MAC 2312
Fall 2019
EXAM 1

Section # ____________ Name ______________________________

UF ID # ____________ TA Name ______________________________

A. Sign your scantron on the back at the bottom in ink.

B. In pencil, write and encode on your scantron in the spaces indicated:

1) Name (last name, first initial, middle initial)
2) UF ID Number
3) Section Number

C. Under "special codes", code in the test ID number 1, 1.

● 2 3 4 5 6 7 8 9 0

● 2 3 4 5 6 7 8 9 0

D. At the top right of your answer sheet, for "Test Form Code", encode A.

● B C D E

E.
1) There are twelve 2.2-points multiple choice questions, plus two free response questions of 14 points for a total of 40.4/40 points.
2) The time allowed is 90 minutes.
3) You may write on the test.
4) Raise your hand if you need more scratch paper or if you have a problem with your test. DO NOT LEAVE YOUR SEAT UNLESS YOU ARE FINISHED WITH THE TEST.

F. **KEEP YOUR SCANTRON COVERED AT ALL TIMES.**

G. When you are finished:

1) Before turning in your test, check for transcribing errors. Any mistakes you leave in are there to stay.
2) Bring your test, scratch paper, and scantron to your proctor to turn them in. Be prepared to show your UF ID card.
3) Answers will be posted in E-Learning after the exam.

The Honor Pledge: "On my honor, I have neither given nor received unauthorized aid in doing this exam."

Student's Signature:_______________

1. Evaluate the integral $\int 5x \underbrace{\cos^2(5x)}_{\frac{1}{2}+\frac{1}{2}\cos(10x)} \, dx$.

A. $\frac{5}{4}\,x^2 + \frac{1}{20}\,x\cos(5x) + \frac{1}{20}\,\cos(5x) + C$

B. $\frac{9}{4}\,x^2 + \frac{1}{20}\,x\sin(20x) - \frac{1}{200}\,\cos(20x) + C$

C. $\frac{15}{4}\,x^2 + \frac{1}{10}\,x\sin(5x) + \frac{1}{200}\,\sin(5x) + C$

D. $\frac{15}{4}\,x^2 + \frac{1}{4}\,x\sin(10x) - \frac{1}{40}\,\cos(10x) + C$

E. $\frac{5}{4}\,x^2 + \frac{1}{4}\,x\sin(10x) + \frac{1}{40}\,\cos(10x) + C$

IBP

$5x$	$+$	$\frac{1}{2}+\frac{1}{2}\cos(10x)\,dx$
5	$-$	$\frac{1}{2}x+\frac{1}{20}\sin(10)$
0	$+$	$\frac{1}{4}x^2 - \frac{1}{200}\cos(10x)$

$I = \frac{5}{2}x^2 + \frac{1}{4}x\sin(10x) - \frac{5}{4}x^2 + \frac{1}{40}\cos(10x) + C$

$= \frac{5}{4}x^2$.

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2. Evaluate the integral $\int_0^1 x^8 e^{x^3}\, dx$.

$= \int x^6 \cdot e^{x^3} \cdot x^2 dx$

A. $2+e$ B. $e-2$ C. $\frac{1}{3}(e-2)$

D. $2-e$ E. $e-3$

u-sub $\left[u \Big|_0^1 = x^3 \Big|_0^1, \ \frac{1}{3}du = x^2dx \right]$

$= \frac{1}{3}\int u^2 e^u du$

IBP

u^2	$+$	$e^u du$
$2u$	$-$	e^u
2	$+$	e^u
0	$-$	e^u

$= \frac{1}{3}e^u(u^2 - 2u + 2)\Big|_0^1$

$= \boxed{\frac{1}{3}e - \frac{2}{3}}$

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3. Evaluate the integral $\int_0^1 \frac{\cos^3(\sqrt{x})}{\sqrt{x}}\, dx$.

3 odd → u-sub

$= 2\int \cos^3 t\, dt = 2\int \cos^2 t \cdot \cos t\, dt$

$= 2\int (1-\sin^2 t) \cos t\, dt$

u, du.

