Exam 3A:

	Test A	Test B
1	B	C
3	A	B
4	C	A
5	C	A
6	A	C
8	B	C
9	A	E
10	E	D
11	E	E
12	C	B

Exam 3B:

	Test A	Test B
3	D	B
7	D	B
9	B	B

Calculus I: MAC2311 **Name:** ____________________
Midterm 3 A

Part II Instructions: free response questions

For Instructor Use Only:

FR 1	
FR 2	
FR 3	
FR 4	
FR 5	
Total Points	

1pt for finding f' ↓

3. (7 pts) Find the location and value of the absolute maximum and minimum of the function $f(x) = 2x^3 - 15x^2 + 24x$ on $[0, 5]$. (Write your answer as a coordinate pair (x, y)).

$f'(x) = 6x^2 - 30x + 24$

$0 = 6(x^2 - 5x + 4)$

$0 = 6(x-4)(x-1)$

$x = 1, 4$

↗ 1pt for finding critical values

1pt each →

$f(0) = 0$

$f(1) = 2 - 15 + 24 = 11$

$f(4) = 2(4)^3 - 15(4)^2 + 24(4)$
$= -16$

$f(5) = 2(5)^3 - 15(5)^2 + 24(5)$
$= -5$

abs max: $(1, 11)$

abs min: $(4, -16)$

$\frac{1}{2}$ pt each for giving final answers as coordinate pairs

4. (7pts) Evaluate $\lim_{x\to\pi} (\pi - x)^{\tan(x)}$ (0^{∞})

$$\lim_{x\to\pi} (\pi - x)^{\tan(x)} = e^{\lim_{x\to\pi} \ln\left((\pi - x)^{\tan(x)}\right)}$$ ← 1pt for taking ln

$$= e^{\lim_{x\to\pi} \tan(x)\ln(\pi - x)}$$ ← 1pt for using log laws to rewrite $(\infty \cdot 0)$

$$= e^{\lim_{x\to\pi} \frac{\ln(\pi - x)}{\cot(x)}}$$ $\left(\frac{0}{0}\right)$ ← 1pt for rewriting as quotient

$$\overset{L'H}{=} e^{\lim_{x\to\pi} \frac{\frac{1}{\pi - x}}{-\csc^2(x)}}$$ ← 1pt for correctly applying L'Hopital's rule

$$= e^{\lim_{x\to\pi} \frac{-\sin^2(x)}{\pi - x}}$$ $\left(\frac{0}{0}\right)$ ← 1pt for rewriting

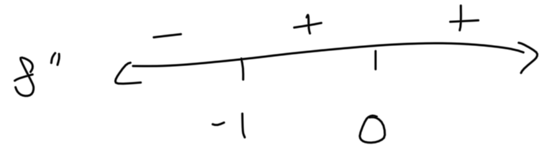
$$\overset{L'H}{=} e^{\lim_{x\to} \frac{-2\sin(x)\cos(x)}{-1}}$$ ← 1pt for applying L'Hopital's rule again

$$= e^0 = 1$$ ← 1pt for correct answer

5. (7 pts) If $f(x) = 3x^5 + 5x^4$, find all of the inflection points of the function **and** the intervals on which the graph is concave up and concave down. (Write your inflection points as coordinate pairs (x, y))

$f'(x) = 15x^4 + 20x^3$ ← 1 pt

$f''(x) = 60x^3 + 60x^2$ ← 1 pt



↑ 1 pt for checking the sign of f'' in the intervals

$0 = 60x^3 + 60x^2$

$0 = 60x^2(x+1)$

$x = 0 \quad x = -1$ ← 1 pt for solving $f''(x) = 0$

$f(-1) = -3 + 5 = 2$

1 pt each

IP: $(-1, 2)$

CD: $(-\infty, -1)$

CU: $(-1, 0) \cup (0, \infty)$

Calculus I: MAC2311
Midterm 3 B

Part II Instructions: free response questions

For Instructor Use Only:

FR 1	
FR 2	
FR 3	
FR 4	
FR 5	
Total Points	

2. A farmer plans to build a rectangular enclosure for his pigs that is adjacent to a river. The enclosure is to be 3,200 square meters. What dimensions would require the least amount of fencing if no fencing is needed along the river?

Solution

Since the river can be used as a side of the enclosure, we only need fencing for the other three sides. Let x be the length of the side parallel to the river, and y be the length of the sides perpendicular to the river. Then we have

$$xy = 3200 \Rightarrow y = \frac{3200}{x}.$$

We need to minimize the amount of fencing used for this enclosure, which is

$$F = x + 2y = x + 2\left(\frac{3200}{x}\right) = x + \frac{6400}{x}.$$

$F' = 1 - \dfrac{6400}{x^2} = 0 \Rightarrow x = \pm\sqrt{6400} = \pm 80$. These are the critical points of F, which can potentially be local extrema. We only test the positive value $x = 80$ because the side length of the enclosure cannot be negative.

