Calculus I: MAC2311
Spring 2024
Exam 1 A
1/31/2024
Time Limit: 100 Minutes

Name: Key

Section: ______________

UF-ID: ______________

Scantron Instruction: This exam uses a scantron. Follow the instructions listed on this page to fill out the scantron.

A. Sign your scantron **on the back** at the bottom in the white area.

B. Write **and code** in the spaces indicated:

1) Name (last name, first initial, middle initial)

2) UFID Number

3) 4-digit Section Number

C. Under *special codes*, code in the test numbers 1, 1:

- • 2 3 4 5 6 7 8 9 0
- • 2 3 4 5 6 7 8 9 0

D. At the top right of your scantron, fill in the *Test Form Code* as A .

- • B C D E

E. This exam consists of 14 multiple choice questions and 5 free response questions. Make sure you check for errors in the number of questions your exam contains.

F. The time allowed is 100 minutes.

G. WHEN YOU ARE FINISHED:

1) Before turning in your test check for **transcribing errors**. Any mistakes you leave in are there to stay!

2) You must turn in your scantron and free response packet to your proctor. **Be prepared to show your proctor a valid GatorOne ID or other signed ID.**

It is your responsibility to ensure that your test has **19 questions**. If it does not, show it to your proctor immediately. You will not be permitted to make up any problems omitted from your test after the testing period ends. There are a total of 105 points available on this exam.

Part I Instructions: 14 multiple choice questions. Complete the scantron sheet provided with your information and fill in the appropriate spaces to answer your questions. Only the answer on the scantron sheet will be graded. Each problem is worth five (5) points for a total of 70 points on Part I.

1. Find the domain of the function $f(x) = \sqrt{x} + \sqrt{4-x}$.

(A) $(-\infty, 0) \cup (4, \infty)$

(B) $[0, 4]$

(C) $(0, \infty)$

(D) $[0, 4] \cup [4, \infty)$

We need $x \geq 0$ and $4 - x \geq 0$.

So, $x \geq 0$ and $4 \geq x$.

2. Find the vertical asymptotes of $f(x) = \frac{x^2+3x}{x^2-9}$. $\rightarrow \dfrac{x\cancel{(x+3)}}{(x-3)\cancel{(x+3)}}$

(A) $x = -3$

(B) $x = -3$, $x = 3$

(C) $x = 3$

(D) $x = 1$

(E) There are no vertical asymptotes.

$X = -3$ is a hole in the graph.

$X = 3$ is the vertical asymptote.

3. Evaluate $\lim_{x\to 1} e^{-7x^4-\sin\pi x+2}$

(A) e^{-9} (B) e^{-6} (C) e^{-8} (D) e^{-5} (E) Does not exist.

This is continuous, so plug in $x=1$

$$\lim_{x\to 1} e^{-7x^4-\sin\pi x+2} = e^{-7-\sin\pi+2}$$

$$= e^{-5-0} = \boxed{e^{-5}}$$

4. Evaluate $\lim_{x\to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x}$

use conjugate.

(A) 0 (B) $\frac{1}{2\sqrt{2}}$ (C) $-\frac{1}{2\sqrt{2}}$ (D) $\frac{1}{2}$ (E) $-\frac{1}{2}$

$$\lim_{x\to 0}\left(\frac{\sqrt{2+x}-\sqrt{2}}{x}\right)\frac{(\sqrt{2+x}+\sqrt{2})}{(\sqrt{2+x}+\sqrt{2})}$$

$$= \lim_{x\to 0}\frac{(2+x)-2}{x(\sqrt{2+x}+\sqrt{2})} = \lim_{x\to 0}\frac{x}{x(\sqrt{2+x}+\sqrt{2})}$$

$$= \lim_{x\to 0}\frac{1}{(\sqrt{2+x}+\sqrt{2})} = \frac{1}{\sqrt{2}+\sqrt{2}} = \frac{1}{2\sqrt{2}}$$

5. An object moves along a straight line with position function given by $s(t) = 2t^2 - 3t + 1$, where $s(t)$ is measured in feet and t in seconds. What is the average velocity in feet per second of the object over the interval $[2, 4]$?

