Calculus I: MAC2311 **Name:** _____
Midterm 3 A

Part I Instructions: multiple choice questions.

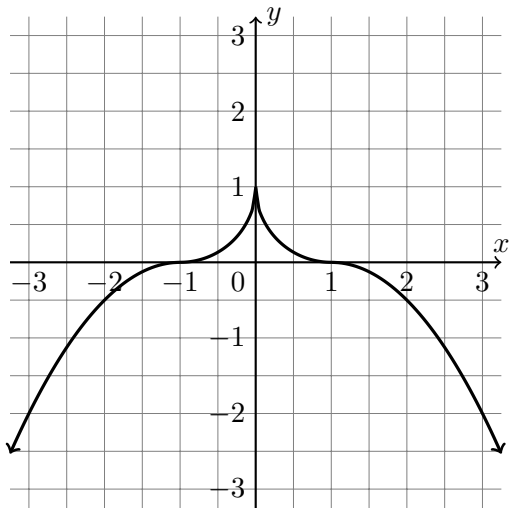
1. The linearization of $f(x) = \sqrt[3]{1+3x}$ at $a = 0$ is given by $L(x) = x + 1$. Use this to approximate $\sqrt[3]{1.03}$.

(A) 2.03 (B) 1.01 (C) 1.03 (D) 1.003 (E) None of the above

3. The elevation h (in feet above the ground) of a stone dropped from a height of 1000 ft is modeled by the equation $h(t) = 1000 - 16t^2$, where t is measured in seconds and air resistance is neglected. Use differentials to approximate the change in elevation over the interval $3 \leq t \leq 3.1$ seconds.

(*A*) $\Delta h \approx -9.6$ ft (*B*) $\Delta h \approx -4.8$ ft (*C*) $\Delta h \approx -846.4$ ft (*D*) $\Delta h \approx -118$ ft (*E*) None of the above

4. How many of the following statements are true concerning the graph of $f(x)$ given below



(*i*) $f''(x) \geq 0$ for all x-values where it exists

(*ii*) $f(x)$ has exactly three critical numbers

(*iii*) $f(x)$ has exactly one local maximum

(*iv*) $f'(x) \geq 0$ for all $x \leq 0$

(*A*) 0 (*B*) 1 (*C*) 2 (*D*) 3 (*E*) 4

5. Evaluate $\lim_{x \to 1} \dfrac{\ln(x)}{1 - x}$

(*A*) 0 (*B*) 1 (*C*) -1 (*D*) ∞ (*E*) Does not exist

6. If $f(x) = -x^2 + 4x + 3 + 2e^x$, then how many of the following are true:

(i) The graph of the function has an inflection point at $x = 1$.

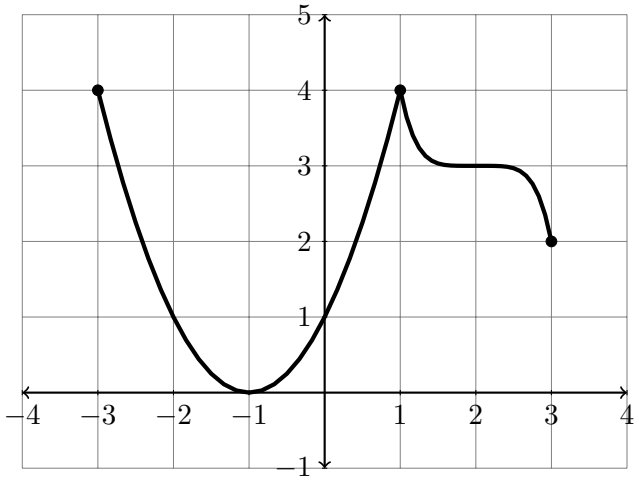
(ii) The function is decreasing at $x = 0$

(iii) The function has two inflection points

(iv) The function is concave downward at $x = \ln 2$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

8. Using the graph of $f(x)$ below, what are the critical points of $f(x)$?



(A) $x = -3, -1, 1, 2, 3$ (B) $x = -1, 1, 2$ only (C) $x = -1, 2$ only (D) $x = -1$ only (E) None of the above

9. If $f(x) = 3x\ln(x)$, then the function is increasing on which of the following intervals:

(A) (e^{-1}, ∞) (B) $(1, e)$ (C) $(0, e)$ (D) $(0, \infty)$ (E) $(0, 1)$

10. Determine if the Mean Value Theorem applies to the function $f(x) = x^3 - 2x$ on the interval $[-2, 2]$. If not, state why. If so, find all values of c guaranteed to exist by the Mean Value Theorem.

