arctan $\sqrt{x}$ $\qquad \frac{d}{dx}\arctan y = \frac{1}{y^2+1}$

① $\int (2x+1)\underbrace{\tan^{-1}(\sqrt{x})}\,dx$

poly, trig
IPET
inverse, exponetial

u	dv
$\tan^{-1}(\sqrt{x})$	$2x+1$

$\boxed{a}$

② A-D

$$\left[\frac{A}{x} + \cdots \frac{D}{(x-1)^2} + \cdots = \frac{-x+7}{x(x+7)(x-1)^2(x+1)}\right](x-1)^2$$

@ $x=0$ $\qquad$ @ $x=1$

$A = \frac{-0+7}{7(-1)^2(1)} = 1 \qquad D = \frac{-1+7}{1(8)(2)} = \frac{6}{16} = \frac{3}{8}$

$\frac{8}{8} - \frac{3}{8} = \frac{5}{8}$ $\qquad \boxed{a}$

③ $\int_0^{\pi/2} \sin^3(x)\cos^3(x)\,dx$ $\qquad \cos^2 x = 1-\sin^2 x$

$u = \sin x$ $\quad \frac{\pi}{2} \to 1$, $\quad 0 \to 0$
$du = \cos x\,dx$

$\int \sin^3 x\,(1-\sin^2 x)\underbrace{\cos x\,dx}$

$\int_0^1 u^3(1-u^2)\,du$

$\int_0^1 u^3 - u^5\,du$

$\left.\frac{u^4}{4} - \frac{u^6}{6}\right|_0^1 = \frac{1}{4} - \frac{1}{6} - 0 = \frac{1}{12}$ $\qquad \boxed{e}$

④ $\int_2^\infty \frac{1}{x^3-x^2}\,dx$

$$\left[\frac{1}{\cancel{x^2(x-1)}} = \frac{A}{\cancel{x}} + \frac{B}{\cancel{x^2}} + \frac{C}{\cancel{x-1}}\right] x^2(x-1)$$

$1 = Ax(x-1) + B(x-1) + Cx^2$

@ $x=0$ $\qquad$ @ $x=1$ $\qquad$ @ $x=2$

$1 = B(-1)$ $\qquad C=1$ $\qquad 1 = A(2)(2-1) + \underbrace{B(1)}_{-1} + \underbrace{C(2)^2}_{4}$
$B = -1$ $\qquad\qquad 1 = 2A + 3$
$-2 = 2A$
$A = -1$

$$\int_2^\infty \frac{-1}{x} + \frac{-1}{x^2} + \frac{1}{x-1}\,dx$$

$$\lim_{n\to\infty} \left.\underbrace{-\ln x + -x^{-1}(-1) + \ln(x-1)}\right|_2^n$$

$$\lim_{n\to\infty} \left.\ln\left(\frac{x-1}{x}\right) + \frac{1}{x}\right|_2^n = \lim_{n\to\infty}\left[\ln\left(1-\cancel{\frac{1}{x}}^{\,0}\right) + \cancel{\frac{1}{x}}^{\,0}\right] - \left[\ln\frac{2-1}{2} + \frac{1}{2}\right]$$

$0 - \underbrace{\ln\frac{1}{2}} - \frac{1}{2}$

$\ln\left(\frac{1}{2}\right)^{-1}$

$0 - \underbrace{\ln \tfrac{1}{2}} - \tfrac{1}{2}$

$\ln\left(\tfrac{1}{2}\right)^{-1}$

$\shortparallel$

$\boxed{\ln 2 - \tfrac{1}{2}}$

b

5

$$\int \frac{8x^2}{x^4-16}\,dx$$

$\uparrow$

$(x^2-4)(x^2+4)$

$(x+2)(x-2)$

$(x+2)(x-2)(x^4+4)$

$$\left[\frac{8x^2}{x^4-16} = \frac{A}{x+2} + \frac{B}{x-2} + \frac{Cx+D}{x^2+4}\right] x^4-16$$

