Part I: Multiple Choice

1. Which of the following series can we conclude diverges using the Test for Divergence?

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{n}$

   (b) $\displaystyle\sum_{n=0}^{\infty} \ln\left(\frac{3n^2+1}{4n^2+n+1}\right)$

   (c) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right)$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2-\cos\left(\frac{1}{n}\right)}$

   (e) (b) and (d)

2. Which of the following series converge absolutely?

   i) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n e^n}{\sqrt{n!}}$ *ii)* $\displaystyle\sum_{n=2}^{\infty} \frac{2-\sin(n!)}{\sqrt[3]{n}}$

   (a) Neither i or ii (b) Both i and ii (c) Only ii (d) Only i

3. Given that the following sequence is monotonically increasing and bounded above, find the limit of the sequence.

$$a_n = \sqrt{1+a_{n-1}},\ a_1 = \frac{1}{2}$$

   (a) 1 (b) $\sqrt{2}$ (c) 5 (d) $\dfrac{1+\sqrt{5}}{2}$ (e) Diverges by TFD

4. Which of the following series' sums may be found using techniques from this class?

   P. $\displaystyle\sum_{n=5}^{\infty} \frac{1}{n^3-n}$ Q. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$ R. $\displaystyle\sum_{n=2}^{\infty} \frac{\sqrt{3}}{e^n \pi}$

   (a) P, Q, and R (b) P and R (c) P and Q (d) P only (e) Q only

5. Determine the values of k for which the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^{1/4}}{\sqrt{n^k + 9n}}$ will converge.

   (a) $k \in \left(\dfrac{5}{2}, \infty\right)$

   (b) $k \in \left[\dfrac{5}{2}, \infty\right)$

   (c) $k \in (3, \infty)$

   (d) $k \in [3, \infty)$

   (e) $k \in \left(\dfrac{8}{3}, \infty\right)$

6. Which of the statements below is true for the following two series?

$$A = \sum_{k=1}^{\infty} \left(\frac{k}{k+1}\right)^{k^2} \qquad\qquad B = \sum_{k=1}^{\infty} \frac{(k!)^2}{(2k)!}$$

   (a) A converges and B diverges.

   (b) A diverges and B converges.

   (c) Both diverge.

   (d) Both converge.

   (e) The convergence of these series cannot be determined.

7. How many of the following statements are true for a series $\sum a_n$ with positive terms?

   - If $\displaystyle\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$, then the series converges conditionally.
   - If $\displaystyle\lim_{n\to\infty} a_n = 0$, then the series converges.
   - If $\displaystyle\lim_{n\to\infty} (a_n)^{\frac{1}{n}} = 0$, then the series converges absolutely.
   - If $\displaystyle\lim_{n\to\infty} (a_n)^{\frac{1}{n}} = 1$, then the series diverges.

   (a) 0 (b) 1 (c) 2 (d) 3 (e) 4

8. Which of the series below are convergent using the limit comparison test?

A. $\displaystyle\sum_{n=10}^{\infty} \tan\left(\frac{1}{n}\right)$

B. $\displaystyle\sum_{n=10}^{\infty} \sin^3\left(\frac{1}{n}\right)$

(a) Both A and B
(b) A only
(c) B only
(d) Neither A nor B
(e) The limit comparison test is inconclusive for these series.

9. For which of the following sequences b_n would the series $\displaystyle\sum_{n=1}^{\infty}(-1)^n b_n$ converge conditionally (and not absolutely)?

(a) $b_n = \dfrac{1}{n \ln n}$
(b) $b_n = \dfrac{2^n}{e^n}$
(c) $b_n = \dfrac{(n+1)!}{n^n}$
(d) (a), (b), and (c)
(e) None of the above

10. Suppose that $0 \leq h(x) \leq f(x) \leq g(x)$ for all $x \geq 1$ and all of the functions are continuous and decreasing. Which of the following would guarantee that $\displaystyle\sum_{n=1}^{\infty} f(n)$ converges?

(a) $\displaystyle\int_1^{\infty} h(x)\, dx$ converges
(b) $\displaystyle\int_1^{\infty} f(x)\, dx$ converges
(c) $\displaystyle\int_1^{\infty} g(x)\, dx$ converges
(d) (a), (b), and (c)
(e) (b) and (c)

Name: ____

Part II: Free Response

FR Scores	
1	/4
2	/6
3	/6
4	/4
FR Total	/20

1. Give examples of the following (you do not need to justify your answers):

   (a) A sequence that is neither monotonic nor bounded.

   (b) A sequence that is monotonic but unbounded

   (c) A sequence that is not monotonic but is bounded

   (d) A sequence that is monotonic and bounded.

2. Find the sum of the following series, or if it diverges, state that it diverges. You may leave your answer as a sum of fractions. **You must show ALL work to receive full credit.**

$$\sum_{n=2}^{\infty} \frac{1}{n^2+5n+6} + \frac{5}{4^n}.$$

3. (**You must show ALL work to receive full credit.**)

(a) Determine if the series $\displaystyle\sum_{n=10}^{\infty} \frac{10 - \sin n}{n^4}$ is convergent or divergent. Clearly indicate what test you are using to justify your conclusion and state why it applies.

Circle one: (Convergent, Divergent), by ________________________ Test.

(b) Suppose that you choose to estimate $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{2 + 3^n}$ by adding the first 4 terms together. According to the Alternating Series Error Estimation Theorem, how large can the error of doing this estimate possibly be?

4. Determine whether the following series converges or diverges. Clearly indicate what test you are using to justify your conclusion and state why it applies. **You must show ALL work to receive full credit.**

$$1+\frac{1\cdot 2}{1\cdot 3}+\frac{1\cdot 2\cdot 3}{1\cdot 3\cdot 5}+\frac{1\cdot 2\cdot 3\cdot 4}{1\cdot 3\cdot 5\cdot 7}+\ldots+\frac{n!}{1\cdot 3\cdot 5\cdot\ldots\cdot(2n-1)}+\ldots$$

1. For which of the following series is the **ratio test** **inconclusive**?

A. $\displaystyle\sum_{n=5}^{\infty} \frac{e^n}{5^n}$

B. $\displaystyle\sum_{n=8}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n+1)}{n^n}$

C. $\displaystyle\sum_{n=3}^{\infty} \frac{n!}{e^{n^2}}$

D. $\displaystyle\sum_{n=8}^{\infty} \frac{n^2}{n^7+1}$

E. $\displaystyle\sum_{n=5}^{\infty} \frac{n}{(n+1)!}$

2. Determine if the series $\displaystyle\sum_{n=5}^{\infty} \frac{3-(-1)^n}{n}$ converges or diverges.

(a) Which test did you use? ____

(b) Show your work in determining whether the series converges or diverges.

(c) The series (converges, diverges). (circle one of the choices)

3. Determine if the series $\displaystyle\sum_{n=5}^{\infty} \frac{(-1)^n}{\sqrt{n}\ln n}$ converges absolutely, ,conditionally or diverges.

(a) Which test(s) did you use? ______________________________

(b) Show your work.

(c) The series (converges absolutely, converges conditionally, diverges). (circle one)

4. Let $s = \sum_{n=1}^{\infty} a_n$ and the partial sum $S_N = \sum_{n=1}^{N} a_n = 3 - \frac{1}{N!}$. Find $a_3 + a_4$.

A. $-\frac{7}{6}$

B. $\frac{11}{24}$

C. $\frac{5}{12}$