It is your responsibility to ensure that your test has **19 questions**. If it does not, show it to your proctor immediately. You will not be permitted to make up any problems omitted from your test after the testing period ends. There are a total of 105 points available on this exam.

Part I Instructions: 14 multiple choice questions. Complete the scantron sheet provided with your information and fill in the appropriate spaces to answer your questions. Only the answer on the scantron sheet will be graded. Each problem is worth five (5) points for a total of 70 points on Part I.

1. Find the slope of the tangent line to $f(x) = \ln\left(\dfrac{e^{2x}\sqrt{x+3}}{x^2+1}\right)$ at $x = 1$.

$(A)\ \dfrac{5}{4}$ $(B)\ -\dfrac{3}{8}$ $(C)\ \dfrac{7}{4}$ $(D)\ \dfrac{13}{8}$ $(E)\ \dfrac{9}{8}$

2. Suppose $f(x) = e^{2x} + 2x^3 - \sin(x)$. Calculate $f'''(1)$.

$(A)\ e^2 - \cos(1) + 12$ $(B)\ 8e^2 + \cos(1) + 12$ $(C)\ 8e^2 - \cos(1) + 12$ $(D)\ e^2 + \cos(1) + 12$

3. The vertical position of your jetpack-wearing Calculus instructor is given by $s(t) = t^3 - 9t^2 - 21t + 16$ for $t \geq 0$. On which of the following intervals are they slowing down?

(A) $(0, 3)$ (B) $(3, 7)$ (C) $(1, 5)$ (D) $(0, 1)$ (E) $(7, \infty)$

4. Find the derivative of $f(x) = x^2 + \arccos(x + 1)$.

(A) $f'(x) = \dfrac{1}{\sqrt{-x^2 - 2x}}$

(B) $f'(x) = 2x + \dfrac{1}{\sqrt{-x^2 - 2x}}$

(C) $f'(x) = 2x - \dfrac{1}{\sqrt{-x^2 - 2x}}$

(D) $2x - \dfrac{1}{\sqrt{x^2 + 2x}}$

5. Let $f(x) = 4^x + \ln(x) + 4x^3$. what is the value of $f'(\frac{1}{2})$?

(A) 7 (B) $\ln(4) + 5$ (C) $4\ln(4) + 5$ (D) $2\ln(4) + 5$ (E) $2\ln(8) + 5$

6. Use implicit differentiation to find $\dfrac{dy}{dx}$ for $6x^3 + 7y^3 = 13xy$.

(A) $\dfrac{dy}{dx} = \dfrac{13y - 18x^2}{21y^2 - 13x}$

(B) $\dfrac{dy}{dx} = \dfrac{18y - 13x^2}{21y^2 - 13x}$

(C) $\dfrac{dy}{dx} = \dfrac{13y - 18x^2}{13y^2 - 21x}$

(D) $\dfrac{dy}{dx} = \dfrac{13y - 18x^2}{13y - 21x^2}$

7. Let $f(1) = 2$ and $g(x) = \dfrac{f(x) - 2}{f(x) + 1}$. If $g'(1) = 2$, then which of the following is equal to $f'(1)$?

$(A)\ -\dfrac{2}{3}$ $(B)\ 4$ $(C)\ 3$ $(D)\ -3$ $(E)\ 6$

8. Assume that $f(x)$ and $g(x)$ are differentiable functions such that

$$f'(x) = -g(x) \text{ and } g'(x) = -f(x).$$

Let $h(x) = (f(x))^2 - (g(x))^2$. Which of the following is equal to $h'(x)$?

$(A)\ 4f(x)g(x)$ $(B)\ 0$ $(C)\ 4f(x)f'(x)$ $(D)\ 2g(x)f(x)$ $(E)\ 2g'(x)f'(x) + (f(x))^2$

9. How many of the following statements are necessarily true?

- $\dfrac{d}{dx}\left[f(x)g(x)\right] = \dfrac{d}{dx}[f(x)] \cdot \dfrac{d}{dx}[g(x)]$
- $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{g(x) \cdot \dfrac{d}{dx}[f(x)] - f(x) \cdot \dfrac{d}{dx}[g(x)]}{[g(x)]^2}$
- $\dfrac{d}{dx}\left[f(x)g(x)\right] = \dfrac{d}{dx}[f(x)] \cdot g(x) + f(x) \cdot \dfrac{d}{dx}[g(x)]$
- $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{\dfrac{d}{dx}[f(x)]}{\dfrac{d}{dx}[g(x)]}$