$F'' = -\dfrac{6400}{x^3}(-2) = \dfrac{12800}{x^3}$. Then $F''(80) => 0$, and by the Second Derivative Test, we can conclude that $x = 80$ is a local minimum. In this case, $y = \dfrac{3200}{x} = 40$.

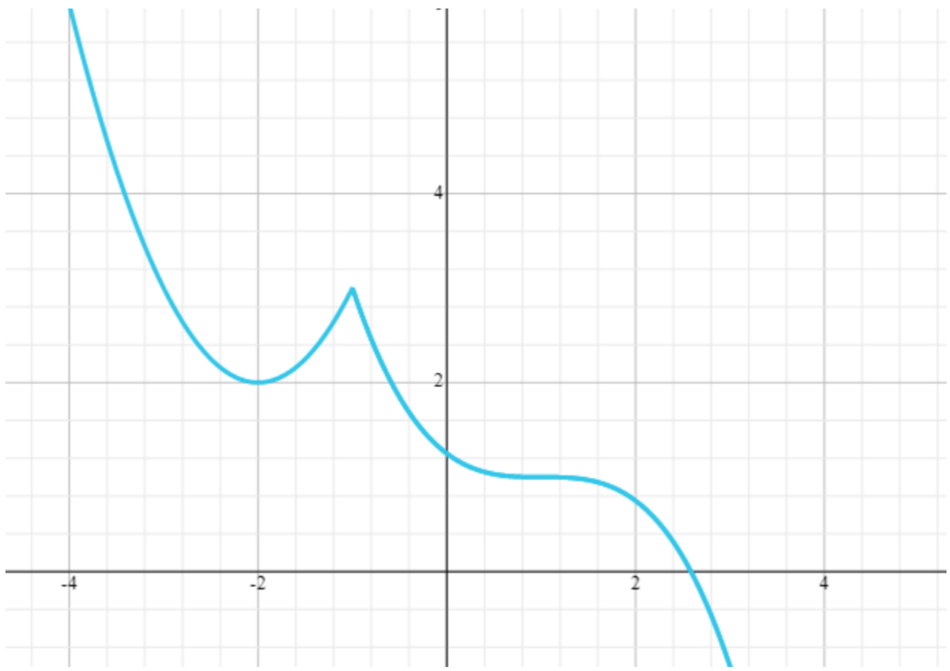
So the dimensions of $80ft \times 40ft$ would require the least amount of fencing, where 80 is the length of the side parallel to the river.

3. Sketch the graph of a function $f(x)$ that has to following properties:

- Local minimum value of $f(-2) = 2$
- Local maximum value of $f(-1) = 3$
- Point of inflection at the point $(1, 1)$
- Increasing on the intervals $(-2, -1)$
- Decreasing on the interval $(-\infty, -2)$ and $(-1, \infty)$
- Concave upward on the intervals $(-\infty, -1)$ and $(-1, 1)$
- Concave downward on the interval $(1, \infty)$

Solution

The graph may vary. Here is an example that could work.



4. Evaluate

$$\lim_{x\to 0}(1+2x)^{3/x}.$$

Solution

$$\begin{aligned}
\lim_{x\to 0}(1+2x)^{3/x} &= \lim_{x\to 0}\exp(\ln((1+2x)^{3/x})) \\
&= \lim_{x\to 0}\exp\left(\frac{3}{x}\ln((1+2x))\right) \\
&= \exp\left(\lim_{x\to 0}\frac{3\ln(1+2x)}{x}\right)
\end{aligned}$$

No that the limit is in $\dfrac{0}{0}$ form, so we use L'Hopital's rule to evaluate it.

$$\begin{aligned}
&= \exp\left(\lim_{x\to 0}\frac{3\left(\frac{2}{1+2x}\right)}{1}\right) \\
&= \exp\left(\lim_{x\to 0}\frac{6}{1+2x}\right) \\
&= \exp\left(\frac{6}{1}\right) \\
&= e^6
\end{aligned}$$

5. You are driving on an interstate highway which has a speed limit of 65 mph. At 2:00 PM you drive past a state trooper at milepost 110 while driving 63 mph. At 5:00 PM you drive past another state trooper at milepost 320 while driving 59 mph. You did not drive past any other state troopers on your trip. Two weeks later you get a speeding ticket in the mail. Explain how the state troopers could use the Mean Value Theorem to determine that you were speeding.

Solution

Use $s(t)$ to denote the position of you at a given time t, with $t = 0$ at 2:00 PM in hours, then $s(0) = 110$ and $s(3) = 320$. Then by the Mean Value Theorem, there exists $0 < c < 3$ such that

$$s'(c) = \frac{s(3) - s(0)}{3 - 0} = \frac{320 - 110}{3} = \frac{210}{3} = 70.$$

This implies that there was a time c between 2:00 PM and 5:00 PM such that you were traveling at 70 mph at time c, which was above the speed limit of 65 mph. Therefore, you were indeed speeding.