Calculus I: MAC2311
Fall 2022
Exam 3 A
11/16/2022
Time Limit: 90 Minutes

Name: ____________________

Section: ____________________

UF-ID: ____________________

Scantron Instruction: This exam uses a scantron. Follow the instructions listed on this page to fill out the scantron.

A. Sign your scantron **on the back** at the bottom in the white area.

B. Write **and code** in the spaces indicated:

1) Name (last name, first initial, middle initial)

2) UFID Number

3) 4-digit Section Number

C. Under *special codes*, code in the test numbers 3, 1:

1 2 • 4 5 6 7 8 9 0
• 2 3 4 5 6 7 8 9 0

D. At the top right of your scantron, fill in the *Test Form Code* as A.

• B C D E

E. This exam consists of 14 multiple choice questions and 5 free response questions. Make sure you check for errors in the number of questions your exam contains.

F. The time allowed is 90 minutes.

G. WHEN YOU ARE FINISHED:

1) Before turning in your test check for **transcribing errors**. Any mistakes you leave in are there to stay!

2) You must turn in your scantron to your proctor. **Be prepared to show your GatorID with a legible signature.**

It is your responsibility to ensure that your test has **19 questions**. If it does not, show it to your proctor immediately. You will not be permitted to make up any problems omitted from your test after the testing period ends. There are a total of 105 points available on this exam.

Part I Instructions: 14 multiple choice questions. Complete the scantron sheet provided with your information and fill in the appropriate spaces to answer your questions. Only the answer on the scantron sheet will be graded. Each problem is worth five (5) points for a total of 70 points on Part I.

1. Evaluate $\lim_{x \to 0^+} x^{\sqrt{x}}$.

(A) 0 (B) 1 (C) e (D) $-\infty$ (E) ∞

2. The volume of a cylinder is given by $V = \pi r^2 h$, where V, r, and h are functions of time, t. If $\frac{dV}{dt}(5) = 6\pi$, $\frac{dh}{dt}(5) = 1$, $\frac{dr}{dt}(5) = -\frac{1}{4}$, and $h(5) = 2$, what is the value of $r(5)$?

(A) 1 (B) -2 (C) 4 (D) 2 (E) 3

3. Which of the following statements must be true for $f(x) = \dfrac{1}{2}x^2 - x - 2\ln(x)$?

P. $f(x)$ has exactly one critical number.

Q. $f(x)$ is decreasing on $(0, 2)$.

R. $f(x)$ is increasing on $(2, \infty)$.

(A) None of P, Q, or R (B) P and Q only (C) Q and R only (D) P and R only (E) P, Q, and R

4. Suppose that a continuous function $f(x)$ has horizontal tangent lines at $x = -2$, $x = 0$, and $x = 1$. If $f''(x) = 3x^2 + 2x - 2$, then according to the Second Derivative Test $f(x)$ which of the following is true for $f(x)$?

(A) $f(x)$ has a local maximum at $x = 1$ and local minima at $x = -2$ and $x = 0$

(B) $f(x)$ has local maxima at $x = -2$ and $x = 0$ and a local minimum at $x = 1$

(C) $f(x)$ has a local maximum at $x = 0$ and a local minimum at $x = -2$ only

(D) $f(x)$ has local maxima at $x = -2$ and $x = 1$ and a local minimum at $x = 0$

(E) $f(x)$ has a local maximum at $x = 0$ and local minima at $x = -2$ and $x = 1$