(A) 18 (B) -18 (C) 9 (D) -9 (E) None of these.

$$\text{Average Velocity} = \frac{s(4)-s(2)}{4-2} \quad \text{(slope formula)}$$

$$= \frac{2(4)^2-3(4)+1-(2\cdot 2^2-3(2)+1)}{4-2}$$

$$= \frac{32-12+1-(8-6+1)}{2} = \frac{21-3}{2} = \boxed{9}$$

6. Let

$$f(x) = \frac{ax\left(x^2+x-1\right)}{3bx^3-10x+1}.$$

For which of the following values of a and b will $\lim_{x\to\infty} f(x) = 4$?

We need to consider the ratio of leading coefficients.

(A) $a = 6,\ b = 1$ (B) $a = 6,\ b = 2$ (C) $a = 12,\ b = 1$

(D) $a = 12,\ b = 2$ (E) $a = 12,\ b = 4$

$\therefore \frac{a}{3b} = 4$. Try all answer choices until one works.

7. Let

$$f(x) = \begin{cases} x^2 - k^2 & \text{if } x < 4 \\ kx + 20 & \text{if } x \geq 4 \end{cases}$$

Plug in $x=4$ to each part.

Calculate the value of k for which $f(x)$ is continuous at $x = 4$.

(A) $k = 0$ (B) $k = 1$ (C) $k = 2$

(D) $k = -1$ (E) $k = -2$

Need $\lim_{x \to 4^-} f(x) = \lim_{x \to 4^+} f(x)$.

$16 - K^2 = 4K + 20$

$0 = K^2 + 4K + 4$

$0 = (K+2)^2$

$\boxed{K = -2}$

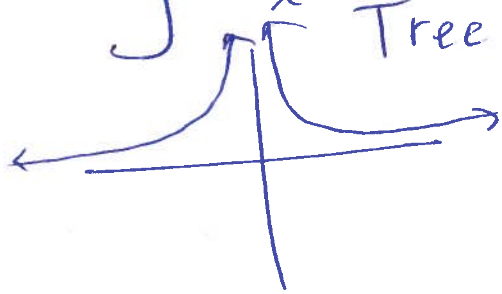
8. For which of the following functions is $\lim_{x \to 0^+} f(x) = \infty$?

(A) $f(x) = \ln(x)$ (B) $f(x) = e^x$ (C) $f(x) = x^2$

(D) $f(x) = \sin(x)$ (E) $f(x) = \frac{1}{x^2}$

Draw them!

$y = \frac{1}{x^2}$ Tree!



9. Calculate $\lim_{x \to 2^-} \frac{4}{x-2}$. "$\frac{4}{0}$".

(A) $\frac{5}{2}$ (B) 0 (C) ∞ (D) $-\infty$ (E) Does not exist.

Test $X = 1.9$:

$$\frac{4}{X-2} \to \frac{4\,(+)}{(-)} \to -\infty$$

10. What type of discontinuity does the function $f(x) = \frac{x^2-4}{x-2}$ have at $x = 2$?

(A) An infinite discontinuity. (B) A removable discontinuity. (hole)

(C) A jump discontinuity. (D) There is no discontinuity at $x = 2$.

$$f(X) = \frac{X^2-4}{X-2} = \frac{\cancel{(X-2)}(X+2)}{\cancel{X-2}} = X+2 \; ; \; X \neq 2$$

11. Let $f(x) = \frac{x-1}{|x-1|}$.

Which of the following statements are **true**? Multiple statements may be true.