(A) 0 $\quad$ (B) 1 $\quad$ (C) 2 $\quad$ (D) 3 $\quad$ (E) 4

10. Let $f(x) = \sin(x) + \cos(x) + \tan(x) + \csc(x)$. What is the value of $f'(\frac{\pi}{4})$?

(A) $2 - 2\sqrt{2}$ $\quad$ (B) $\sqrt{2} - 2$ $\quad$ (C) $2 - \sqrt{2}$ $\quad$ (D) $2 + \sqrt{2}$ $\quad$ (E) $2 + 2\sqrt{2}$

11. Which of the following is an equation for the tangent line to the graph of $f(x) = \arctan(2x)$ at $x = 0$?

(A) $y = x$ (B) $y = x + 1$ (C) $y = 2x$ (D) $y = x - 1$ (E) $y = 2x - 1$

12. Consider $f(x) = x^3 \ln(x) + \dfrac{x}{x^2 + 1}$. What is $f'(1)$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

13. If $g(x) = x^{1/5}(x-1)^{3/5}$, find the domain of $g'(x)$.

(A) $(-\infty, 2) \cup (2, 3) \cup (3, \infty)$

(B) $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

(C) $(-\infty, \infty)$

(D) $(0, \infty)$

14. If $h(x) = [f(x)g(x) + x]^2$, what is $h'(0)$ if $f(0) = 2$, $g(0) = 1$, $f'(0) = 0$, and $g'(0) = -1$?

(A) -2

(B) -4

(C) 6

(D) -3

(E) 4

1. Jacques Monod modeled the per capita growth rate R of *Escherichia coli* bacteria by the function

$$R(N) = \frac{N}{3+N}$$

where N is the concentration of the nutrient.

(a) Determine $\frac{dR}{dN}$.

(b) Calculate $\frac{dR}{dN}$ at $N = 8$. Using this answer, is the growth rate rising or falling at $N = 8$? Explain why.

2. The vertical position of a yo-yo at time t, for $0 \leq t \leq \pi$, is given by

$$s(t) = \sqrt{3}\sin(t) - \cos(t).$$

(a) What is the displacement of the yo-yo from $t = 0$ to $t = \dfrac{\pi}{2}$?

(b) What is the total distance traveled by the yo-yo from $t = 0$ to $t = \pi$?

3. Let $f(x) = x^{\sin(x)}$. Calculate $f'(x)$.

4. Let $x^4 - x^2y + y^4 = 1$

(a) Find the slope of the tangent line at the point $(-1, 1)$.

(b) Write an equation for the tangent line to the curve at the point $(-1, 1)$.

5. The table below gives the values of $f(x)$, $f'(x)$, $g(x)$, and $g'(x)$ for various values of x. Use the table to answer the following questions.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	0	4	2	3
2	1	-1	2	-2
3	6	5	4	3

(a) Let $A(x) = \ln(g(x))$. What is $A'(2)$?

(b) Let $B(x) = f(x)(g(x))^2$. What is $B'(3)$?

(c) Let $C(x) = \dfrac{(f(x))^3}{g(x)}$. What is $C'(2)$?

14. The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$ with radius r. Suppose the sphere expands as time passes. Which of the following gives $\frac{dV}{dt}$ in terms of $\frac{dr}{dt}$?

(A) $\frac{dV}{dt} = \frac{4}{3}\pi r^3 \frac{dr}{dt}$

(B) $\frac{dV}{dt} = 4\pi r^2$

(C) $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

(D) $\frac{dV}{dt} = 4\pi \left(\frac{dr}{dt}\right)^2$

(E) None of the above

1. (7 pts) Use logarithmic differentiation to find the derivative of $f(x) = \sqrt{\frac{x^2 \cos^3(x)}{e^x \sqrt{x}}}$.

2. (7 pts) Suppose a 13 foot ladder rests against a wall. If the bottom of the ladder slides away from the wall at a rate of 3 feet per second, at what rate does the angle the ladder makes with the ground change when the top of the ladder is 5 feet from the ground?