Spring 2019 Final Exam A

2) $a(t) = \sin(t) + \cos(t)$

$s(0) = 2 \qquad s(\pi/2) = 3$

Find pos. at $t = \pi$ $\qquad s(\pi)$

$a(t) = \sin(t) + \cos(t)$
$v(t) = -\cos(t) + \sin(t) + C$
$s(t) = -\sin(t) - \cos(t) + Cx + D$

sin
cos
-sin
-cos
sin

$s(0) = 2$

$-\sin(0) - \cos(0) + \cancel{c(0)}^{\,0} + D = 2$

$0 - 1 + D = 2$

$+1 \qquad\qquad +1$

$D = 3$

$s(t) = -\sin(t) - \cos(t) + Cx + 3$

$s(\pi/2) = 3$

$-\sin\left(\frac{\pi}{2}\right) - \cos\left(\frac{\pi}{2}\right) + c\left(\frac{\pi}{2}\right) + 3 = 3$

$-1 - 0 + c\left(\frac{\pi}{2}\right) + 3 = 3$

$2 + \frac{\pi}{2}c = 3$

$-2 \qquad\qquad -2$

$$\cancel{\frac{2}{\pi}} \cdot \cancel{\frac{\pi}{2}}\, c = 1 \cdot \frac{2}{\pi}$$

$$c = \frac{2}{\pi}$$

$$s(t) = -\sin(t) - \cos(t) + \frac{2}{\pi} x + 3$$

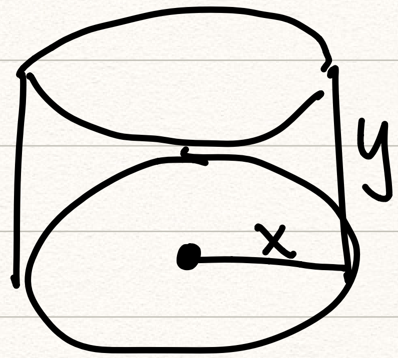
$$s(\pi) = -\sin(\pi) - \cos(\pi) + \frac{2}{\cancel{\pi}}(\cancel{\pi}) + 3$$

$$= 0 - (-1) + 2 + 3$$

$$= 1 + 2 + 3$$

$$\boxed{s(\pi) = 6}$$

Final 2019 #2

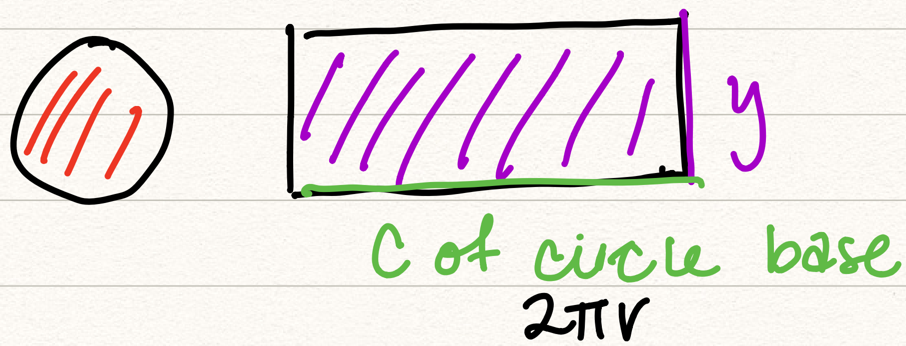


10π in^2 of material

$$V = \pi r^2 h$$

$$V = \pi x^2 y$$

$$SA = \underbrace{\pi x^2} + \underbrace{2\pi x y} = 10\pi$$



$$\pi x^2 + 2\pi x y = 10\pi$$

$$\frac{2\pi x y}{2\pi x} = \frac{10\pi - \pi x^2}{2\pi x}$$

$$y = \frac{10\cancel{\pi} - \cancel{\pi} x^2}{2\cancel{\pi} x}$$

$$\boxed{V = \pi x^2 \left(\frac{10 - x^2}{2x}\right)}$$

B

9. The graph of a function $f(x)$ is shown below on the interval $[2, 4]$ [-2,4]. Let the positive numbers A, B, and C represent the areas of the shaded regions. How many of the following statements below are true?

