MAC 2311 Exam 3 Review

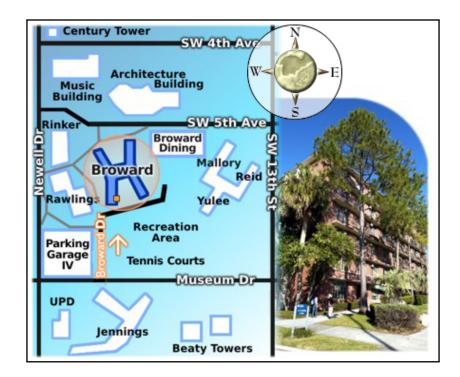


Spring 2018

This review, produced by the Broward Teaching Center, contains a collection of questions which are representative of the type you may encounter on the exam. Other resources made available by the Teaching Center include:

- Walk-In tutoring at Broward Hall
- Private-Appointment, one-on-one tutoring at Broward Hall
- Walk-In tutoring in LIT 215
- Supplemental Instruction
- Video resources for Math and Science classes at UF
- Written exam reviews and copies of previous exams

The teaching center is located in the basement of Broward Hall:



You can learn more about the services offered by the teaching center by visiting https://teachingcenter.ufl.edu/

- 1. Find the average rate of change in the area of a circle as its radius changes from
  - (a) 2 to 3 cm.
  - (b) 2 to 2.1 cm.
  - (c) 2.5 to 2 cm.
- 2. Two resistors with resistance  $R_1$  and  $R_2$  (measured in Ohms, denoted  $\Omega$ ) are wired in parallel. The net resistance, R, of the two obeys the rule

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Suppose  $R_1$  and  $R_2$  are increasing at rates  $0.3\Omega/s$  and  $0.2\Omega/s$  respectively. How fast is R changing at the moment when  $R_1 = 80\Omega$  and  $R_2 = 100\Omega$ ?

- 3. A kite flying 100 feet above the ground moves horizontally at a speed of 8 feet per second. At what rate is the angle between the string and the horizontal changing when 200 feet of string has been let out?
- 4. Use linear approximation to estimate the following numbers. Hint: interpret the numbers as values of a differentiable function.
  - (a)  $\sqrt{143.9}$
  - (b)  $(2018.1)^{-1}$
  - (c)  $(2018.1)^5$
- 5. Find the linearization for each of the functions at the prescribed place.

(a) 
$$f(x) = \cos(x)$$
 at  $x = \pi/4$ .  
(b)  $g(x) = \ln(x - 1)$  at  $x = e$ .  
(c)  $h(x) = \frac{1}{x^2}$  at  $x = 2$ .

- 6. Create a list of critical numbers for each function below.
  - (a)  $f(x) = \sqrt[3]{x^2}(2x-1)$ (b)  $g(t) = 6t - 4\cos(3t)$
  - (c)  $h(x) = 10xe^{3-x^2}$
- 7. Locate and describe any local extrema for the functions from the previous problem.

8. Determine if the functions satisfy the hypotheses of the Mean Value Theorem on the intervals specified. For those that do, list the numbers guaranteed to exist by the theorem; for those that do not, explain why.

(a) 
$$f(x) = x^3 + 2x^2 - x$$
 on  $[-1, 2]$ 

(b) 
$$g(t) = 8t + e^{-3t}$$
 on  $[-2,3]$ 

(c) 
$$h(x) = \begin{cases} x^2 & x \le 2\\ 4x - 4 & x > 2 \end{cases}$$
 on  $[0, 4]$   
(d)  $X(s) = 1 - \frac{1}{s}$  on  $[-1, 2]$ 

- 9. Suppose f(x) is continuous on [6, 15], differentiable on (6, 15), f(6) = -2, and  $f'(x) \le 10$  for all x. What is the largest possible value of f(15)? Hint: use MVT
- 10. Determine the intervals on which each of the following functions is increasing/decreasing.
  - (a)  $f(x) = x^{9/5} x$ (b)  $h(x) = 2\sin^2(x) - 2x$  restricted to  $[0, \pi]$
- 11. Find real numbers A and B such that the function  $f(x) = Ax^3 + Bx^2 + 1$  has an inflection point at (-1, 2).
- 12. Calculate the limits below, employing L'Hôpital's rule as needed.

(a) 
$$\lim_{x \to \infty} \frac{e^{2018x} + 1}{e^{1729x}}.$$
(b) 
$$\lim_{x \to 0} \frac{3^x - 1}{x}$$
(c) 
$$\lim_{x \to 0} \frac{2\sin(x) - \sin(2x)}{x - \sin(x)}$$
(d) 
$$\lim_{t \to \infty} t^2 e^{-t}$$
(e) 
$$\lim_{x \to 0^+} x^x$$
(f) 
$$\lim_{x \to \infty} \sqrt{x^2 + 1} - \sqrt{x + 1}$$

13. Evaluate the limits below. If the limit does not exist, indicate why.

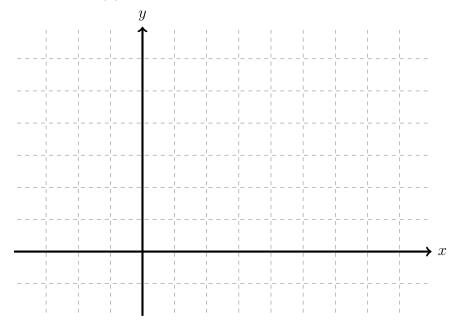
(a) 
$$\lim_{x \to 2} \frac{x^2 + x - 6}{x - 2}$$
  
(b)  $\lim_{x \to 0} \left( \frac{1}{x} - \frac{1}{x^2 + x} \right)$   
(c)  $\lim_{x \to -4} \frac{\sqrt{x^2 + 9} - 5}{x + 4}$ 

14. Evaluate the limits below. If the limit does not exist, indicate why.

(a) 
$$\lim_{x \to \infty} \frac{7x^3 + x^2 - 5}{x^3 - 2x + 1}$$
  
(b)  $\lim_{x \to \infty} \frac{4x^5}{3 - 2x^5}$   
(c)  $\lim_{x \to \infty} \frac{x - 3}{x^2 + 2x + 1}$   
(d)  $\lim_{x \to \infty} \frac{2x + 1}{3x - 5}$   
(e)  $\lim_{x \to \infty} e^{\tan^{-1}(x)}$ 

15. Let  $f(x) = x(6-x)^{2/3}$ .

- (a) Find any zeros for f(x), and determine where f is positive/negative.
- (b) Determine the function's end behavior.
- (c) Determine where f is increasing/decreasing, and find any local extrema.
- (d) Determine where f is concave up/concave down, and find any inflection points.
- (e) Sketch a graph of y = f(x), labeling all of the features discussed above.



16. Determine **all** extrema, local and absolute.

- (a)  $f(x) = 2x^3 + 3x^2 12x + 4$  on the interval [-4, 2].
- (b)  $f(x) = 2x^3 + 3x^2 12x + 4$  on the interval [0, 2].
- (c)  $g(t) = 2000 10te^{5-t^2/8}$  on the interval [0, 10].