Calculus I: MAC2311 **Name:** ______________________
Final Exam A

Instructions: multiple choice questions.

1. Approximate the area under the graph $f(x) = 1 + \sin^2(\pi x)$ on the interval $[0, 1/2]$ using a right-endpoint Riemann sum with 3 subintervals of equal length.

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{5}{6}$ (D) $\frac{9+\sqrt{3}}{12}$ (E) 1

2. Given $f'(x) = e^x + \cos(x)$ and $f(0) = 0$, find $f(x)$.

(A) $f(x) = e^x + \sin(x) - 1$ (B) $f(x) = e^x + \sin(x)$

(C) $f(x) = e^x - \sin(x) - 1$ (D) $f(x) = \ln(\sin(x))$ (E) none of the above

3. Consider the function $f(x) = \sqrt{3-x}$. Which of the following is the definition of $f'(-1)$?

(A) $f'(-1) = \lim\limits_{x \to -1} \dfrac{\sqrt{3-x} - 2}{x+1}$

(B) $f'(-1) = \lim\limits_{x \to \infty} \dfrac{\sqrt{3-x} - 2}{x+1}$

(C) $f'(-1) = \lim\limits_{h \to 0} \dfrac{\sqrt{3-x-h} - \sqrt{3-x}}{h}$

(D) $f'(-1) = \lim\limits_{h \to 0} \dfrac{h}{\sqrt{3-x-h} - \sqrt{3-x}}$

4. Water flows from the bottom of a storage tank at a rate $r(t) = 200 - 4t$ liters per minute, where $0 \le t \le 50$. If at time $t = 0$ there are 5,000 gallons of water in the tank, how many of the following are true?

(i) Between the times $t = 5$ and $t = 10$, 850 gallons leave the tank

(ii) At $t = 5$, there are only 850 gallons still in the tank.

(iii) At $t = 10$, 1800 gallons have left the tank.

(iv) At $t = 50$, the tank is empty.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

5. Evaluate $\displaystyle\int_1^4 \left(\frac{3}{x} + e \cdot e^{-x} - x^{-3/2}\right) dx$

(A) $\ln(64) - \dfrac{1}{e^3}$ (B) $-\dfrac{19}{8} + \dfrac{1}{e^3}$ (C) $\dfrac{151}{64} - \dfrac{1}{e^3}$ (D) $\ln(81) + 3e - \dfrac{1}{e^4}$ (E) $3\ln(4) + \dfrac{1}{e^3} - \dfrac{4}{3}$

6. Find $\int \sin(\cos(x))\sin(x)\ dx$.

$(A)\ -\cos(\sin(x))+C$ $(B)\ \cos(\sin(x))+C$ $(C)\ \cos(\cos(x))+C$ $(D)\ -\cos(\cos(x))+C$ (E) none of the abov

7. Evaluate $\lim_{x\to 0} x^4 \sin\left(\frac{1}{x}\right)$

$(A)\ \infty$ $(B)\ 0$ $(C)\ 1$ (D) The limit does not exist

8. If $\int_0^1 f(x)\ dx = 5$ and $\int_0^1 g(x)\ dx = -3$, which of the following must be true?

(i) $\int_0^1 f(x)g(x)\ dx = -15$

(ii) $\int_0^1 [f(x)+g(x)]dx = 2$

(iii) $f(x) \geq g(x)$ for all x in $[0,1]$

(A) (i) and (ii) (B) (ii) (C) (iii) (D) (ii) and (iii) (E) (i), (ii), and (iii)

9. A ladder 13 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 2 ft/s, how fast is the angle that the ladder makes with the ground changing when the top of the ladder is 5 ft from the ground?

