Calculus I: MAC2311 **Name:** _____
Fall 2019
Midterm 3 A **Section:** _____
11/14/2019
Time Limit: 1 Hour 30 Minutes **UF-ID:** _____

It is your responsibility to ensure that your test has **17 questions**. If it does not, show it to your proctor immediately. You will not be permitted to make up any problems omitted from your test after the testing period ends. There are a total of 105 points available on this exam.

Part I Instructions: 14 multiple choice questions. Complete the scantron sheet provided with your information and fill in the appropriate spaces to answer your questions. Only the answer on the scantron sheet will be graded. Each problem is worth five (5) points for a total of 70 points on Part I.

1. If $f(x) = \sin(-\pi x^2)$, which of the following is true at $x = 1$?

(A) The graph of $f(x)$ is increasing and concave upward.

(B) The graph of $f(x)$ is decreasing and concave upward.

(C) The graph of $f(x)$ is increasing and concave downward.

(D) The graph of $f(x)$ is decreasing and concave downward.

(E) None of the above

2. Consider the equation $\ln(x + y) = xy$ where both x and y are functions of t. What is the value of $\frac{dx}{dt}$ when $\frac{dy}{dt} = 1$, $y = 0$, and $x = 1$.

(A) -1 (B) 1 (C) 0 (D) Does not exist (E) None of these

3. If $f(x)$ has a local minimum at $x = c$, is it necessarily true that $f'(c) = 0$?

(A) Yes (B) No

4. Which of the following functions satisfies the hypotheses of Rolle's Theorem on the interval $[-3, 1]$?

(A) $f(x) = x^2 + 2x + 7$ (B) $f(x) = |x + 1|$ (C) $f(x) = x^3 + 3x - 9$ (D) $f(x) = 9x - 2$

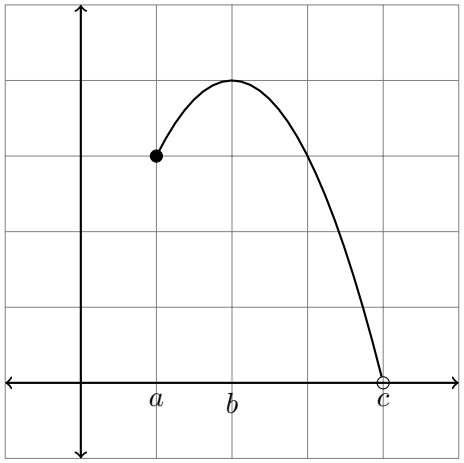
5. Using the linearization of $\sqrt[3]{x}$ at $a = 27$, which of the following is an approximation of $\sqrt[3]{26.8}$? Which of the following functions satisfies all four properties?

(A) $\dfrac{-0.2}{27} + 3$ (B) $\dfrac{0.2}{27} + 3$ (C) $\dfrac{\sqrt[3]{26.8} - 27}{27} + 3$ (D) $-\dfrac{\sqrt[3]{26.8} - 27}{27} + 3$ (E) None of these

6. Use dy to approximate Δy if $y = \sin(x) + x^2$ and x changes from 0 to 0.2.

(A) 0.2 (B) $0.2(\cos(0.2) + 0.4)$ (C) $-0.2(\cos(0.2) + 0.4)$ (D) -0.2 (E) 0

7. The figure below is a graph of the function $y = f(x)$. Use the figure to identify the values (if any) in the interval $[a, c]$ where the function $f(x)$ has an absolute maximum or an absolute minimum.



(A) absolute maximum at $x = b$ and absolute minimum at $x = c$

(B) absolute maximum at $x = a$ and absolute minimum at $x = c$

(C) absolute maximum at $x = b$ and no absolute minimum

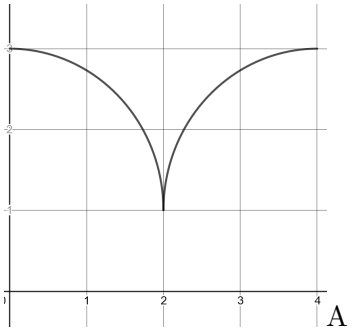
(D) no absolute maximum and absolute minimum at $x = c$

8. Find the value of c that satisfies the conclusion of the Mean Value Theorem for the function $f(x) = x + \dfrac{24}{x}$ on the interval $[3, 4]$.

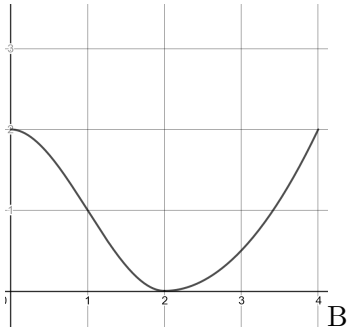
$(A)\ -1$ $(B)\ 2\sqrt{3}$ $(C)\ \dfrac{7}{2}$ $(D)\ 0$

9. The graphs of four functions on the interval $[0, 4]$ are given below. Which of them has all of the following properties:

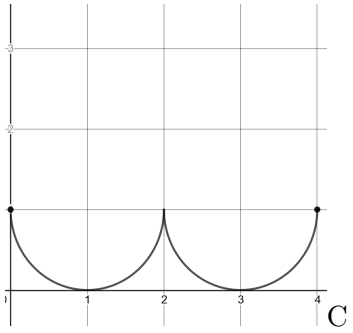
- $f'(x) < 0$ on $(0, 2)$
- $f''(x) > 0$ on $(1, 3)$
- $f(x)$ has no points of inflection.