P. $f(x)$ has a jump discontinuity at $x = 1$.

Q. $\lim_{x \to 1^+} f(x) = 1$

R. $x = 1$ is a vertical asymptote on the graph of $f(x)$. False. A jump is not a vertical asymptote.

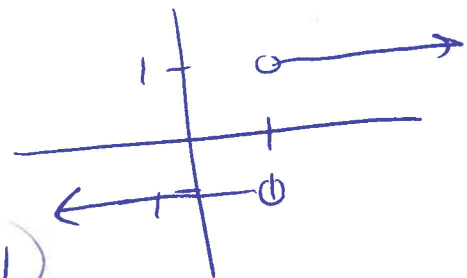
S. $f(x)$ is continuous from the right and continuous from the left, but not continuous at $x = 1$. False, both are open circles at $x=1$

(A) P only (B) P and Q (C) Q and R (D) P, R, and S (E) P, Q, and S

Graph: $f(x) = \frac{x-1}{|x-1|}$

$$\lim_{x\to1^+} \frac{x-1}{|x-1|} = \lim_{x\to1^+} \frac{x-1}{x-1} = 1$$

$$\lim_{x\to1^-} \frac{x-1}{|x-1|} = \lim_{x\to1^-} \frac{x-1}{-(x-1)} = -1$$



12. Which of the following statements is **true**? (Only one statement below is true.)

A. Every continuous function $f(x)$ is also differentiable. False. $y=|x|$ is not differentiable

B. Rational functions $h(x) = \frac{f(x)}{g(x)}$ are always continuous. False. $y = \frac{1}{x-1}$ is not continuous

C. Every function $f(x)$ has one or two horizontal asymptotes. False Some functions have zero asymptotes

D. A function $f(x)$ can have infinitely many vertical asymptotes. True.

(A) Choice A (B) Choice B (C) Choice C (D) Choice D

13. Suppose $\lim_{x\to1} f(x) = 2$, $\lim_{x\to1} g(x) = 0$, $\lim_{x\to1} h(x) = \infty$, $\lim_{x\to1} j(x) = -2$, and $\lim_{x\to1} k(x) = -\infty$. Which of the following limits does **not** result in an indeterminate form?

(A) $\lim_{x\to1} \dfrac{g(x)}{f(x)+j(x)} = "\dfrac{0}{0}"$ (B) $\lim_{x\to1} \dfrac{j(x)-k(x)}{h(x)} = "\dfrac{\infty}{\infty}"$ (C) $\lim_{x\to1} h(x)g(x) = "\infty \cdot 0"$

(D) $\lim_{x\to1} \dfrac{f(x)j(x)}{k(x)} = "\dfrac{-4}{-\infty}"$ (E) None of these.

$\dfrac{-4}{-\infty} \to 0$, which is not indeterminate.

derivative.

14. Find the slope of the tangent line to $f(x) = \frac{1}{x}$ at $x = 3$.

(A) $\frac{1}{9}$ (B) $-\frac{1}{9}$ (C) $\frac{1}{3}$ (D) $-\frac{1}{3}$

$$\text{slope} = f'(x) = \lim_{h\to0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \lim_{h\to0} \frac{\frac{x-(x+h)}{x(x+h)}}{h} = \lim_{h\to0} \frac{\frac{-h}{x(x+h)}}{h} = \lim_{h\to0} \frac{-1}{x(x+h)}$$

$$= \frac{-1}{x^2}.$$

So, $f'(x) = \dfrac{-1}{x^2}$

$f'(3) = \dfrac{-1}{9}$

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Part II Instructions: 5 free response questions. Neatly give a complete solution to each problem and show all work and intermediate steps. We are grading the work and notation as well as the answer. Each problem is worth seven (7) points. A total of 35 points is possible on Part II. **No credit will given without proper work.** If we cannot read it and follow it, you will receive no credit for the problem.

For Instructor Use Only:

FR 1	
FR 2	
FR 3	
FR 4	
FR 5	
Total Points	

1. Evaluate $f'(x)$ for $f(x) = \frac{1}{x-2}$ using the limit definition of the derivative. **NO CREDIT** will be given if the limit definition of the derivative is not used. You can use either definition 1 or definition 2.