$(A)\ -\frac{1}{5}$ $(B)\ -\frac{10}{169}$ $(C)\ -\frac{1}{6}$ $(D)\ -\frac{2}{5}$

10. You are asked to find the minimal cost of materials to construct a rectangular storage container with an open top having a volume of 10 m^3. The length of the base is twice the width. Material for the base cost 15 per square meter, and material for the sides costs 5 per square meter. Let C represent the cost, w the width of the box, l the length of the box, and h the height of the box. Which of the following equations should you optimize to find the minimal cost of materials to construct the box?

$(A)\ C = \frac{150}{w} + 30w^2$ $(B)\ C = \frac{150}{w} + 60w^2$ $(C)\ C = \frac{30}{w} + 2w^2$ $(D)\ C = \frac{30}{w} + w^2$ (E) none of the above

11. Which of the following is the Riemann sum for the function $f(x) = \ln(x)$ over the interval $[2, 4]$ using left-endpoints and 4 subintervals of equal length?

(A) $\sum_{i=1}^{4} \ln\left(2 + \frac{1}{2}i\right)\frac{1}{2}$

(B) $\sum_{i=0}^{3} \ln\left(2 + \frac{1}{2}i\right)\frac{1}{2}$

(C) $\sum_{i=1}^{4} \ln\left(\frac{1}{2}i\right)\frac{1}{2}$

(D) $\sum_{i=0}^{3} \ln\left(\frac{1}{2}i\right)\frac{1}{2}$

12. Find the values of m and k such that $f(x)$ is continuous everywhere.

$$f(x) = \begin{cases} 2x^3 + x + 7, & x < -1 \\ m(x+1) + k, & -1 \leq x \leq 2 \\ x^2 + 2, & 2 < x \end{cases}$$

(A) $m = \frac{2}{3}$ and $k = 4$ (B) $m = \frac{1}{2}$ and $k = 3$ (C) $m = \frac{2}{3}$ and $k = -4$ (D) $m = -\frac{2}{3}$ and $k = -4$

13. Let $f(x) = x^2 g(\cos(2x))$. Find $f'(x)$.

(A) $2xg(\cos(2x)) - 2x^2 \sin(2x)g'(\cos(2x))$ (B) $-2xg'(2\sin(2x))$

(C) $2xg(\cos(2x)) + x^2 \sin(2x)g'(\cos(2x))$ (D) $2x - 2\sin(2x)g'(\cos(2x))$ (E) none of the above

14. If $xy^2 + 2xy = 8$ then at the point $(1, 2)$, y' is

(A) $-\frac{5}{2}$ (B) $-\frac{4}{3}$ (C) -1 (D) $-\frac{1}{2}$ (E) 0

15. The sum of two positive numbers is 16. What is the smallest possible value of the sum of their squares?

(A) 128 (B) 162 (C) 130 (D) 256 (E) none of the above

16. Evaluate $\displaystyle\int_1^e \frac{\ln(x)}{x} dx$.

(A) $\frac{1}{3}$ (B) $\frac{1}{e^2} - 1$ (C) $\frac{1}{2}$ (D) 0 (E) none of the above

17. Evaluate $\lim\limits_{x\to\infty} x^4 e^{-x}$

(A) 0 (B) 24 (C) ∞ (D) The limit does not exist

18. How many of the following are necessarily true?

(i) The Net Change Theorem can be stated as follows:

$$\int_a^b F'(x)\ dx = F(b) - F(a)$$

(ii) An indefinite integral is a function

(iii) A definite integral is a number.

(iv) If $v(t)$ is the velocity function of a particle moving along a straight line, then $\int_a^b v(t)\ dt$ gives the difference in the position of the particle between times $t = a$ and $t = b$.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

19. Evaluate $F'(2)$, if $F(x) = \displaystyle\int_1^{x^2} \frac{\sin(t)}{t} dt$.

(A) $\sin(4)$ (B) $\dfrac{\sin(4)}{4}$ (C) $2\sin(2)$ (D) $\dfrac{2\cos(2) - \sin(2)}{4}$ (E) 0