✓ (i) $\int_{-2}^{4} f(x)\,dx = A - B + C$ ← below x-axis

✓ (ii) $\int_{-2}^{4} |f(x)|\,dx = A + B + C$ ← since | | abs. value

✗ (iii) $\int_{0}^{2} f(x)\,dx = B$ ← should be neg

✓ (iv) $F(4) - F(2) = C$, where $F(x)$ is any antiderivative of $f(x)$

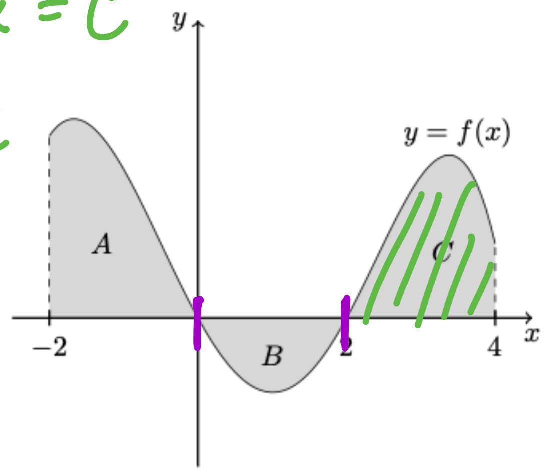
$\int_2^4 f(x)\,dx = C$

$F(x)\Big|_2^4 = C$

$F(4) - F(2) = C$

f(x)
↳ antideriv
F(x)

f'(x)
↳ antideriv
f(x)



(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Fall 2019 Final B

13) $\int_0^3 f(x)\,dx = 4$ $\int_0^5 f(x)\,dx = -6$

$\int_3^0 f(x)\,dx = -4$

$\int_3^5 f(x)\,dx = -4 + -6 = -10$

$$\int_3^5 (3f(x)-1)\,dx$$

$$\int_3^5 3f(x) + \int_3^5 -1\,dx$$

$$3\int_3^5 f(x) - \int_3^5 1\,dx$$

$$3(-10) - [x]_3^5$$

$$-30 - [5-3]$$

$$-30 - 2 = -32$$

A

Fall 2019 Final A #8

$f(x)$ defined $\mathbb{R}$

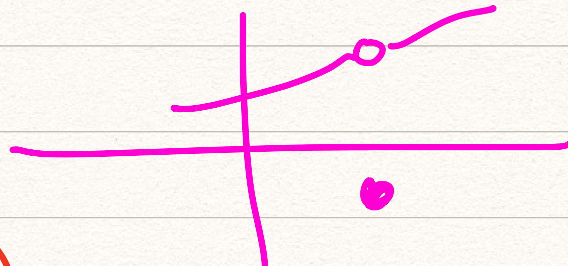
$f(-2) = 7$ $\qquad$ $f(5) = -3$

$f(x) = y$

$\Rightarrow \exists c \quad (-2, 5) \qquad f(c) = 6$

$$f'(c) = \cancel{\frac{f(5) - f(-2)}{5 - (-2)} = \frac{-3 - 7}{7}} = \frac{-10}{7}$$

False may not be cont (could be a hole)



may not be cont.

Question from Chat

Find $f(x)$

$f''(x) = -\cos x + \sin x \qquad f(0) = 1,\ f(\pi) = 0$

$f'(x) = -\sin x - \cos x + C$

$f(x) = \cos x - \sin x + Cx + D$

sin
cos
-sin
-cos
sin

Use $f(0) = 1$

$f(0) = \cos(0) - \sin(0) + C(0) + D = 1$

$= 1 - 0 + 0 + D = 1$

$1 + D = 1$

$D = 0$

$f(x) = \cos(x) - \sin(x) + Cx$

Use $f(\pi) = 0$

$f(\pi) = \cos(\pi) - \sin(\pi) + C\pi = 0$

$-1 - 0 + C\pi = 0$

$C\pi = 1$

$C = \dfrac{1}{\pi}$

$f(x) = \cos x - \sin x + \frac{1}{\pi} x$

1st Exam 9

$$f(x) = \begin{cases} 4^{x-1} + 3 & -\infty < x < 2 \\ 3^{x-1} + 4 & 2 \leq x < \infty \end{cases}$$