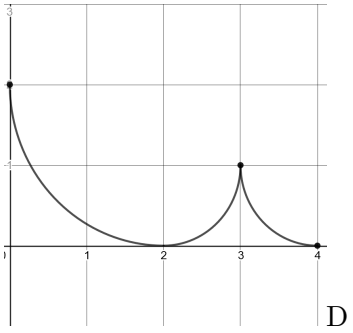
A



B



C



D

10. Compute the limit

$$\lim_{x\to 0}\left(\frac{e^x-1}{x^2}-\frac{1}{x}\right)$$

(A) 0 (B) ∞ (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$ (E) Does not exist

11. The volume of a circular cylinder of radius r and height h is $V=\pi r^2 h$. If the cylinder expands as time passes, which of the following gives $\frac{dV}{dt}$ in terms of $\frac{dr}{dt}$ and $\frac{dh}{dt}$?

(A) $\frac{dV}{dt}=\pi\left(\frac{dr}{dt}\right)^2\frac{dh}{dt}$

(B) $\frac{dV}{dt}=2\pi r\frac{dr}{dt}h+\pi r^2\frac{dh}{dt}$

(C) $\frac{dV}{dt}=2\pi rh+\pi r^2$

(D) $\frac{dV}{dt}=2\pi rh+\pi r^2$

12. If $f(x)=(\ln(x))^2$ at how many of the following x values is the function decreasing and concave upward?

(*i*) $x=0$

(*ii*) $x=e$

(*iii*) $x=e^2$

(*iv*) $x=e^3$

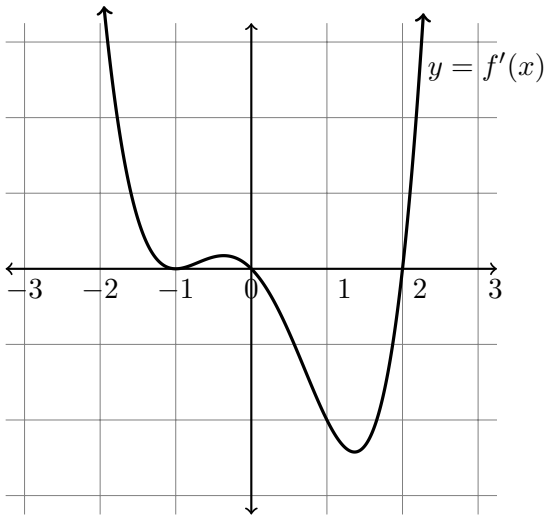
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

13. Compute the limit

$$\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^{6x}$$

(A) e^{18} (B) 1 (C) ∞ (D) $e^{\frac{1}{2}}$ (E) Does not exist

Use the following graph to answer question 14.



14. The graph of $f'(x)$, (the **derivative** of $f(x)$) is given above. How many of the statements below are true?

(*i*) $f(x)$ is increasing on the intervals $(-\infty,-1)\cup(-1,0)$.

(*ii*) $f(x)$ has a point of inflection somewhere in the interval $(1,2)$.

(*iii*) $f(x)$ is concave down from $(-\infty,-1)$.

(*iv*) $f(x)$ does not have a local extremum at $x=-1$.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

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Part II Instructions: 3 free response questions. Neatly give a complete solution to each problem and show all work and intermediate steps. We are grading the work and notation as well as the answer. A total of 35 points is possible on Part II. **No credit will given without proper work.** If we cannot read it and follow it, you will receive no credit for the problem.

For Instructor Use Only:

FR 1	
FR 2	
FR 3	
Total Points	

1. (7 pts) Find the absolute maximum and absolute minimum of the function $f(x) = x\sqrt{4-x^2}$ on the interval $[-2, 1]$. **Write your answers as coordinate pairs** (x, y).

2. (7 pts) A spotlight on the ground shines on a wall 12 m away. If a man 2 m tall walks from the spotlight toward the building at a speed of 4 m/s, how fast is the height of his shadow on the building changing when he is 4 m from the building?

3. Consider the function $y = f(x)$, where

$$f(x) = \frac{x^2 - 3}{(x+1)^2} \qquad f'(x) = \frac{2(x+3)}{(x+1)^3} \qquad f''(x) = \frac{-4(x+4)}{(x+1)^3}$$

(*a*) (1 pts) What is the domain of $f(x)$? Write your answer in interval notation.

(*b*) (2 pts) What are the vertical and horizontal asymptotes of $f(x)$? Write your answer as a line (either $x = c$ or $y = d$).

(*c*) (3 pts) List the critical point(s) of $f(x)$ (write them as coordinate pair(s) (x, y)).

(*d*) (3 pts) On what intervals is $f(x)$ increasing? decreasing?

(*e*) (2 pts) At which coordinates does the graph of $f(x)$ have local maximum(s) or local minimum(s)? Write your answer as coordinate pair(s) (x, y).

(f) (3 pts) On what intervals is $f(x)$ concave up? concave down?

(g) (2 pts) List the point(s) of inflection (write them as coordinate pair(s) (x, y)).

(h) (5 pts) Sketch the graph of $f(x)$ on the following graph. Label all horizontal and vertical asymptotes, all local maximum and minimum, and all inflection points.