A. $2\left(\sin 1 - \frac{1}{3}\sin^3 1\right)$ B. $\frac{1}{2}\left(\sin 1 - \frac{1}{3}\sin^3 1\right)$ C. $\frac{2}{3}\sin^3 1$

D. $\frac{2}{3}\sin 1$ E. $\sin 1 + \frac{1}{3}\sin^3 1$

t-sub $\left[t\Big|_0^1 = \sqrt{x}\Big|_0^1, \ dt = \frac{1}{2\sqrt{x}}\, dx \right]$

u-sub $\left[u\Big|_0^{\sin 1} = \sin t\Big|_0^1 \right]$

$= 2\int (1-u^2)\, du$

$= 2\left[u - \frac{u^3}{3}\right]_0^{\sin 1}$

$= \boxed{2\left[\sin 1 - \frac{1}{3}\sin^3 1\right]}$

PFD

$u = e^x$, $\quad = \int \frac{u \cdot}{(u-1)(u^2+1)} du$.

6. When calculating $\displaystyle\int \frac{e^{2x}}{(e^x - 1)(e^{2x} + 1)}\, dx$, which statements below are true?

X P. Arcsecant functions appear as a term of the solution.

✓ Q. After appropriate substitution, $\displaystyle\int \frac{u\, du}{(u-1)(u^2+1)}$ is an equivalent integral.

✓ R. One of the constant coefficients that appears when you do the partial fraction decomposition is $\frac{1}{2}$.

A. Only R B. Only P and Q C. Only Q and R

D. Only P E. Only Q

PFD: $\underbrace{(u-1)}_{\text{case 1}}\underbrace{(u^2+1)}_{\text{case 3}}$ $\quad \frac{u}{(u-1)(u^2+1)} = \frac{\overset{\frac{1}{2}}{A}}{u-1} + \frac{Bu + \cancel{C}\,\frac{1}{2}}{u^2+1}$ (split top)

coverup: $A = \frac{1}{2}$

let $u=0$: $0 = -\frac{1}{2} + C \Rightarrow C = \frac{1}{2}$

$\int \frac{e^{2x}}{(e^x-1)(e^{2x}+1)} dx = \int \frac{u}{(u-1)(u^2+1)} du$.

$= \frac{1}{2}\int \frac{1}{u-1} + \frac{Bu}{u^2+1} + \frac{\frac{1}{2}}{u^2+1}$

↑ ln. ↑ (u-sub) ln ↑ arctan.

7. When you calculate $\displaystyle\int \frac{4x + 5}{x^2 + 2x + 5}\, dx$, which of the following appears as a term of the solution?

$5 = 4 + 1$

$b^2 - 4ac < 0 \to$ 2 steps:

A. $2\ln(x^2 + 2x + 5)$ B. $\cancel{\frac{1}{2}} \arctan\left(\cancel{\frac{x}{2}}\right)$ C. $\cancel{2} \arctan\left(\frac{x+1}{2}\right)$

D. $\cancel{\frac{1}{2}}\ln(x^2 + 2x + 5)$ E. $\cancel{\ }\ln(x^2 + 2x + 5)$

① $u = x^2+2x+5$

$du = (2x+2)\,dx$

$2du = (4x+4)\,dx$

② □: x^2+2x+5

$= (x+1)^2 + 2^2$

$I = \int \frac{4x+4}{x^2+2x+5} dx + \int \frac{1}{(x+1)^2+2^2} dx$

$= 2\int \frac{1}{u} du + \frac{1}{2}\arctan\left(\frac{x+1}{2}\right)$

$= 2\ln(x^2+2x+5) + \frac{1}{2}\arctan\left(\frac{x+1}{2}\right) + C$

cover up
$= -1$

11. The partial fraction decomposition of $\dfrac{4x-1}{x(x^2+1)^2}$ has the form $\dfrac{A}{x}+\dfrac{Bx+C}{x^2+1}+\dfrac{Dx+E}{(x^2+1)^2}$. Find A+B.

A. 0 B. 1 C. 2 D. -4 E. -1

$4x-1 = A(x^2+1)^2+(Bx+C)x(x^2+1)+(Dx+E)x$
$= \underbrace{(A+B)}_{=0}x^4 + \cdots$

12. Evaluate the integral $\displaystyle\int \frac{4x^2+8}{(x+1)(x-1)^2}\,dx$.

A. $3\ln\left|\dfrac{x-1}{x+1}\right| - \arctan\left(\dfrac{6}{x-1}\right)+C$

B. $3\ln\left|\dfrac{x+1}{x-1}\right| - \arctan\left(\dfrac{6}{x-1}\right)+C$

C. $3\ln\left|\dfrac{x-1}{x+1}\right| - \dfrac{6}{x-1}+C$

D. $3\ln|x+1|+\ln|x-1| - \dfrac{6}{x-1}+C$

E. $3\ln\left|\dfrac{x+1}{x-1}\right| + \dfrac{6}{x-1}+C$

PFD:

$\dfrac{4x^2+8}{(x+1)(x-1)^2} = \dfrac{A}{x+1}+\dfrac{B}{x-1}+\dfrac{C}{(x-1)^2}$ 3 1 6.

case 1 case 2

cover up: $A = \dfrac{4+8}{(-2)^2} = 3$

$C = \dfrac{4+8}{2} = 6$

let $x=0$:

$\dfrac{8}{1} = 3 - B + 6$

$B = 1$

$= \displaystyle\int \frac{3}{x+1} + \frac{1}{x-1} + \frac{6}{(x-1)^2}$

(power rule)

$= \boxed{3\ln|x+1| + \ln|x-1| - \frac{6}{x-1} + C}$

1. Evaluate the indefinite integral $\displaystyle\int \frac{x^2}{\sqrt{-8x+x^2}}\,dx$.

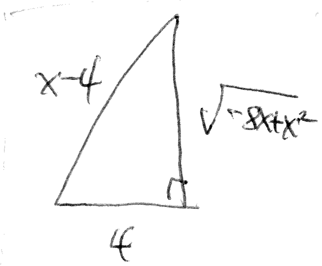
First, complete the square and write $-8x+x^2$ as a difference of two squares.

$x^2-8x+16-16=(x-4)^2-4^2$

$-8x+x^2 =$ $\boxed{(x-4)^2-4^2}$

Trig-sub: $x-4=4\sec\theta$,

$$\int \frac{(4\sec\theta+4)^2\cdot \cancel{4}\sec\theta\,\cancel{\tan\theta}\,d\theta}{\cancel{4}\,\cancel{\tan\theta}}$$



$$=\int(16\sec^2\theta+32\sec\theta+16)\sec\theta\,d\theta$$

$$=\int(16\sec^3\theta+32\sec^2\theta+16\sec\theta)\,d\theta$$

$$=16\cdot\tfrac{1}{2}(\sec\theta\tan\theta+\ln|\sec\theta+\tan\theta|)+32\tan\theta+16\ln|\sec\theta+\tan\theta| + C$$

$$=8\sec\theta\tan\theta+24\ln|\sec\theta+\tan\theta|+32\tan\theta+C$$

$$=\cancel{8}^{2}\cdot\frac{x-4}{\cancel{4}}\cdot\frac{\sqrt{\ }}{\cancel{4}\,2}+24\ln\left|\frac{x-4}{4}+\frac{\sqrt{\ }}{4}\right|+32\cdot\frac{\sqrt{\ }}{4}+c$$

$$=\frac{(x-4)\sqrt{-8x+x^2}}{2}+24\ln\left|x-4+\sqrt{-8x+x^2}\right|+8\sqrt{-8x+x^2}+c$$

$$\int \frac{x^2}{\sqrt{-8x+x^2}}\,dx = \underline{\hspace{4cm}} + C$$

2. Evaluate the improper integral $\int_0^\infty e^{-\sqrt{x}}\,dx$.

Begin with an appropriate u−sub. $u =$ $\sqrt{x}$

$u^2 = x, \quad 2u\,du = dx$

$= \boxed{\int 2u e^{-u}\,du}$ (+1)

IBP:
$$\begin{bmatrix} 2u & + & e^{-u}\,du \\ 2 & - & -e^{-u} \\ 0 & + & e^{-u} \end{bmatrix}$$
(+1)

$= e^{-u}(-2u-2)$ (+0.5)

or

$= \dfrac{-2\sqrt{x}-2}{e^{\sqrt{x}}}$

(+0.5) (+2.5)

$$\lim_{t\to\infty} \frac{-2\sqrt{x}-2}{e^{\sqrt{x}}}\Bigg|_0^t = \lim_{t\to\infty}\left[\cancel{\frac{-2\sqrt{t}-2}{e^{\sqrt{t}}}}^{\,\to 0} - \frac{-2}{1}\right]$$

(+1)

$= \boxed{2}$

$\int e^{-\sqrt{x}}\,dx =$ $\dfrac{-2\sqrt{x}-2}{e^{\sqrt{x}}}$ +C (+2.5)

Does the improper integral $\int_0^\infty e^{-\sqrt{x}}\,dx$ converge? Circle one. (Yes/No) (+1)

If it converges, what does it converge to? Or say 'DIV' if it diverges. $\boxed{2}$. (+0.5)