$$8x^2 = A(x-2)(x^2+4) + B(x+2)(x^2+4) + (Cx+D)(x^2-4)$$

x^3	x^2	x	0
① $0 = A+B+C$	② $8 = -2A-2B+D$	③ $0 = 4A+4B-4C$	④ $0 = -8A+8B+4D$

4 ①

$+$ ③

$C = 0$ $\quad A = -1$ $\quad B = 1$ $\quad D = 4$

$$\frac{1}{\left(\frac{x}{2}\right)^2 + 1}$$

$u = \frac{x}{2}$

$du = \frac{1}{2}dx$

$2\,du = dx$

$$\int \frac{-1}{x+2} + \frac{1}{x-2} + \frac{4}{x^2+4}\,dx \quad \int \frac{\cancel{4}}{\cancel{4}\left(\left(\frac{x}{2}\right)^2+1\right)}\,dx$$

$$2\int \frac{1}{u^2+1}\,du$$

$$2\arctan\frac{x}{2}$$

$$-\ln x+2 + \ln x-2 + \underbrace{2\arctan\frac{x}{2}} + C$$

$\boxed{c}$

6

$$\int \frac{1}{(4x-x^2)^{3/2}}\,dx$$

$-(x^2-4x+4)+4$

$4-(x-2)^2$

$\left[4\left[1-\left(\frac{x-2}{2}\right)^2\right]\right]$

$\uparrow$

$\frac{x-2}{2} = \sin\theta$

$\frac{1}{2}dx = \cos\theta\,d\theta$

$dx = 2\cos\theta\,d\theta$

$$\int \frac{2\cos\theta\,d\theta}{\left(4\left[1-\sin^2\theta\right]\right)^{3/2}}$$

$1-\sin^2\theta = \cos^2\theta$

$$\frac{2\cos\theta}{8(\cos^2\theta)^{3/\cancel{2}}}$$

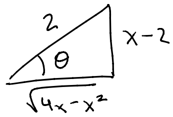
$$\int \frac{\cancel{\cos\theta}}{4\cos^{\cancel{3}2}\theta}\,d\theta$$

$\frac{1}{\cos^2\theta} = \sec^2\theta$

$\frac{1}{4}\tan\theta + C$

$\frac{1}{\cdot} \cdot \frac{x-2}{2} + C$

$\frac{1}{4}\tan\theta + C$



$$\frac{1}{4}\cdot\frac{x-2}{\sqrt{4x-x^2}}+C$$

$\boxed{d}$

7

$$\int x^2 e^{x/3}\,dx$$

u	dv
x^2 +	$e^{x/3}$
$2x$ −	$3e^{x/3}$
2 +	$9e^{x/3}$
0	$27e^{x/3}$

$$x^2(3e^{x/3})$$
$$-2x(9e^{x/3})+2(27e^{x/3})+C$$

$\boxed{b}$

8

$$\frac{x^4+10x^2}{(x^2+1)(x^2+9)}$$

$$x^4+10x^2+9$$

$$x^4+10x^2+9\,\overline{)\,x^4+10x^2\,}\quad 1$$
$$-\;x^4+10x^2+9$$
$$-9$$

$$1-\frac{9}{(x^2+1)(x^2+9)}$$

$$\left(\frac{-9}{(x^2+1)(x^2+9)}=\boxed{\frac{Cx+D}{x^2+1}}+\frac{Ax+B}{x^2+9}\right)(x^2+1)(x^2+9)$$

$$-9=\cancel{Ax^3}+\cancel{Bx^2}+\cancel{Ax}+B+\cancel{Cx^3}+\cancel{Dx^2}+\cancel{9Cx}+9D$$

x^3	x^2	x	0
$0=A+C$	$0=B+D$	$0=A+9C$	$-9=B+9D$
$-A=C$	$B=-D$	$-C+9C$	$-9=-D+9D$
$A=0$	$B=\frac{9}{8}$	$C=0$	$-\frac{9}{8}=D$

$\boxed{1}$ $\boxed{2}$ 3

$\boxed{c}$

9

$$\int \sec x\tan^2 x\,dx$$

$$\int \sec x(\sec^2 x-1)\,dx$$

$\int \sec^3 x - \sec x \, dx$

$\frac{1}{2}\left[\sec x \tan x + \ln(\sec x + \tan x)\right] - \ln(\sec x + \tan x)$

$\frac{1}{2}\sec x \tan x + \underline{\frac{1}{2}\ln(\sec x + \tan x) - \frac{2}{2}\ln(\sec x + \tan x)}$

