1. Find the sum of the telescoping series.

$$\sum_{n=5}^{\infty}\left[\sec\left(\frac{1}{n}\right)-\sec\left(\frac{1}{n+2}\right)\right]$$

(a) Determine the nth partial sum s_n of the series.

$$\sum_{n=5}^{\infty}\left[\sec\left(\frac{1}{n}\right)-\sec\left(\frac{1}{n+2}\right)\right]=\sum_{n=5}^{\infty}\left[\sec\left(\frac{1}{n}\right)-\sec\left(\frac{1}{n+1}\right)+\sec\left(\frac{1}{n+1}\right)-\sec\left(\frac{1}{n+2}\right)\right]$$

$$S_N=\sec\left(\tfrac{1}{5}\right)-\sec\left(\tfrac{1}{6}\right)+\sec\left(\tfrac{1}{6}\right)-\sec\left(\tfrac{1}{7}\right)$$

$$+\sec\left(\tfrac{1}{6}\right)-\sec\left(\tfrac{1}{7}\right)+\sec\left(\tfrac{1}{7}\right)-\sec\left(\tfrac{1}{8}\right)$$

$$\vdots$$

$$+\sec\left(\tfrac{1}{N}\right)-\sec\left(\tfrac{1}{N+1}\right)+\sec\left(\tfrac{1}{N+1}\right)-\sec\left(\tfrac{1}{N+2}\right)$$

$s_n =$ $\sec\left(\frac{1}{5}\right)+\sec\left(\frac{1}{6}\right)-\sec\left(\frac{1}{N+1}\right)-\sec\left(\frac{1}{N+2}\right)$

(b) Determine the sum of the series.

$$S=\lim_{N\to\infty}S_N=\lim_{N\to\infty}\sec\left(\tfrac{1}{5}\right)+\sec\left(\tfrac{1}{6}\right)-\sec\left(\tfrac{1}{N+1}\right)-\sec\left(\tfrac{1}{N+2}\right)$$

Sum= $\sec\left(\frac{1}{5}\right)+\sec\left(\frac{1}{6}\right)-2$

2. Which of the following series $\sum a_n$ can we conclude converges or diverges using the direct comparison test (DCT)? What b_n do you compare a_n to?

If DCT can not be used, what test can we use?

(You may also use abbreviations: TFD(test for divergent), AST (alternating series test), LCT (limit comparison test), INT (integral test)).

You do not have to show work.

(a) $\sum_{n=7}^{\infty} \frac{8+\cos n}{n^2}$.

i. Circle one: (convergent, divergent)

ii. If DCT: $b_n =$ $\frac{9}{n^2}$

iii. If Not DCT, test: ______.

or (ii) $\frac{K}{n^2}$, $K \geq 9$.

(b) $\sum_{n=7}^{\infty} \frac{1}{1+n^2}$.

i. Circle one: (convergent, divergent)

ii. If DCT: $b_n =$ $\frac{1}{n^2}$

iii. If Not DCT, test: ______.

or (ii) $\frac{K}{n^2}$, $K \geq 1$

(c) $\sum_{n=5}^{\infty} \frac{1}{n^2(\ln n)}$

i. Circle one: (convergent, divergent)

ii. If DCT: $b_n =$ $\frac{1}{n^2}$

iii. If Not DCT, test: ______.

(d) $\sum_{n=5}^{\infty} \frac{1}{n(\ln n)^2}$

i. Circle one: (convergent, divergent)

ii. If DCT: $b_n =$ ______

iii. If Not DCT, test: INT.

(e) $\sum_{n=7}^{\infty} \frac{5+(-1)^n}{n-1}$.

i. Circle one: (convergent, divergent),

ii. If DCT: $b_n =$ $\frac{4}{n-1}$

iii. If Not DCT, test: ______.

(ii) or $b_n = \frac{4}{n}$

1. Find the sum of the telescoping series.

$$\sum_{n=8}^{\infty}\left[\cos\left(\frac{\pi}{n}\right)-\cos\left(\frac{\pi}{n+2}\right)\right]$$

(a) Determine the nth partial sum s_n of the series.

$$\sum_{n=8}\left[\cos\left(\frac{\pi}{n}\right)-\cos\left(\frac{\pi}{n+2}\right)\right]=\sum_{n=8}^{\infty}\left[\cos\left(\frac{\pi}{n}\right)-\cos\left(\frac{\pi}{n+1}\right)+\cos\left(\frac{\pi}{n+1}\right)-\cos\left(\frac{\pi}{n+2}\right)\right]$$

$$S_N = \cos\left(\frac{\pi}{8}\right)-\cos\left(\frac{\pi}{9}\right)+\cos\left(\frac{\pi}{9}\right)-\cos\left(\frac{\pi}{10}\right)$$

$$+\cos\left(\frac{\pi}{9}\right)-\cos\left(\frac{\pi}{10}\right)+\cos\left(\frac{\pi}{10}\right)-\cos\left(\frac{\pi}{11}\right)$$

$$\vdots$$

$$+\cos\left(\frac{\pi}{N}\right)-\cos\left(\frac{\pi}{N+1}\right)+\cos\left(\frac{\pi}{N+1}\right)-\cos\left(\frac{\pi}{N+2}\right)$$