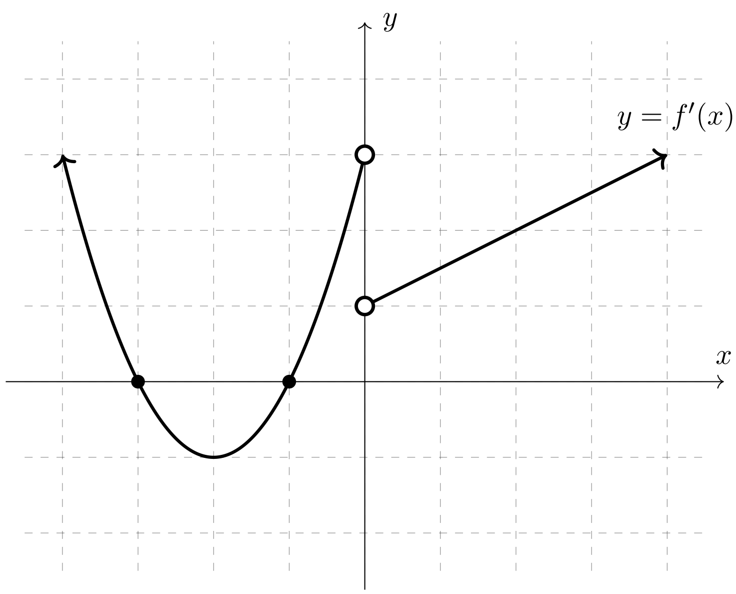
5. Consider the function $f(x)$ where

$$f(x) = \frac{x}{x^2-1}, \quad f'(x) = \frac{-(x^2+1)}{(x^2-1)^2}, \quad f''(x) = \frac{2x(x^2+3)}{(x^2-1)^3}.$$

On what interval(s) is $f(x)$ concave upward?

(A) $(-\infty,-1)\cup(0,1)$ (B) $(-1,0)\cup(0,\infty)$ (C) $(-1,0)\cup(1,\infty)$ (D) $(-\infty,-1)$ (E) $(-\infty,0)$

6. Suppose that $f(x)$ is continuous on $(-\infty,\infty)$ and that the graph of its **derivative** $y=f'(x)$ is given below.



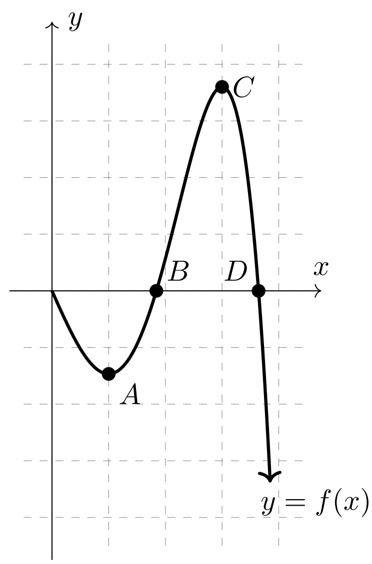
Where is $f(x)$ decreasing?

(A) $(-\infty,0)$ (B) $(-3,-1)$ (C) $(-\infty,-2)$ (D) $(0,\infty)$ (E) $f(x)$ is never decreasing

7. Let $y = 3x^2 + x + 1$. Suppose x increases from 1 to 1.1. Which of the following is $\Delta y - dy$?

(A) 0 (B) 0.01 (C) 0.02 (D) 0.03 (E) 0.04

8. At which of the labeled points on the graph below is $f''(x) > f'(x) > f(x)$?



(A) A (B) B (C) C (D) D

9. The base damage, B, dealt by a character in a certain MMORPG is given by

$$B = \frac{1}{2}D^2 + 3SD + S^2$$

where D is the character's Dexterity score and S is the character's Strength score. If a character has 6 total points to assign between Strength and Dexterity, what combination of Strength and Dexterity will maximize B?

(A) $S = 2$, $D = 4$ (B) $S = 4$, $D = 2$ (C) $S = 1$, $D = 5$ (D) $S = 3$, $D = 3$ (E) $S = 5$, $D = 1$

10. Which of the following value(s) of x satisfy the conclusion of the Mean Value Theorem for $f(x) = x^3 - 2x^2 - 4x + 1$ on the interval $[0, 2]$?