Def 1.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{(x+h)-2} - \frac{1}{x-2}}{h} = \lim_{h\to 0} \frac{\frac{x-2-[(x+h)-2]}{[(x+h)-2](x-2)}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{-h}{[(x+h)-2](x-2)}}{h} = \lim_{h\to 0} \frac{-1}{[(x+h)-2](x-2)}$$

$$= \frac{-1}{(x-2)(x-2)} = \boxed{\frac{-1}{(x-2)^2}}$$

Def 2.

$$f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\frac{1}{x-2} - \frac{1}{a-2}}{x-a} = \lim_{x\to a} \frac{\frac{a-2-(x-2)}{(x-2)(a-2)}}{x-a}$$

$$= \lim_{x\to a} \frac{\frac{a-x}{(x-2)(a-2)}}{x-a} = \lim_{x\to a} \frac{\overset{-1}{\cancel{a-x}}}{(x-2)(a-2)} \cdot \frac{1}{\cancel{x-a}}$$

$$= \lim_{x\to a} \frac{-1}{(x-2)(a-2)} = \frac{-1}{(a-2)(a-2)} = \frac{-1}{(a-2)^2}$$

$$\boxed{f'(x) = \frac{-1}{(x-2)^2}}$$

2. Given the two functions below, explain why you **can** or **cannot** use the Intermediate Value Theorem to determine whether there is a root on the given interval. If you can use the Intermediate Value Theorem to show the existence of a root on the given interval, do so. Explain using complete sentences.

(i) $f(x) = 2x^2 + 3x - 1$ on the interval $(0, 1)$.

f(x) is a continuous function since it is a polynomial. We can apply IVT:

$f(0) = -1 < 0$

$f(1) = 4 > 0$

Since $f(0) < 0 < f(1)$, there exists some c in $(0,1)$ such that $f(c) = 0$ by IVT.

(ii) $g(x) = \frac{1}{x-1}$ on the interval $(-1, 1)$.

We cannot use IVT since g(x) is <u>not</u> continuous at $x=1$.

3. Calculate

$$\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right).$$

Show all work and **state any appropriate theorem(s)** used in your reasoning.

We know that for all x near 0,

$$-1 \leq \sin\left(\frac{1}{x}\right) \leq 1$$

multiply through by $x^2 \geq 0$:

$$-x^2 \leq x^2 \sin\left(\frac{1}{x}\right) \leq x^2$$

Notice that

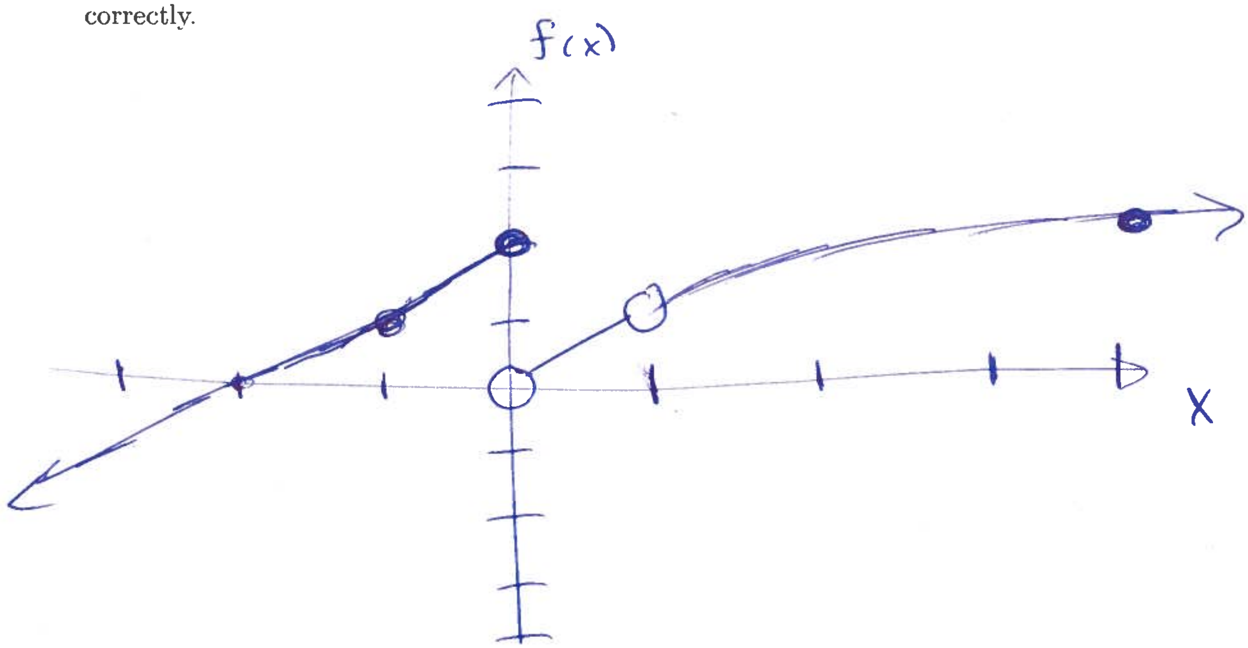
$\lim_{x \to 0} -x^2 = \lim_{x \to 0} x^2 = 0$. Therefore,