IVT $[-1, 4]$ $\boxed{\text{yes}}$

cont. on the closed interval

$$\lim_{x \to 2^-} 4^{x-1} + 3 = 4^{2-1} + 3$$
$$= 4 + 3 = 7$$

$$\lim_{x \to 2^+} 3^{x-1} + 4 = 3^{2-1} + 4$$
$$= 3 + 4 = 7$$

$$f(-1) = 4^{-1-1} + 3$$
$$= 4^{-2} + 3$$
$$= \frac{1}{4^2} + 3$$

+

$$f(4) = 3^{4-1} + 4$$
$$= 3^3 + 4$$

+

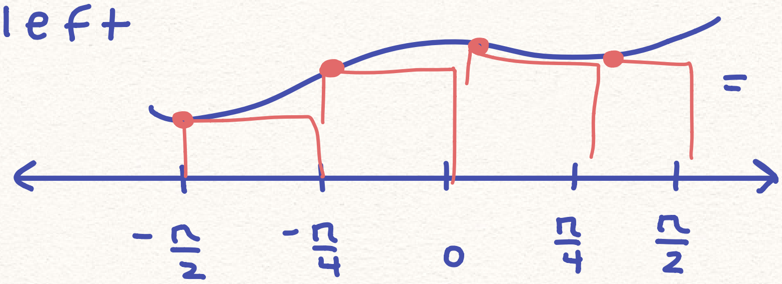
Final 2019 # 7

$f(x) = 12\cos(x)$ $\qquad \Delta x = \dfrac{x_2 - x_1}{n}$

$\left[\overset{x_1}{-\frac{\pi}{2}}, \overset{x_2}{\frac{\pi}{2}} \right]$ $\qquad n = 4 \qquad = \dfrac{\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)}{4}$

left

$= \dfrac{\pi}{4}$



$$\frac{\pi}{4}\left[f\left(-\frac{\pi}{2}\right) + f\left(-\frac{\pi}{4}\right) + f(0) + f\left(\frac{\pi}{4}\right) \right]$$

$$\frac{\pi}{4}\left[\cancelto{0}{12\cos\left(-\frac{\pi}{2}\right)} + 12\cos\left(-\frac{\pi}{4}\right) + 12\cos(0) + 12\cos\left(\frac{\pi}{4}\right) \right]$$

$$= \frac{\pi}{4}\left[12\left(\frac{\sqrt{2}}{2}\right) + 12(1) + 12\left(\frac{\sqrt{2}}{2}\right) \right]$$

$$= \frac{\pi}{4}\left(6\sqrt{2} + 12 + 6\sqrt{2} \right)$$

$$= \frac{\pi}{4}\left(12 + 12\sqrt{2} \right)$$

$$= \frac{12\pi}{4} + \frac{12\sqrt{2}}{4}\pi$$

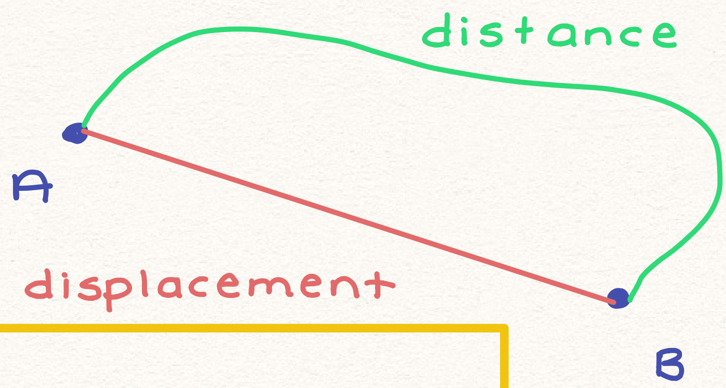
$$= \boxed{3\pi + 3\sqrt{2}\pi} \quad E$$