(A) $x = 0, \frac{4}{3}$ (B) $x = \frac{4}{3}$ only (C) $x = 2$ (D) $x = 0$ only (E) $x = \frac{4}{3}, -\frac{4}{3}$

11. Suppose $f'(x) = x^3 - 2x^2 - 3x$. Which of the following must be **False**?

(A) $f(x)$ has a local minimum at $x = -1$

(B) $f(x)$ has a local minimum at $x = 3$

(C) $f(x)$ has a local minimum at $x = 0$

(D) $f(x)$ is increasing on $(3, \infty)$

12. What is the absolute maximum of $f(x) = e^{\sin(x)}$ on $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$?

(A) 1 (B) e (C) $e^{\sqrt{2}/2}$ (D) $e^{\sqrt{3}/2}$ (E) $e^{1/2}$

13. Suppose $f(x)$ is a differentiable function such that $\lim_{x\to 0} f(x) = 0$ and $\lim_{x\to 0} f'(x) = 1$. Evaluate

$$\lim_{x\to 0} \frac{f(x)}{x}.$$

(A) -1 (B) 0 (C) 1 (D) ∞ (E) $-\infty$

14. Use the linear approximation of $f(x) = \sqrt[3]{x}$ at $a = 8$ to approximate $\sqrt[3]{7.76}$.

(A) 1.975 (B) 1.98 (C) 1.985 (D) 1.99 (E) 1.995

Calculus I: MAC2311
Fall 2022
Exam 3 A
11/16/2022
Time Limit: 90 Minutes

Name: ____________________

Section: ____________________

UF-ID: ____________________

<u>Part II Instructions</u>: 5 free response questions. Neatly give a complete solution to each problem and show all work and intermediate steps. We are grading the work and notation as well as the answer. Each problem is worth seven (7) points. A total of 35 points is possible on Part II. **No credit will given without proper work.** If we cannot read it and follow it, you will receive no credit for the problem.

For Instructor Use Only:

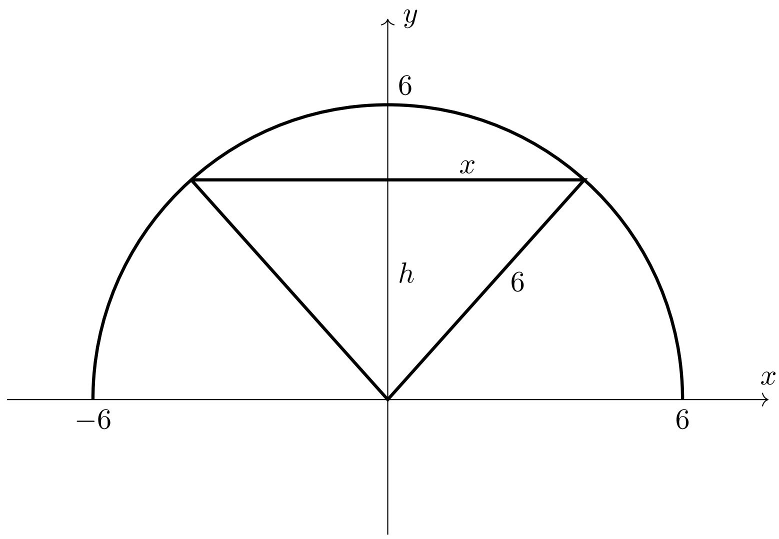
FR 1	
FR 2	
FR 3	
FR 4	
FR 5	
Total Points	

1. Use the linear approximation of the function $f(x) = \sqrt{4 + \ln(x)}$ at $x = 1$ to approximate the value of $f(1.4)$.

2. Sand pouring from a chute forms a conical pile whose radius is always equal to twice its height. If the volume increases at a constant rate of 12π cubic feet per minute, at what rate is the height of the pile changing when the radius is 2 feet? (The volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$).

