Calculus I: MAC2311 **Name:** Key

Exam 2

Part I Instructions: 14 multiple choice questions

1. Find $f''(x)$ for the function $f(x) = \dfrac{x}{e^x}$

(A) $\dfrac{-x+2}{e^x}$ (B) $\dfrac{x+2}{e^x}$ (C) $\dfrac{-x-2}{e^x}$ $\boxed{(D)\ \dfrac{x-2}{e^x}}$

Need Quotient Rule $\dfrac{gf' - fg'}{g^2}$

$f(x) = \dfrac{x}{e^x}$ $f = x$ $g = e^x$

$f' = 1$ $g' = e^x$

$$f'(x) = \frac{e^x(1) - xe^x}{(e^x)^2} = \frac{e^x(1-x)}{(e^x)^2} = \frac{1-x}{e^x}$$

Quotient Rule again:

$f'(x) = \dfrac{1-x}{e^x}$ $f = 1-x$ $g = e^x$

$f' = -1$ $g' = e^x$

$$f''(x) = \frac{e^x(-1) - (1-x)(e^x)}{(e^x)^2} = \frac{e^x(-1-(1-x))}{(e^x)^2}$$

$$= \frac{-1-(1-x)}{e^x} = \frac{-1-1+x}{e^x} = \boxed{\frac{x-2}{e^x}}$$

2. Suppose $g(x)$ is differentiable for any real number x. Let $f(x) = \dfrac{e^{g(x)}}{x^2+3}$. Find $f'(x)$.

(A) $\dfrac{2xe^{g'(x)} - (x^2+3)e^{g(x)}g'(x)}{(x^2+3)^2}$ (B) $\dfrac{2xe^{g(x)} - 2xe^{g(x)}g'(x)}{(x^2+3)^2}$ (C) $\dfrac{2xe^{g(x)} - (x^2+3)e^{g(x)}g'(x)}{(x^2+3)^2}$

(D) $\dfrac{2xe^{g(x)}g'(x) - 2xe^{g(x)}}{(x^2+3)^2}$ $\boxed{(E)\ \dfrac{(x^2+3)e^{g(x)}g'(x) - 2xe^{g(x)}}{(x^2+3)^2}}$

$$f(x) = \frac{e^{g(x)}}{x^2+3}$$

Need Quotient Rule: $\dfrac{gf' - fg'}{g^2}$

$f = e^{g(x)}$ $\qquad g = x^2+3$

$f' = e^{g(x)} \cdot g'(x)$ $\qquad g' = 2x$

$$f'(x) = \frac{(x^2+3)(e^{g(x)} \cdot g'(x)) - e^{g(x)}(2x)}{(x^2+3)^2}$$

3. Determine the derivative of $f(x) = e^{\cos^2(x)}$.

(A) $e^{-2\sin(x)\cos(x)}$ (B) $-2e^{\cos^2(x)}\sin(x)$ (C) $2e^{\cos^2(x)}\cos(x)$

$\boxed{(D)\ -2e^{\cos^2(x)}\cos(x)\sin(x)}$ (E) $e^{-2\sin(x)\cos(x)}\sin(x)$

$$f(x) = e^{\cos^2(x)}$$

$$f'(x) = e^{\cos^2(x)}\left(\underline{\frac{d}{dx}\left(\cos^2(x)\right)}\right)$$

$$\frac{d}{dx}\cos^2(x) = \frac{d}{dx}\left(\cos(x)\right)^2 = 2(\cos(x))\cdot(-\sin(x)) = -2\cos(x)\sin(x)$$