(A) Mean Value Theorem does not apply because $f(x)$ is not continuous on $[-2, 2]$.

(B) Mean Value Theorem does not apply because $f(x)$ is not differentiable on $(-2, 2)$.

(C) Mean Value Theorem does not apply because $f(-2) \neq f(2)$.

(D) Mean Value Theorem applies; $c = \pm\sqrt{\frac{2}{3}}$.

(E) Mean Value Theorem applies; $c = \pm\frac{2}{\sqrt{3}}$.

11. Consider $f(x) = \dfrac{1}{(x-2)^2}$. For which of the following intervals can we apply Rolle's Theorem to $f(x)$?

(A) only $[1, 3]$ (B) only $[2, 4]$ (C) $[2, 4]$ and $[3, 7]$ (D) only $[3, 7]$ (E) None of the above

12. Evaluate $\lim_{x \to -1} \dfrac{x-3}{\sin(\pi x) + x^2 + 1}$

(A) 0 (B) 1 (C) -2 (D) $-\infty$ (E) None of the above

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Midterm 3 A

Part II Instructions: free response questions

For Instructor Use Only:

FR 1	
FR 2	
FR 3	
FR 4	
FR 5	
Total Points	

3. (7 pts) Find the location and value of the absolute maximum and minimum of the function $f(x) = 2x^3 - 15x^2 + 24x$ on $[0, 5]$. (Write your answer as a coordinate pair (x, y)).

4. (7pts) Evaluate $\lim_{x \to \pi} (\pi - x)^{\tan(x)}$

5. (7 pts) If $f(x) = 3x^5 + 5x^4$, find all of the inflection points of the function **and** the intervals on which the graph is concave up and concave down. (Write your inflection points as coordinate pairs (x, y))

Calculus I: MAC2311
Midterm 3 B

Name: _____

Part I Instructions: multiple choice questions

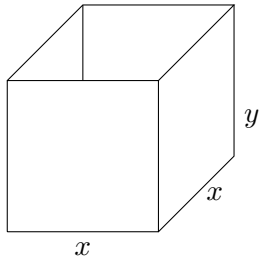
3. Which of the following is the area of a rectangle with perimeter 20 and maximum area.

(*A*) 20 (*B*) 1 (*C*) 0 (*D*) 25 (*E*) None of the above

7. Find the point on the line $y = x + 1$ that is closest to the origin $(0, 0)$.

(A) $(0, 1)$ (B) $(\frac{1}{2}, -\frac{1}{2})$ (C) $(0, 0)$ (D) $(-\frac{1}{2}, \frac{1}{2})$ (E) None of the above

9. Suppose you are trying to construct a box (drawn below) with a square bottom and an open top out of 20 square inches of material with maximum volume. Which of the following is the function you will want to maximize?



(A) $V = \dfrac{20 - x^2}{4x}$ (B) $V = x^2 \cdot \dfrac{20 - x^2}{4x}$ (C) $V = x \cdot \dfrac{20 - x^2}{4x}$ (D) $V = x^3$ (E) None of the above

Calculus I: MAC2311
Midterm 3 B

Part II Instructions: free response questions

For Instructor Use Only:

FR 1	
FR 2	
FR 3	
FR 4	
FR 5	
Total Points	

2. A farmer plans to build a rectangular enclosure for his pigs that is adjacent to a river. The enclosure is to be 3,200 square meters. What dimensions would require the least amount of fencing if no fencing is needed along the river?

3. Sketch the graph of a function $f(x)$ that has to following properties:

   - Local minimum value of $f(-2) = 2$
   - Local maximum value of $f(-1) = 3$
   - Point of inflection at the point $(1, 1)$
   - Increasing on the intervals $(-2, -1)$
   - Decreasing on the interval $(-\infty, -2)$ and $(-1, \infty)$
   - Concave upward on the intervals $(-\infty, -1)$ and $(-1, 1)$
   - Concave downward on the interval $(1, \infty)$

4. Evaluate

$$\lim_{x \to 0} (1 + 2x)^{3/x}.$$

5. You are driving on an interstate highway which has a speed limit of 65 mph. At 2:00 PM you drive past a state trooper at milepost 110 while driving 63 mph. At 5:00 PM you drive past another state trooper at milepost 320 while driving 59 mph. You did not drive past any other state troopers on your trip. Two weeks later you get a speeding ticket in the mail. Explain how the state troopers could use the Mean Value Theorem to determine that you were speeding.