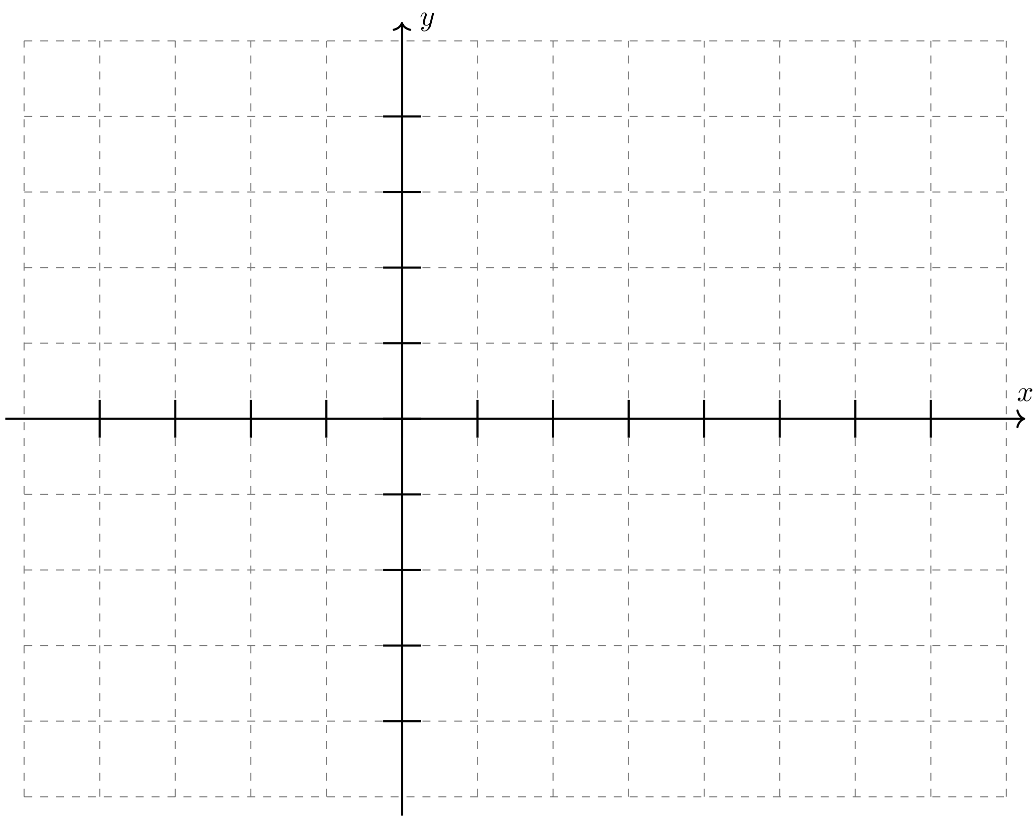
3. Find all value(s) of x which satisfy the conclusion of the Mean Value Theorem for the function $g(x) = ax^2 + x + k$ on the interval $[0, 1]$. Assume that a and k are nonzero real numbers.

4. Find the maximum area of an isosceles triangle inscribed in a semicircle of radius 6, if one vertex of the triangle is on the midpoint of the diameter of the semicircle (A diagram is provided below). Be sure to provide justification that your answer is a maximum value.



5. Sketch a graph of the function $f(x)$ that has the following properties:

- $f(x)$ is continuous on $(-\infty,-1)\cup(-1,\infty)$
- $f(x)$ has a vertical asymptote $x=-1$
- $\lim_{x\to\infty} f(x)=1$ and $\lim_{x\to-\infty} f(x)=2$
- $f(x)$ is increasing on $(-\infty,-1)\cup(-1,2)$
- $f(x)$ is decreasing on $(2,\infty)$
- $f(x)$ has a local maximum at $(2,3)$ and no local minimums
- $f(x)$ is concave upward on $(-\infty,-1)\cup(4,\infty)$
- $f(x)$ is concave downward on $(-1,4)$
- $f(x)$ has an inflection point $(4,2)$



Calculus I: MAC2311
Fall 2022
Exam 3 B
11/16/2022
Time Limit: 90 Minutes

Name: ____________________

Section: ____________________

UF-ID: ____________________

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A. Sign your scantron **on the back** at the bottom in the white area.

B. Write **and code** in the spaces indicated:

1) Name (last name, first initial, middle initial)

2) UFID Number

3) 4-digit Section Number

C. Under *special codes*, code in the test numbers 3, 2:

1 2 • 4 5 6 7 8 9 0
1 • 3 4 5 6 7 8 9 0

D. At the top right of your scantron, fill in the *Test Form Code* as B.

A • C D E

E. This exam consists of 14 multiple choice questions and 5 free response questions. Make sure you check for errors in the number of questions your exam contains.