$s_n =$ $\cos\left(\frac{\pi}{8}\right)+\cos\left(\frac{\pi}{9}\right)-\cos\left(\frac{\pi}{N+1}\right)-\cos\left(\frac{\pi}{N+2}\right)$

(b) Determine the sum of the series.

$$S=\lim_{N\to\infty} S_N = \lim_{N\to\infty}\cos\left(\frac{\pi}{8}\right)+\cos\left(\frac{\pi}{9}\right)-\cos\left(\frac{\pi}{N+1}\right)-\cos\left(\frac{\pi}{N+2}\right)$$

Sum= $\cos\left(\frac{\pi}{8}\right)+\cos\left(\frac{\pi}{9}\right)-2$

2. Which of the following series $\sum a_n$ can we conclude converges or diverges using the direct comparison test (DCT)? What b_n do you compare a_n to?

If DCT can not be used, what test can we use?

(You may also use abbreviations: TFD(test for divergent), AST (alternating series test), LCT (limit comparison test), INT (integral test)).

You do not have to show work.

(a) $\sum_{n=1}^{\infty} \frac{1}{1+2^n}$.

i. Circle one: (convergent, divergent)

ii. If DCT: $b_n =$ $\frac{1}{2^n}$

iii. If Not DCT, test: —.

(c) $\sum_{n=5}^{\infty} \frac{1}{n(\ln n)^2}$

i. Circle one: (convergent, divergent)

ii. If DCT: $b_n =$ —

iii. If Not DCT, test: INT.

(b) $\sum_{n=5}^{\infty} \frac{4-\sin n}{n^4}$.

i. Circle one: (convergent, divergent)

ii. If DCT: $b_n =$ $\frac{5}{n^4}$

iii. If Not DCT, test: —.

or (ii) $\frac{K}{n^4}$, $K \geq 5$.

(d) $\sum_{n=5}^{\infty} \frac{1}{n^2(\ln n)}$

i. Circle one: (convergent, divergent)

ii. If DCT: $b_n =$ $\frac{1}{n^2}$

iii. If Not DCT, test: —.

(e) $\sum_{n=8}^{\infty} \frac{2+(-1)^n}{1+n^2}$.

i. Circle one: (convergent, divergent),

ii. If DCT: $b_n =$ $\frac{3}{1+n^2}$

or ii) $b_n = \frac{3}{n^2}$

iii. If Not DCT, test: —.

Version C.

#1. $\displaystyle\sum_{n=7}^{\infty}\left[\cos\left(\frac{\pi}{n+2}\right)-\cos\left(\frac{\pi}{n+1}\right)+\cos\left(\frac{\pi}{n+1}\right)-\cos\left(\frac{\pi}{n}\right)\right]$

$$S_N = \cos\left(\frac{\pi}{9}\right)-\cos\left(\frac{\pi}{8}\right)+\cos\left(\frac{\pi}{8}\right)-\cos\left(\frac{\pi}{7}\right)$$

$$+\cos\left(\frac{\pi}{10}\right)-\cos\left(\frac{\pi}{9}\right)+\cos\left(\frac{\pi}{9}\right)-\cos\left(\frac{\pi}{8}\right)$$

$$\vdots$$

$$+\cos\left(\frac{\pi}{N+2}\right)-\cos\left(\frac{\pi}{N+1}\right)+\cos\left(\frac{\pi}{N+1}\right)-\cos\left(\frac{\pi}{N}\right)$$

$$\boxed{S_N = -\cos\left(\frac{\pi}{8}\right)-\cos\left(\frac{\pi}{7}\right)+\cos\left(\frac{\pi}{N+2}\right)+\cos\left(\frac{\pi}{N+1}\right)}$$

$$\boxed{S = \lim_{N\to\infty} S_N = 2-\cos\left(\frac{\pi}{8}\right)-\cos\left(\frac{\pi}{7}\right)}$$

Version C

#2.

(a) $\sum \frac{\ln n}{n}$.

~~convergent~~ divergent

DCT, $b_n = \frac{1}{n}$

(c) $\sum \frac{1}{n(\ln n)^2}$

convergent,

INT.

(b) $\sum \frac{8+\cos n}{n^2}$

convergent

DCT, $b_n = \frac{9}{n^2}$.

or $b_n = \frac{K}{n^2}$, $K \geq 9$.

(d) $\sum \frac{1}{n^2(\ln n)}$

convergent

DCT, $b_n = \frac{1}{n^2}$

(e) $\sum \frac{4+(-1)^n}{2^n}$

convergent

DCT, $b_n = \frac{5}{2^n}$.

or $b_n = \frac{K}{2^n}$, $K \geq 5$.