20. At $x = 0$, for how many of the following is the function both increasing and concave up?

(i) $f(x) = -2\cos(x)$

(ii) $g(x) = \ln(x+1)$

(iii) $h(x) = \tan^{-1}(x)$ $\quad (h(x) = \arctan(x))$

(iv) $k(x) = e^x + e^{-x}$

(A) 0 $\quad$ (B) 1 $\quad$ (C) 2 $\quad$ (D) 3 $\quad$ (E) 4

21. If $\int_{-1}^{0} f(x)\ dx = -1$ and $\int_{0}^{1} f(x)\ dx = 2$, how many of the following must be true?

(i) $\int_{-1}^{1} f(x)\ dx = 1$

(ii) $\int_{1}^{1} f(x)\ dx = 0$

(iii) $\int_{0}^{2} f(x)\ dx = 4$

(iv) $\int_{1}^{-1} f(x)\ dx = 1$

(A) 0 $\quad$ (B) 1 $\quad$ (C) 2 $\quad$ (D) 3 $\quad$ (E) 4

22. A **magic square** is a 3×3 square of numbers from 1 to 9 in which the sum of any row, column, and diagonal is the same. The following square is missing a number. What number must be filled in in order for this square to be a magic square?

2	7	6
9	5	
4	3	8

(A) 1 $\quad$ (B) Look left $\quad$ (C) Look left again $\quad$ (D) Look further left $\quad$ (E) Try the first one again

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Final Exam B

Instructions: multiple choice questions.

1. If $\int_a^b f(x)\ dx = -7$ and $\int_a^b g(x)\ dx = 3$, then $\int_a^b [-3f(x) - 2g(x)]dx$ is equal to:

(A) -27 (B) 27 (C) -15 (D) 15 (E) None of the above

2. The acceleration of a particle moving along the horizontal axis is given by $a(t) = \sin(t) - \cos(t)$. The particles position at time $t = 0$ is 2 and time $t = \frac{\pi}{2}$ is 3, find the position at time $t = \pi$.

(A) 6 (B) 2 (C) 0 (D) $5 + \frac{3\pi}{2}$ (E) $5 + 2\pi$

3. Which of the following is the area of a rectangle with perimeter 24 and maximum area.

(A) 30 (B) 36 (C) 0 (D) 1 (E) None of the above

4. If $f(x)$ is defined and continuous for $a \leq x \leq b$, divide $[a, b]$ into n subintervals of equal lenght $\Delta x = (b - a)/n$. Let $x_0(= a), x_1, x_2, \ldots, x_n(= b)$ be the endpoints of these subintervals and let x_i^* be any sample point in the subinterval $[x_{i-1}, x_i]$. The definite integral of $f(x)$ from a to b is

$$\int_a^b f(x)\ dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

provided the limit exists.

(A) True (B) False

5. Estimate the area A under the curve $f(x) = 1 + x^2$ from $x = -1$ to $x = 2$ using three rectangles and right endpoints.

(A) 3 (B) 5 (C) 6 (D) 8 (E) None of the above

6. Letting $u = \sin(x)$, which of the following is equal to $\displaystyle\int_0^{\pi/2} (\sin(x))^4 \cos(x)\ dx$

(A) $\int_0^1 u^4\ du$ (B) $\int_1^0 u^4\ du$ (C) $4\int_0^1 u^4\ du$ (D) $\int_0^{-1} u^4\ du$ (E) None of the above

7. Find the point on the line $y = 1 - x$ that is closest to the origin $(0, 0)$.

(A) $(1, 0)$ (B) $(\frac{1}{2}, \frac{1}{2})$ (C) $(0, 0)$ (D) $(\frac{1}{2}, -\frac{1}{2})$ (E) None of the above

8. Evaluate $\displaystyle\int_0^1 \frac{3x^2}{1 + x^3} dx$

(A) 1 (B) $\ln(2)$ (C) $\ln(3) - \ln(2)$ (D) 0 (E) None of the above