F. The time allowed is 90 minutes.

G. WHEN YOU ARE FINISHED:

1) Before turning in your test check for **transcribing errors**. Any mistakes you leave in are there to stay!

2) You must turn in your scantron to your proctor. **Be prepared to show your GatorID with a legible signature.**

It is your responsibility to ensure that your test has **19 questions**. If it does not, show it to your proctor immediately. You will not be permitted to make up any problems omitted from your test after the testing period ends. There are a total of 105 points available on this exam.

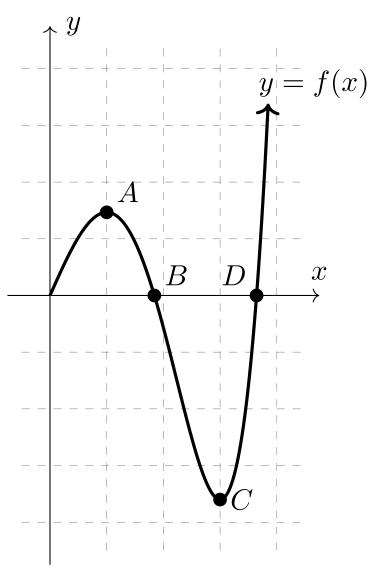
Part I Instructions: 14 multiple choice questions. Complete the scantron sheet provided with your information and fill in the appropriate spaces to answer your questions. Only the answer on the scantron sheet will be graded. Each problem is worth five (5) points for a total of 70 points on Part I.

1. Suppose $f(x)$ is a differentiable function such that $\lim_{x\to 0} f(x) = 0$ and $\lim_{x\to 0} f'(x) = -1$. Evaluate

$$\lim_{x\to 0} \frac{f(x)}{x}.$$

(A) -1 (B) 0 (C) 1 (D) ∞ (E) $-\infty$

2. At which of the labeled points on the graph below is $f(x) > f'(x) > f''(x)$?



(A) A (B) B (C) C (D) D

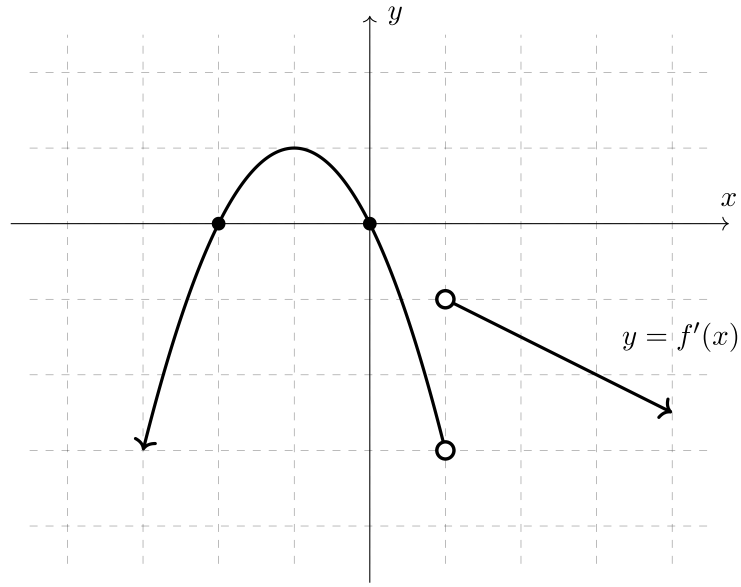
3. Let $y = 2x^2 + x + 1$. Suppose x increases from 1 to 1.1. Which of the following is equal to $\Delta y - dy$?

(A) 0 (B) 0.01 (C) 0.02 (D) 0.03 (E) 0.04

4. Which of the following value(s) of x satisfy the conclusion of the Mean Value Theorem for $f(x) = x^3 - 2x^2 - 4x + 1$ on the interval $[0, 1]$?