by the Squeeze Theorem,

$$\boxed{\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) = 0.}$$

4. Let $f(x) = \begin{cases} x+2 & \text{if } x \le 0 \\ |x| & \text{if } 0 < x < 1 \\ \sqrt{x} & \text{if } x > 1 \end{cases}$

(i) With the space provided below, carefully graph $f(x)$. Take care to label open and closed circled correctly.



(ii) Does $f(x)$ have any removable, jump, or infinite discontinuities? If so, explain why (using limit statements) and provide the x-values for every discontinuity $f(x)$ possesses.

$\lim_{x\to 0^-} f(x) = 2$ and $\lim_{x\to 0^+} f(x) = 0$.

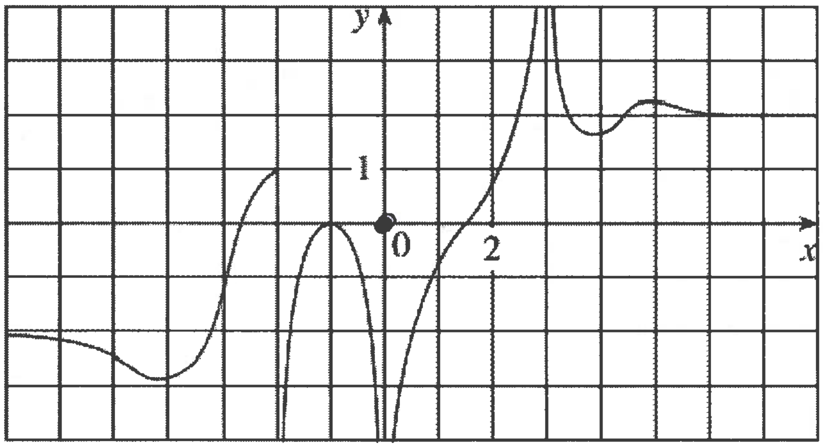
Therefore f(x) has a jump discontinuity at $x = 0$.

$\lim_{x\to 1^-} f(x) = 1$ and $\lim_{x\to 1^+} f(x) = 1$.

However, f(1) is undefined.

Therefore, f(x) has a removable discontinuity at $x = 1$.

5. Use the given graph of f to state the value of each quantity, if it exists. If it does not exist, write "DNE". You do not need to provide an explanation for these.



1. $f(-1) =$ 0
2. $\lim_{x \to 3^-} f(x) =$ ∞
3. $\lim_{x \to 3^+} f(x) =$ ∞
4. $\lim_{x \to 3} f(x) =$ ∞
5. $\lim_{x \to -2^-} f(x) =$ 1
6. $\lim_{x \to -2} f(x) =$ DNE
7. $\lim_{x \to -\infty} f(x) =$ -2
8. $\lim_{x \to \infty} f(x) =$ 2