$\frac{1}{2}\left[\sec x \tan x - \ln(\sec x + \tan x)\right]$

$\boxed{b}$

10

$\int_0^2 \frac{x}{x^4+1} dx$

$(x^2)^2$

$u = x^2$ $\quad 4$, 0

$du = 2x\,dx$

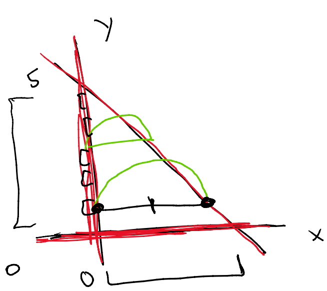
$\frac{1}{2} du = x\,dx$

$\frac{1}{2}\int_0^4 \frac{du}{u^2+1}$

$\frac{1}{2}\arctan u \Big|_0^4 = \boxed{\frac{1}{2}\arctan 4} - 0$

$\boxed{c}$

1



$x + y = 5$

$x = 5 - y$

$\int A \, dy$

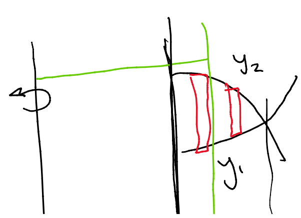
$A = \frac{1}{2}\pi r^2 \leftarrow \left(\frac{x}{2}\right)$

$\frac{\pi}{8}\int_0^5 (5-y)^2 dy$

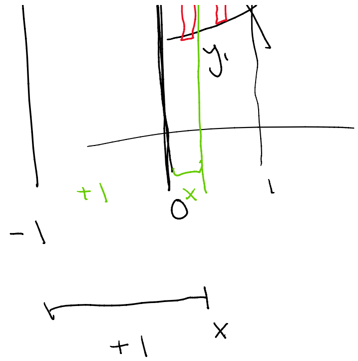
$\frac{1}{2}\pi \frac{x^2}{4} = \frac{\pi}{8} x^2$

b

2



$2\pi \int_a^b \text{radius (height)} \, dx$



$$2\pi \int_a^b \text{radius (height)}\, dx$$

$$2\pi \int_a^b (x+1)(y_2 - y_1)\, dx$$

$$2\pi \int_0^1 (x+1)(4 - x^2 - x^3 - 2)\, dx$$

$$(-x^3 - x^2 + 2)$$

$\boxed{c}$

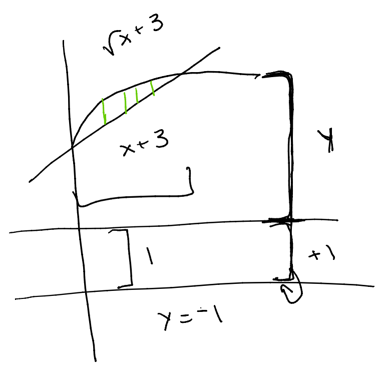
$y_2 = 4 - x^2$

$y_1 = x^3 + 2$

$4 - x^2 = x^3 + 2$

$y_1 = y_2 \quad x = 1$

③



$$\pi \int_a^b [OR^2 - IR^2]\, dx$$

$$\pi \int_0^1 (\sqrt{x}+3+1)^2 - (x+3+1)^2\, dx$$

$$(\sqrt{x}+4)^2 - (x+4)^2$$

$$x + 8\sqrt{x} + \cancel{16} - (x^2 + 8x + \cancel{16})$$

$$\pi \int_0^1 -x^2 - 7x + 8\sqrt{x}\quad dx$$

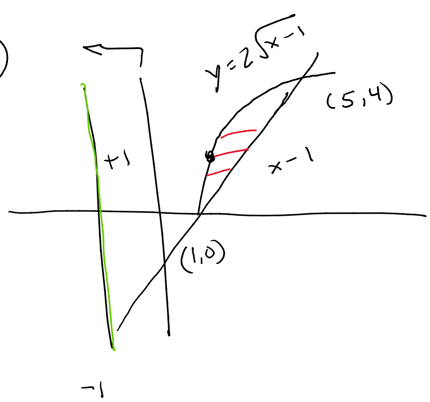
$\sqrt{x} + 3 = x + 3$

$\sqrt{x} = x$

$x = 1$

$\boxed{e}$

④



$$\pi \int (OR^2 - IR^2)\, dy$$

$$\pi \int_0^4 (y+2)^2 - \left(\frac{y^2}{4} + 2\right) dy$$

$\boxed{d}$

$y = x - 1$

$y + 1 = x$

$y = 2\sqrt{x-1}$

$\left(\frac{y}{2}\right)^2 = x-1$

$\frac{y^2}{4}+1 = x$

(1) $\int e^{-2x}\sin x\,dx$

IBP (ET)