A) $\frac{3\pi}{2}\left(\sqrt{2} + 2\right)$

$$= \frac{3\pi\sqrt{2}}{2} + \frac{3\pi(\cancel{2})}{\cancel{2}}$$

$$= \frac{3\pi\sqrt{2}}{2} + 3\pi$$

Fall 2019 Final B #19



$$\boxed{\int_0^5 s'(t)\,dt}$$

D

Practice Final Exam #12

$$f(x) = \frac{g(x)}{\sin(x)} \qquad f'\left(\frac{\pi}{3}\right) = ?$$

$$g\left(\frac{\pi}{3}\right) = 1 \qquad g'\left(\frac{\pi}{3}\right) = 0$$

$$f'(x) = \frac{g'(x)\sin(x) - g(x)\cos(x)}{[\sin(x)]^2}$$

$$f'\left(\frac{\pi}{3}\right) = \frac{\cancelto{0}{g'\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{3}\right)} - g\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}\right)}{[\sin(\pi/3)]^2}$$

$$= \frac{-(1)\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{-\frac{1}{2}}{\frac{3}{4}}$$

B

$$= -\frac{1}{2} \times \frac{4}{3} = \frac{-4}{6} = \boxed{\frac{-2}{3}}$$

Question from chat

1. $\lim_{x \to 0} x^{10} \sin\left(\frac{4}{x}\right)$

$$-1 \leq \sin\left(\frac{4}{x}\right) \leq 1$$

$$-x^{10} \leq x^{10} \sin\left(\frac{4}{x}\right) \leq x^{10}$$

$$\lim_{x \to 0} -x^{10} \leq \lim_{x \to 0} x^{10} \sin\left(\frac{4}{x}\right) \leq \lim_{x \to 0} x^{10}$$

$$\boxed{0 \leq \lim_{x \to 0} x^{10} \sin\left(\frac{4}{x}\right) \leq 0}$$

2. $f(x) = \ln(x+2)$ $\quad f'(x) = \dfrac{1}{x+2}$

$a = -1$

$\sim \ln(2) = \ln(x+2)$

$x = 0$

$L(x) = f(a) + f'(a)(x-a)$

$= \ln(-1+2) + \left(\dfrac{1}{-1+2}\right)(0-(-1))$

$= \cancel{\ln(1)}^{\,0} + (1)(1) = \boxed{1}$

$L(x) = \cancel{\ln(1)}^{\,0} + (1)(x+1)$

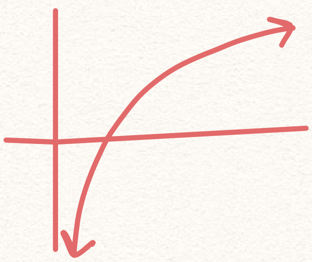
$$\boxed{L(x) = x+1}$$

3. $\lim\limits_{x \to 0^+} x^{\sqrt{x}} = 0^0$

indeterminate form

$e^{\ln(x)} = x$

$\lim\limits_{x \to 0^+} e^{\ln\left(x^{\sqrt{x}}\right)}$



$\sqrt{x}\,\ln(x) = 0(-\infty)$

$$= \frac{\ln(x)}{\frac{1}{\sqrt{x}}} = \frac{-\infty}{\infty}$$

$\frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$

$$= \frac{\frac{1}{x}}{-\left(\frac{1}{2}\right)x^{-\frac{3}{2}}}$$

$$= \frac{\frac{1}{x}}{-\frac{1}{2x^{\frac{3}{2}}}} = \frac{1}{x} \cdot \frac{-2x^{\frac{3}{2}}}{1} = -2x^{\frac{1}{2}}$$

$$\lim_{x \to 0^+} e^{-2x^{\frac{1}{2}}} = e^0 = \boxed{1}$$

$$\lim_{n \to \infty} \sum_{i=\Delta x}^{n} f(x_i)\,\Delta x$$

n : rectangles

$$\Delta x = \frac{x_2 - x_1}{n}$$

Questions from chat

#1

What is the slope of the line tangent to the curve $x^2y^2 - 3x = 6 - 2y$ at $(2,-2)$

