Part I: Multiple Choice

1. To evaluate the integral $\int (2x+1)\tan^{-1}(\sqrt{x})\,dx$ using integration by parts, we should choose:
   (a) $u = \tan^{-1}\sqrt{x}$ and $v' = 2x+1$
   (b) $u = 2x+1$ and $v' = \tan^{-1}\sqrt{x}$
   (c) $u = \sqrt{x}$ and $v' = \tan^{-1}\sqrt{x}$
   (d) $u = \tan^{-1} x$ and $v' = 2x+1$
   (e) $u = \sqrt{x}$ and $v' = 2x+1$

2. The partial fraction decomposition of $\dfrac{-x+7}{x(x+7)(x-1)^2(x+1)}$ has the form $\dfrac{A}{x} + \dfrac{B}{x+7} + \dfrac{C}{x-1} + \dfrac{D}{(x-1)^2} + \dfrac{E}{x+1}$. Find $A - D$.
   (a) $\dfrac{5}{8}$ (b) $-\dfrac{7}{6}$ (c) $\dfrac{11}{8}$ (d) $\dfrac{13}{6}$ (e) $\dfrac{3}{8}$

3. Evaluate the definite integral:
$$\int_0^{\pi/2} \sin^3(x)\cos^3(x)\,dx$$
   (a) $-\dfrac{1}{12}$ (b) $-\dfrac{1}{16}$ (c) 0 (d) $\dfrac{1}{16}$ (e) $\dfrac{1}{12}$

4. Evaluate the definite integral
$$\int_2^{\infty} \frac{1}{x^3 - x^2}\,dx$$
   (a) $\ln\left(\dfrac{1}{2}\right)$ (b) $\ln(2) - \dfrac{1}{2}$ (c) $-2 - \ln\left(\dfrac{1}{2}\right)$ (d) $\dfrac{1}{2}$ (e) This integral diverges

5. When you calculate $\int \frac{8x^2}{x^4 - 16}\, dx$, which of the following appears as a term of the solution?

(a) $-\frac{3}{x+2}$ (b) $-\frac{32}{3(x-2)^3}$ (c) $2\arctan\left(\frac{x}{2}\right)$ (d) $\ln|x^2+4|$

6. Evaluate the following indefinite integral:

$$\int \frac{1}{(4x - x^2)^{3/2}}\, dx$$

(a) $\frac{2}{\sqrt{4x - x^2}} + C$

(b) $\frac{2}{5}(4x - x^2)^{5/2} + C$

(c) $\frac{8 - 4x}{\sqrt{4x - x^2}} + C$

(d) $\frac{x - 2}{4\sqrt{4x - x^2}} + C$

7. Evaluate the following indefinite integral:

$$\int x^2 e^{x/3}\, dx$$

(a) $x^3 e^{x/3} + C$

(b) $3x^2 e^{x/3} - 18xe^{x/3} + 54e^{x/3} + C$

(c) $3x^2 e^{x/3} - 18xe^{x/3} - 9e^{x/3} + C$

(d) $\frac{1}{3}x^2 - \frac{2}{9}xe^{x/3} + \frac{1}{27}e^{x/3} + C$

8. Consider $\dfrac{x^4+10x^2}{(x^2+1)(x^2+9)}$. How many of the following statments are **true**?

- The largest coefficient that appears when you do the partial fraction decomposition is $\dfrac{9}{8}$.
- In the $\dfrac{Bx+C}{x^2+1}$ term, $B=0$.
- One of the terms in the partial fraction decomposition is the constant function 1.

(a) 0 (b) 1 (c) 2 (d) 3

9. Evaluate the indefinite integral:

$$\int \sec(x)\tan^2(x)\,dx$$

(a) $\dfrac{1}{2}\Big(\sec(x)\tan(x)+\ln|\sec(x)+\tan(x)|\Big)+C$

(b) $\dfrac{1}{2}\Big(\sec(x)\tan(x)-\ln|\sec(x)+\tan(x)|\Big)+C$

(c) $\dfrac{1}{2}\sec(x)\tan(x)+\dfrac{1}{2}\ln|\tan(x)|+C$

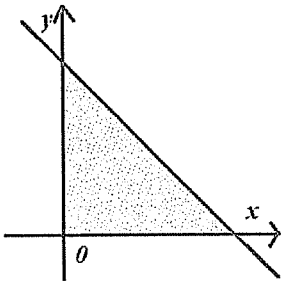
(d) $\sec(x)\tan(x)-\sec^3(x)+C$

10. Evaluate the following definite integral:

$$\int_0^2 \frac{x}{x^4+1}\,dx$$

(a) $\dfrac{\arctan(2)}{2}$ (b) $\dfrac{\ln(17)}{4}$ (c) $\dfrac{\arctan(4)}{2}$ (d) $\dfrac{\ln(17)}{2}$

1. The base of a solid is the triangle enclosed by $x + y = 5$, the $x-$axis, and the $y-$axis. Its cross sections perpendicular to the $y-$axis are semicircles. Which integral below calculates the volume of the solid?



