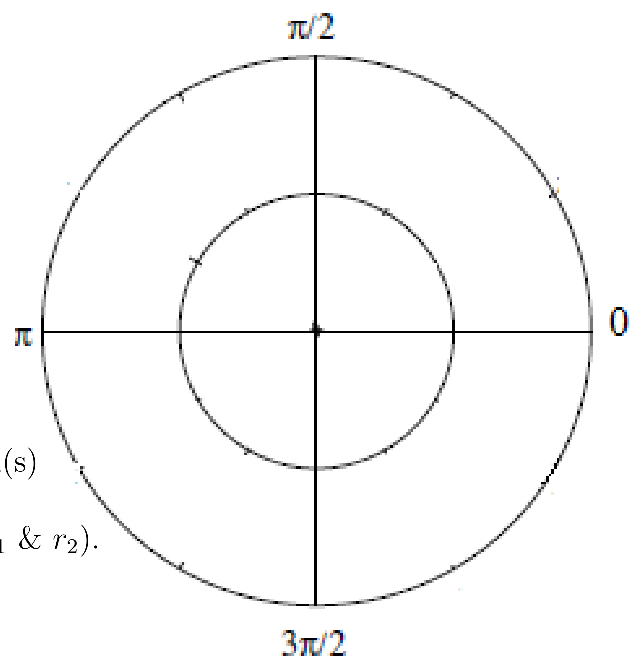
on the curve $r_1 : (\quad, 0), (\quad, \pi/2),$

$(\quad, \pi), (\quad, 3\pi/2), (\quad, 2\pi).$

on the curve $r_2 : (\quad, 0), (\quad, \pi/2),$

$(\quad, \pi), (\quad, 3\pi/2), (\quad, 2\pi).$

(c) Using the information from (a) and (b), **sketch** the two polar curves on the same plane **and** **sketch** and **label** the lines of intersections on the graph.



(d) Set up, (do not evaluate), the integral(s) for the area **inside both curves**.
(you may leave your answer in terms of r_1 & r_2).

Area (set up only) = ____________________________________