**Calculus I: MAC2311**
**Fall 2019**
**Midterm 3 A**
**11/14/2019**
**Time Limit: 1 Hour 30 Minutes**

**Name:** ____________________

**Section:** ____________________

**UF-ID:** ____________________

Part II Instructions: 3 free response questions. Neatly give a complete solution to each problem and show all work and intermediate steps. We are grading the work and notation as well as the answer. A total of 35 points is possible on Part II. **No credit will given without proper work.** If we cannot read it and follow it, you will receive no credit for the problem.

**For Instructor Use Only:**

| | |
|---|---|
| FR 1 | |
| FR 2 | |
| FR 3 | |
| Total Points | |

1. (7 pts) Find the absolute maximum and absolute minimum of the function $f(x) = x\sqrt{4-x^2}$ on the interval $[-2, 1]$. **Write your answers as coordinate pairs** $(x, y)$.
**Solution:**

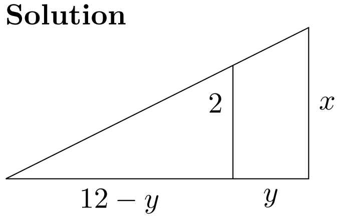
$$\begin{aligned} f'(x) &= \sqrt{4-x^2} + x\left(\frac{1}{2}\right)(4-x^2)^{-\frac{1}{2}}(-2x) \\ &= \sqrt{4-x^2} - \frac{x^2}{\sqrt{4-x^2}} \\ &= \frac{4-x^2-x^2}{2\sqrt{4-x^2}} = \frac{4-2x^2}{2\sqrt{4-x^2}} \end{aligned}$$

| | | |
|---|---|---|
| $f'(x) = 0$ when | $f'(x)$ DNE when | $+\sqrt{2}$ and 2 are not in the domain |
| $4 - 2x^2 = 0$ | $2\sqrt{4-x^2} = 0$ | $f(-2) = (-2)\sqrt{4-4} = 0$ |
| $2x^2 = 4$ | $4 - x^2 = 0$ | $f(-\sqrt{2}) = -\sqrt{2}(\sqrt{4-2}) = -\sqrt{2}\sqrt{2} = -2$ |
| $x^2 = 2$ | $x^2 = 4$ | $f(1) = 1\sqrt{4-1} = \sqrt{3}$ |
| $x = \pm\sqrt{2}$ | $x = \pm 2$ | |

Thus we get absolute min: $(-\sqrt{2}, -2)$ and absolute max: $(1, \sqrt{3})$

2. (7 pts) A spotlight on the ground shines on a wall 12 m away. If a man 2 m tall walks from the spotlight toward the building at a speed of 4 m/s, how fast is the height of his shadow on the building changing when he is 4 m from the building?

**Solution**




We are given $\frac{dy}{dt} = -4\text{m/sec}$ and we want to solve for $\frac{dx}{dt}$ when $y = 4\text{m}$.
Using similar triangles we get $\frac{2}{12-y} = \frac{x}{12}$.
When $y = 4\text{m}$ we get $\frac{x}{12} = \frac{2}{8} = \frac{1}{4}$ so $x = 3\text{m}$.

$$\begin{aligned} \frac{2}{12-y} &= \frac{x}{12} \\ 24 &= 12x - xy \end{aligned}$$

now differentiating both sides gives

$$\begin{aligned} 0 &= 12\frac{dx}{dy} - \left(\frac{dx}{dt}y + x\frac{dy}{dt}\right) \\ 0 &= 12\frac{dx}{dt} - 4\frac{dx}{dt} + 12 \\ -8\frac{dx}{dt} &= 12 \\ \frac{dx}{dt} &= -\frac{12}{8} = -\frac{3}{2}\text{m/sec} \end{aligned}$$

3. Consider the function $y = f(x)$, where

$$f(x) = \frac{x^2 - 3}{(x+1)^2} \qquad f'(x) = \frac{2(x+3)}{(x+1)^3} \qquad f''(x) = \frac{-4(x+4)}{(x+1)^3}$$

(*a*) (1 pts) What is the domain of $f(x)$? Write your answer in interval notation.
**Solution:** $(-\infty, -1) \cup (-1, \infty)$

(*b*) (2 pts) What are the vertical and horizontal asymptotes of $f(x)$? Write your answer as a line (either $x = c$ or $y = d$).

**Solution:**
$\lim_{x \to -1^+} \frac{x^2}{(x+1)^2} = -\infty$ and $\lim_{x \to -1^-} \frac{x^2}{(x+1)^2} = -\infty$ so we have the vertical asymptote $x = -1$

$\lim_{x \to \infty} \frac{x^2}{(x+1)^2} = 1$ and $\lim_{x \to -\infty} \frac{x^2}{(x+1)^2} = 1$ se we have one horizontal asymptote $y = 1$.

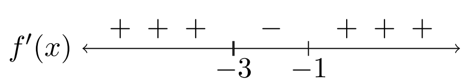
(*c*) (3 pts) List the critical point(s) of $f(x)$ (write them as coordinate pair(s) $(x, y)$).

**Solution:**
$f'(x) = 0$ when $2(x+3) = 0$ so when $x = -3$
$f'(x)$ DNE when $(x+1)^3 = 0$ so when $x = -1$ but that is not in the domain
$f(-3) = \frac{9-3}{2^2} = \frac{6}{4} = \frac{3}{2}$
critical point:$(-3, \frac{3}{2})$

(*d*) (3 pts) On what intervals is $f(x)$ increasing? decreasing?

**Solution:**

Consider the sign chart



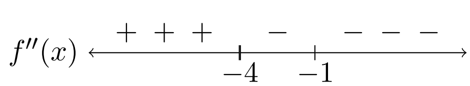

$f(x)$ is increasing on $(-\infty, -3) \cup (-1, \infty)$ and decreasing on $(-3, -1)$

(*e*) (2 pts) At which coordinates does the graph of $f(x)$ have local maximum(s) or local minimum(s)? Write your answer as coordinate pair(s) $(x, y)$.

**Solution:**
local max at $(-3, \frac{3}{2})$ ($-1$ is not in the domain so no extreme value there)

(*f*) (3 pts) On what intervals is $f(x)$ concave up? concave down?

**Solution:**
$f''(x) = 0$ when $-4(x+4) = 0$ so when $x = -4$
$f''(x)$ DNE when $(x+1)^4 = 0$ so when $x = -1$
Consider the sign chart




$f(x)$ is concave up on $(-\infty, -4)$ and concave down on $(-4, -1) \cup (-1, \infty)$

(g) (2 pts) List the point(s) of inflection (write them as coordinate pair(s) $(x, y)$).

**Solution:**
$f(-4) = \frac{16-3}{3^2} = \frac{13}{9}$
$-1$ is not in the domain so $(-4, \frac{13}{9})$ is the only inflection point.

(h) (5 pts) Sketch the graph of $f(x)$ on the following graph. Label all horizontal and vertical asymptotes, all local maximum and minimum, and all inflection points.