A. $\frac{\pi}{2}\int_0^5 (5-y)^2\,dy$

B. $\frac{\pi}{8}\int_0^5 (5-y)^2\,dy$

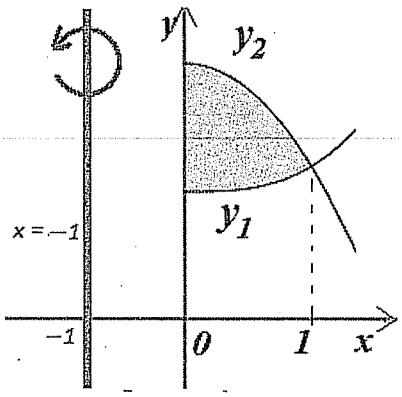
C. $\frac{\pi}{4}\int_0^5 (5-x)^2\,dx$

D. $\int_0^5 (5-y)^2\,dy$

E. $4\int_{-2}^2 (4-y^2)\,dy$

2. Using shell method, which integral below represents the volume of revolution of the shaded region about the line $x = -1$.

$$y_1 = x^3 + 2, y_2 = 4 - x^2$$



A. $2\pi \int_0^1 (x+1)(x^3+x^2-2)\,dx$

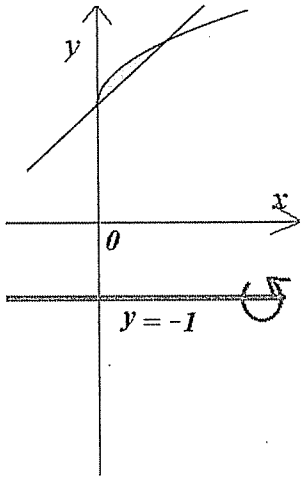
B. $\pi \int_0^1 (-x^3-x^2+2)^2\,dx$

C. $2\pi \int_0^1 (x+1)(-x^3-x^2+2)\,dx$

D. $2\pi \int_0^1 (x-1)(-x^3-x+2)\,dx$

E. $\pi \int_2^4 y(\sqrt{4-y} - \sqrt[3]{y-2})\,dy$

3. Using <u>washer method</u>, find the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x} + 3$ and $y = x + 3$ around the line $y = -1$. Which integral below represents the volume of the solid?



A. $2\pi \int_3^4 (y+1)(-y-\sqrt{y})\, dy$

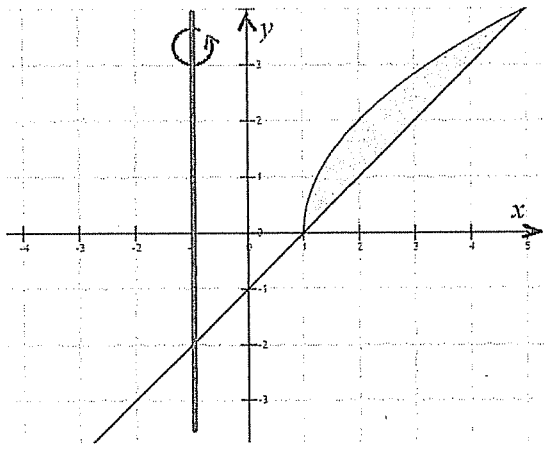
B. $\pi \int_0^1 (x^2 + 7x - 8\sqrt{x})\, dx$

C. $\pi \int_0^1 (x - 2x^{3/2} + x^2)\, dx$

D. $\pi \int_3^4 (3-y)(3-\sqrt{y})\, dy$

E. $\pi \int_0^1 (-x^2 - 7x + 8\sqrt{x})\, dx$

4. Which integral below determines the volume of the solid obtained by rotating the region bounded by $y = 2\sqrt{x-1}$ and $y = x - 1$ about the line $x = -1$?



A. $\pi \int_0^4 \left((x+2)^2 - \left(\frac{x^2}{4} + 2 \right)^2 \right) dx$

B. $\pi \int_0^4 \left((y+1)^2 - \left(\frac{y^2}{4} + 1 \right)^2 \right) dy$

C. $\pi \int_1^5 \left((x+2)^2 - \left(\frac{x^2}{4} + 2 \right)^2 \right) dx$

D. $\pi \int_0^4 \left((y+2)^2 - \left(\frac{y^2}{4} + 2 \right)^2 \right) dy$

Name: _____

Part II: Free Response

FR Scores	
1	/6
2	/4
3	/5
4	/5
FR Total	/20

1. (a) Evaluate the indefinite integral:

$$\int e^{-2x} \sin x \, dx$$

$$\int e^{-2x} \sin(x) \, dx = \_\_\_\_\_$$

(b) Calculate $\int_0^\infty e^{-2x} \sin x \, dx$

2. Use this list of integrals to answer the questions below.

i) $\int x\cos(x^2)\,dx$ *ii)* $\int xe^{x^2}\,dx$ *iii)* $\int \frac{\ln(x)}{x^2}\,dx$

iv) $\int x\cos(x)\,dx$ *v)* $\int e^{x^2}\,dx$ *vi)* $\int \frac{\ln(x)}{x}\,dx$

vii) $\int x^2e^x\,dx$ *viii)* $\int e^x\sin(e^x)\,dx$ *ix)* $\int \frac{1}{(1-x^2)^{3/2}}\,dx$

(a) Which of the above integrals can be done using **only** integration by parts? You do not need to justify your answers.

(b) Which of the integrals above can be done using **only** a u-substitution and/or trigonometric substitution? You do not need to justify your answers.

3. Evaluate the integral:

$$\int \arctan\left(\frac{1}{x-7}\right)\, dx$$

$$\int \arctan\left(\frac{1}{x-7}\right)\, dx = \underline{\qquad\qquad\qquad\qquad\qquad\qquad}$$

4. Evaluate the integral $\int \frac{8x+7}{x^2+2x+2}\,dx$

$$\int \frac{8x+7}{x^2+2x+2}\,dx = \underline{\qquad\qquad\qquad\qquad\qquad\qquad}$$