(A) $x = 1, \frac{1}{3}$ (B) $x = \frac{1}{3}$ only (C) $x = 1$ only (D) $x = 0$ (E) $x = \frac{1}{3}, -\frac{1}{3}$

5. Suppose that $f(x)$ is continuous on $(-\infty, \infty)$ and that the graph of its **derivative** $y = f'(x)$ is given below.



Where is $f(x)$ increasing?

(A) $(-\infty, -2)$ (B) $(-3, -1)$ (C) $(-2, 0)$ (D) $(0, \infty)$ (E) $f(x)$ is never increasing

6. What is the absolute maximum of $f(x) = e^{\cos(x)}$ on $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$?

(A) 1 (B) e (C) $e^{\sqrt{2}/2}$ (D) $e^{\sqrt{3}/2}$ (E) $e^{1/2}$

7. Consider the function $f(x)$ where

$$f(x) = \frac{x}{x^2-1}, \quad f'(x) = \frac{-(x^2+1)}{(x^2-1)^2}, \quad f''(x) = \frac{2x(x^2+3)}{(x^2-1)^3}.$$

On what interval(s) is $f(x)$ concave downward?

(A) $(-\infty,-1)\cup(0,1)$ (B) $(-1,0)\cup(0,\infty)$ (C) $(-1,0)\cup(1,\infty)$ (D) $(-\infty,-1)$ (E) $(-\infty,0)$

8. Evaluate $\lim_{x\to 0^+}(\sqrt{x})^x$.

(A) 0 (B) 1 (C) e (D) $-\infty$ (E) ∞

9. Use the linear approximation of $f(x) = \sqrt[3]{x}$ at $a = 27$ to approximate $\sqrt[3]{26.73}$.

(A) 2.975 (B) 2.98 (C) 2.985 (D) 2.99 (E) 2.995

10. The volume of a cylinder is given by $V = \pi r^2 h$, where V, r, and h are functions of time, t. If $\frac{dV}{dt}(5) = 8\pi$, $\frac{dh}{dt}(5) = 1$, $\frac{dr}{dt}(5) = 2$, and $h(5) = \frac{1}{2}$, what is the value of $r(5)$?

(A) -4 (B) 3 (C) 1 (D) 2 (E) 4

11. The base damage, B, dealt by a character in a certain MMORPG is given by

$$B = D^2 + 3SD + S^2$$

where D is the character's Dexterity score and S is the character's Strength score. If a character has 6 total points to assign between Strength and Dexterity, what combination of Strength and Dexterity will maximize B?

(A) $S = 2$, $D = 4$ (B) $S = 1$, $D = 5$ (C) $S = 3$, $D = 3$ (D) $S = 4$, $D = 2$ (E) $S = 0$, $D = 6$

12. Which of the following statements must be true for $f(x) = \frac{1}{2}x^2 + 2x - 3\ln(x)$?

P. $f(x)$ has exactly one critical number.

Q. $f(x)$ is decreasing on $(0, 1)$.

R. $f(x)$ is increasing on $(1, \infty)$.

(A) None of P, Q, or R (B) P and Q only (C) Q and R only (D) P and R only (E) P, Q, and R

13. Suppose $f'(x) = -x^3 + 2x^2 + 3x$. Which of the following must be **False**?

(A) $f(x)$ has a local maximum at $x = -1$

(B) $f(x)$ has a local maximum at $x = 3$

(C) $f(x)$ has a local maximum at $x = 0$

(D) $f(x)$ is increasing on $(0, 3)$

14. Suppose that a continuous function $f(x)$ has horizontal tangent lines at $x = -2$, $x = 0$, and $x = 1$. If $f''(x) = 2x^2 - 4x + 1$, then according to the Second Derivative Test $f(x)$ which of the following is true for $f(x)$?