u	dv
e^{-2x} +	$\sin x$
$-2e^{-2x}$ −	$-\cos x$
$4e^{-2x}$ +	$-\sin x$

$$\int e^{-2x}\sin x\,dx = -e^{-2x}\cos x - 2e^{-2x}\sin x - 4\int e^{-2x}\sin x\,dx$$

$$5\int e^{-2x}\sin x\,dx = -e^{-2x}\cos x - 2e^{-2x}\sin x$$

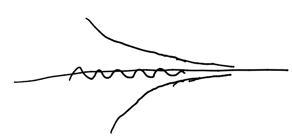
$$\int e^{-2x}\sin x\,dx = \text{a)}\ \boxed{\frac{-e^{-2x}\cos x - 2e^{-2x}\sin x}{5} + C}$$

b) $\int_0^\infty e^{-2x}\sin x\,dx$

$$\lim_{n\to\infty} \frac{-e^{-2x}}{5}(\cos x - 2\sin x)\Big|_0^\infty = 0 - \left(-\frac{e^0}{5}(1-0)\right) = \boxed{\frac{1}{5}}$$

$-1 \le \cos x \le 1$
$-1 \le \sin x \le 1$

$$\lim_{n\to\infty} \frac{\cos x - 2\sin x}{e^{2x}\cdot(-5)}$$



(2) i) $\int x\cos x^2\,dx$ ii) u sub iii) $\frac{\ln x}{x^2}$ IBP

u	dv
$\ln x$	$\frac{1}{x^2}$

(2)

i) $\int x\cos x^2\,dx$ u sub

ii) u sub

iii) $\frac{\ln x}{x^2}$ IBP

u	dv
$\ln x$	$\frac{1}{x^2}$
$\frac{1}{x}$	$-x^{-1}$

$-\frac{\ln x}{x} + \int \frac{1}{x^2}dx$

iv) $\int x\cos x\,dx$ IBP

v) $\int e^{x^2}dx$

vi) $\int \frac{\ln x}{x}dx$ u sub

$u = \ln x$

$du = \frac{1}{x}dx$

$\int u\,du$

vii) $\int x^2 e^x dx$ IBP

viii) $\int e^x \sin e^x dx$ u sub $\int \sin u\,du$

u	dv
$\ln x$	$\frac{1}{x}$
$\frac{1}{x}$	$\ln x$

$\int \frac{\ln x}{x} = \ln^2 x - \int \frac{\ln x}{x}$

ix) $\int \frac{1}{(1-x^2)^{3/2}}dx$ ← trig

v) $\int e^{x^2}dx$

$u = x^2$ ← $x = \sqrt{u}$

$du = 2x\,dx$

$\int \frac{1}{2\sqrt{u}} e^u du$ $\frac{1}{2x}du = dx$

a) IBP b) u sub or trig sub

(3)