① Find $\frac{dy}{dx}$ of $x^2y^2 - 3x = 6 - 2y$

$\frac{d}{dx}(x^2y^2)$ need prod rule $f'g + fg'$

$f = x^2$ $\quad g = y^2$

$f' = 2x$ $\quad g' = 2y\frac{dy}{dx}$

$$\overbrace{2xy^2 + x^2 2y\frac{dy}{dx}} - 3 = -2\frac{dy}{dx}$$

② Plug in $(2,-2)$ and solve for $\frac{dy}{dx}$

$$2(2)(-2)^2 + (2)^2 2(-2)\frac{dy}{dx} - 3 = -2\frac{dy}{dx}$$

$$16 - 16\frac{dy}{dx} - 3 = -2\frac{dy}{dx}$$

$$13 - 16\frac{dy}{dx} = -2\frac{dy}{dx}$$

$$13 = 14\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{13}{14}$$

#2 $h(x) = \int_{2x}^{x^2} \cos(t)\sin(t)\,dt$

$$h'(x) = \frac{d}{dx}\int_{2x}^{x^2} \cos(t)\sin(t)\,dt$$

$$h'(x) = \cos(t)\sin(t)\Big|_{t=2x}^{t=x^2}$$

$$h'(x) = \cos(x^2)\sin(x^2)\cdot\frac{d}{dx}(x^2) - \cos(2x)\sin(2x)\cdot\frac{d}{dx}(2x)$$

$$h'(x) = \cos(x^2)\sin(x^2)\cdot 2x - \cos(2x)\sin(2x)\cdot 2$$

$$h'(x) = 2x\cos(x^2)\sin(x^2) - 2\cos(2x)\sin(2x)$$

* Extra work for the long way to do it attached on next pages *
That might help you see how we got there

12) $n(x) = \int_{2x}^{x^2} \cos(t)\sin(t)\,dt$

find $\frac{d}{dx}\left(\int_{2x}^{x^2} \cos(t)\sin(t)\,dt\right)$

$$\frac{d}{dx}\left(\int_{2x}^{0} \cos(t)\sin(t)\,dt + \int_{0}^{x^2} \cos(t)\sin(t)\,dt\right)$$

Antiderivative:

$\int \cos(t)\sin(t)\,dt$

$u = \sin(t)$

$du = \cos(t)\,dt$

$\int u\,du$

$\frac{u^2}{2} + C$

$\frac{1}{2}\sin^2(t) = \frac{1}{2}(\sin(t))^2$

deriv. $\frac{1}{\cancel{2}}\cancel{2}(\sin(t))(\cos(t))$

$\frac{\sin^2(t)}{2} + C$

$$\frac{d}{dx}\left(\frac{\sin^2(t)}{2}\Big|_{2x}^{0} + \frac{\sin^2(t)}{2}\Big|_{0}^{x^2}\right)$$

$$\frac{d}{dx}\left(\cancel{\frac{\sin^2(0)}{2}} - \frac{\sin^2(2x)}{2} + \frac{\sin^2(x^2)}{2} - \cancel{\frac{\sin^2(0)}{2}}\right)$$

$$\frac{d}{dx}\left(\frac{-\sin^2(2x)}{2} + \frac{\sin^2(x^2)}{2}\right)$$

$$\frac{d}{dx}\left(\frac{-1}{2}(\sin(2x))^2 + \frac{1}{2}(\sin(x^2))^2\right)$$

$$-\cancel{\frac{1}{2}}\cdot\cancel{2}\sin(2x)\cdot\frac{d}{dx}(\sin(2x)) + \cancel{\frac{1}{2}}\cdot\cancel{2}(\sin(x^2))\cdot\frac{d}{dx}(\sin(x^2))$$

$$-\sin(2x)\cdot\cos(2x)\cdot 2 + \sin(x^2)\cos(x^2)\cdot 2x$$

$$-2\sin(2x)\cos(2x) + 2x\sin(x^2)\cos(x^2)$$

More Questions from Chat

#3

$$\int \frac{15x^2}{(x^3+11)^4}\,dx$$

Pick $u = x^3+11$ since it's a function inside function $(\ )^4$

$u = x^3+11$

$\frac{du}{3} = \frac{3x^2\,dx}{3}$

$\frac{1}{3}du = \frac{du}{3} = x^2dx$

Plug in! $\int \frac{15}{u^4}\frac{1}{3}du$

Simplify $\int \frac{5}{u^4}du = \int 5u^{-4}du$

Integrate $\frac{5u^{-3}}{-3} + C$

Plug back in u $\frac{5}{-3}(x^3+11)^{-3} + C$

$$= \frac{5}{-3(x^3+11)^3} + C$$

Good luck on your exam!! :)

-Hannah B. & Sara K.