(A) $f(x)$ has a local maximum at $x = 1$ and local minima at $x = -2$ and $x = 0$

(B) $f(x)$ has local maxima at $x = -2$ and $x = 0$ and a local minimum at $x = 1$

(C) $f(x)$ has a local maximum at $x = 0$ and a local minimum at $x = -2$ only

(D) $f(x)$ has local maxima at $x = -2$ and $x = 1$ and a local minimum at $x = 0$

(E) $f(x)$ has a local maximum at $x = 0$ and local minima at $x = -2$ and $x = 1$

Calculus I: MAC2311
Fall 2022
Exam 3 B
11/16/2022
Time Limit: 90 Minutes

Name: ____________________

Section: ____________________

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For Instructor Use Only:

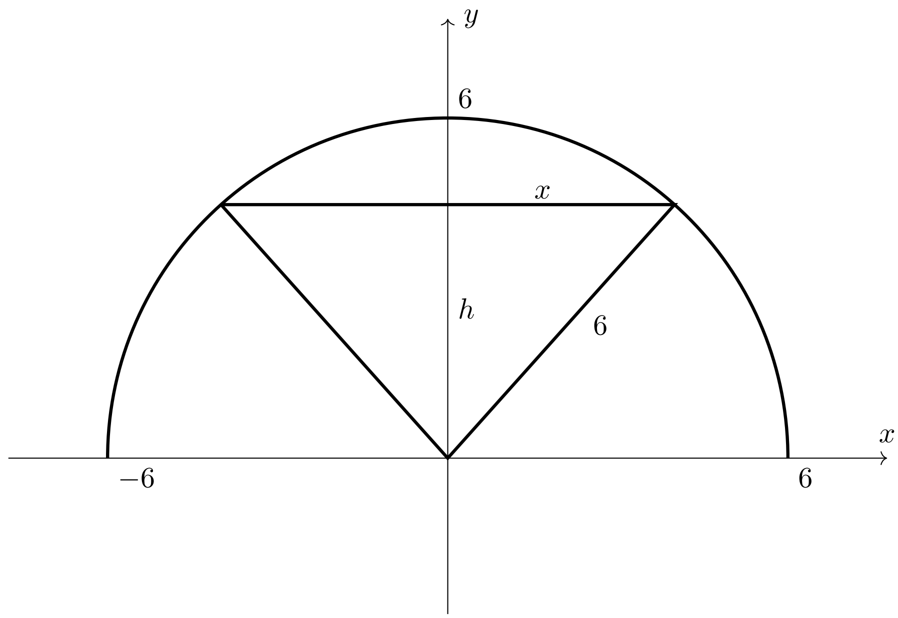
FR 1	
FR 2	
FR 3	
FR 4	
FR 5	
Total Points	

1. Use the linear approximation of the function $f(x) = \sqrt{9 + \ln(x)}$ at $x = 1$ to approximate the value of $f(1.6)$.

2. Sand pouring from a chute forms a conical pile whose radius is always equal to twice its height. If the volume increases at a constant rate of 54π cubic feet per minute, at what rate is the height of the pile changing when the radius is 3 feet? (The volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$).

3. Find all value(s) of x which satisfy the conclusion of the Mean Value Theorem for the function $g(x) = ax^2 + x + k$ on the interval $[0, 1]$. Assume that a and k are nonzero real numbers.

4. Find the maximum area of an isosceles triangle inscribed in a semicircle of radius 6, if one vertex of the triangle is on the midpoint of the diameter of the semicircle (A diagram is provided below). Be sure to provide justification that your answer is a maximum value.



5. Sketch a graph of the function $f(x)$ that has the following properties:

- $f(x)$ is continuous on $(-\infty, -1) \cup (-1, \infty)$
- $f(x)$ has a vertical asymptote $x = -1$
- $\lim_{x\to\infty} f(x) = -1$ and $\lim_{x\to-\infty} f(x) = -2$
- $f(x)$ is decreasing on $(-\infty, -1) \cup (-1, 2)$
- $f(x)$ is increasing on $(2, \infty)$
- $f(x)$ has a local minimum at $(2, -3)$ and no local maximums
- $f(x)$ is concave downward on $(-\infty, -1) \cup (4, \infty)$
- $f(x)$ is concave upward on $(-1, 4)$
- $f(x)$ has an inflection point $(4, -2)$