$\int \arctan \frac{1}{x-7}\,dx$

$y = \arctan \frac{1}{x-7}$

$\tan y = \frac{1}{x-7}$

$u = \frac{1}{x-7}$ $x = \frac{1}{u} + 7$

$du = \frac{-1}{(x-7)^2}dx$

$-(x-7)^2 du = dx$ $\frac{1}{u}+7-7$

$-\int \arctan u \left(\frac{1}{u}+\cancel{7}-\cancel{7}\right)^2 du$

$\int -\frac{1}{u^2}\arctan u\,du$

w	dv
$\arctan u$	$-\frac{1}{u^2}$
$\frac{1}{u^2+1}$	u^{-1}

$\frac{\arctan u}{u} - \int \frac{1}{u(u^2+1)}du$

$\int \arctan \frac{1}{x-7}dx$

$y = \arctan \frac{1}{x-7}$

$\tan y = \frac{1}{x-7}$

$\sec^2 y\,dy = \frac{-1}{(x-7)^2}$

$-(x-7)^2 \sec^2$

$-\cot^2 y \sec^2$

$\int y \cot^2 y \sec^2 y$

$\int y \frac{\cancel{\cos^2 y}}{\sin^2 y}$ —

$\int y\, c$

$\frac{1}{\tan y} = x-7$

dx

$y\, dy = dx$

$y\, dy$

dy

$\frac{1}{\cancel{\cos^2 y}}\, dy$

$sc^2 y\, dy$

u	dv
y	$\csc^2 y$
1	$-\cot y$
0	$-\ln \sin x$

$$\left[\frac{1}{u(u^2+1)} = \frac{A}{u} + \frac{Bu+C}{u^2+1}\right] u(u^2+1)$$

$$Au^2 + A$$

$$1 = A(u^2+1) + Bu^2 + Cu$$

@ $u=0$ $\underline{u^2}$ $\underline{u}$ $\underline{0}$

$A=1$ $0=A+B$ $0=C$ $1=A$

$B=-1$

$$\int \frac{1}{u} - \frac{u}{u^2+1}\, du \qquad z = u^2+1, \quad dz = 2u\, du$$

$$\frac{\text{arctan}\, u}{u} - \ln u + \frac{1}{2}\ln(u^2+1) + C$$

$u = \frac{1}{x-7}$

(4)

$$\int \frac{8x+7}{x^2+2x+2}\, dx$$

$u = x^2+2x+2$ $= 8x+7$

$du = 2x+2\, dx$ $4(2x+2) = 8x+8-1$

$$\int \frac{\overset{8x+8}{4(2x+2)} - 1}{x^2+2x+2}\, dx$$

$$4\int \frac{2x+2}{x^2+2x+2}\, dx - \int \frac{1}{x^2+2x+2}\, dx$$

$\frac{d}{dy}\text{arctan}\, y = \frac{1}{y^2+1}$

$(x+1)^2$

$x^2+2x+2 = (x^2+2x+1)+1$

$x^2+2x+1+1$

$(\ \ +1)^2+1$

$$4\int \frac{1}{u}\, du$$

$$4\ln(x^2+2x+2) - \text{arctan}(x+1) + C$$

$$\int_1^\infty \ln\left(\frac{1}{x}\right) dx$$

u	dv
$\ln\frac{1}{x}$	1
$\frac{1}{}$ -1	x

$\int_1^{\infty} \ln\left(\frac{1}{x}\right) dx$

$\ln \frac{1}{x}$	1
$\frac{1}{\frac{1}{x}} \cdot \frac{-1}{x^2}$	x
$-\frac{1}{x}$	

$x \ln \frac{1}{x} + \int 1\, dx$

$x \ln \frac{1}{x} + x \Big|_1^{\infty}$

$\lim_{x \to \infty} x\left(\ln \frac{1}{x} + 1\right) \quad -1$

$\int e^{x^2} dx$

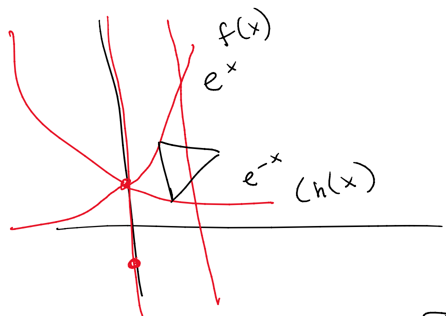
$u = x^2 \leftarrow \sqrt{u} = x$

$du = 2x\, dx$

$\frac{1}{2x}\, du = dx$

$\int \frac{1}{2x} e^u\, du$

$\frac{1}{2} \int u^{-1/2} e^u\, du$



$y = e^{-x}$

$\int A\, dx$

$A = \frac{\sqrt{3}}{4} s^2$

$\int_0^1 \frac{\sqrt{3}}{4} (f(x) - h(x))^2\, dx$

$\int_0^1 \frac{3}{4} (e^x - e^{-x})^2\, dx$

$e^{2x} - 2 + e^{-2x}$

$\int e^{2x} - e^{-2x}\, dx$

$\left(\frac{1}{2} e^{2x} - 2x - \frac{1}{2} e^{-2x}\right)\Big|_0^1 = \frac{e^2}{2} - 2 -$

$\frac{3}{4}\left(\frac{1}{2} e^{2x} + \frac{1}{2} e^{-2x}\right)\Big|_0^1 = \frac{1}{2} e^2 + \frac{1}{2e^2} - \left(\frac{1}{2} + \frac{1}{2}\right)$

$\frac{\sqrt{3}}{4}\left(\frac{e^2}{2} + \frac{1}{2e^2} - 1\right)$

$$\frac{e^{-2}}{2} - \left(\cancel{\frac{1}{2}} - 0 - \cancel{\frac{1}{2}}\right)$$

$$\frac{\sqrt{3}}{2}\left(e^{2}-1\right)$$