$$f'(x) = e^{\cos^2(x)}\left(-2\cos(x)\sin(x)\right)$$

$$f'(x) = -2e^{\cos^2(x)}\cos(x)\sin(x)$$

* Answer key was incorrect *

4. If $f(x) = \cot^{-1}(x)$ ($f(x) = \text{arccot}(x)$), then find an expression for $f''(x)$.

$\boxed{(A)\ f''(x) = \dfrac{2x}{(1+x^2)^2}}$

(B) $f''(x) = \dfrac{-2x}{(1+x^2)^2}$

(C) $f''(x) = \dfrac{1-2x+x^2}{(1+x^2)^2}$

(D) $f''(x) = \dfrac{-x^2+2x-1}{(1+x^2)^2}$

$$f(x) = \cot^{-1}(x)$$

$$f'(x) = \frac{-1}{1+x^2} \quad \text{(Memorize)}$$

OR Rewrite as $-1(1+x^2)^{-1}$ and use Chain Rule

Quotient Rule $\dfrac{gf' - fg'}{g^2}$ for $f''(x)$

$f = -1 \qquad g = 1+x^2$

$f' = 0 \qquad g' = 2x$

$$f''(x) = \frac{(1+x^2)(0) - (-1)(2x)}{(1+x^2)^2} = \frac{2x}{(1+x^2)^2}$$

5. Use implicit differentiation to find y' for the equation $xy = \sin(y)$.

$\boxed{(A)\ y' = \frac{-y}{x - \cos(y)}}$ $(B)\ y' = \frac{y}{x - \cos(y)}$ $(C)\ y' = \frac{x - \cos(y)}{-y}$ $(D)\ y' = \frac{x - \cos(y)}{y}$

$$\frac{d}{dx}(xy) = \frac{d}{dx}(\sin(y))$$

Prod. Rule $gf' + fg'$

$f = x$ $g = y$

$f' = 1$ $g' = 1\frac{dy}{dx}$

chain rule

$$y(1) + (x)\left(1\frac{dy}{dx}\right) = \cos(y)\underbrace{\left(\frac{dy}{dx}\right)}$$

$$y + x\frac{dy}{dx} = \cos(y)\frac{dy}{dx}$$

$$x\frac{dy}{dx} - \cos(y)\frac{dy}{dx} = -y$$

$$\frac{dy}{dx}(x - \cos(y)) = -y$$

$$\frac{dy}{dx} = \frac{-y}{x - \cos(y)}$$

6. Which of the following is equal to the derivative of $g(x) = 3x^2 + \sqrt[3]{x^2} + 4$

(A) $g'(x) = 6x + \dfrac{1}{3\sqrt[3]{x}}$ (B) $g'(x) = 3x + \dfrac{1}{3\sqrt[3]{x}}$ (C) $g'(x) = 3x + \dfrac{2}{3\sqrt[3]{x}} + 4$ $\boxed{(D)\ g'(x) = 6x + \dfrac{2}{3\sqrt[3]{x}}}$

$g(x) = 3x^2 + x^{2/3} + 4$

$\frac{2}{3} - \frac{3}{3} = \frac{-1}{3}$

$g'(x) = 6x + \frac{2}{3}x^{-1/3}$

$g'(x) = 6x + \dfrac{2}{3\sqrt[3]{x}}$

Need chain rule

7. Let $f(x) = (1+\sqrt{x})^{\frac{1}{3}}$. What is $f'(x)$?

(A) $\frac{1}{3}\left(1+x^{\frac{1}{2}}\right)^{-\frac{2}{3}}$ $\boxed{(B)\ \frac{1}{6}x^{-\frac{1}{2}}\left(1+x^{\frac{1}{2}}\right)^{-\frac{2}{3}}}$ (C) $\frac{1}{3}\left(1+\frac{1}{2}x^{-\frac{1}{2}}\right)^{-\frac{2}{3}}$ (D) $\frac{1}{3}\left(\frac{1}{2}x^{-\frac{1}{2}}\right)^{-\frac{2}{3}}$ (E) $\frac{1}{2}x^{-\frac{1}{2}}\left(1+x^{\frac{1}{2}}\right)^{-\frac{2}{3}}$

$$f(x) = (1+\sqrt{x})^{1/3}$$

$$f(x) = (1+x^{1/2})^{1/3}$$

deriv. of inside

$$f'(x) = \frac{1}{3}(1+x^{1/2})^{-\frac{2}{3}}\left(\frac{1}{2}x^{-1/2}\right)$$

$$f'(x) = \frac{1}{6}x^{-1/2}(1+x^{1/2})^{-2/3}$$

$$\frac{1}{3} - \frac{3}{3} = \frac{-2}{3}$$

8. What is the slope of the line tangent to the graph of $y = 2^x - 3^x + 4^x$ when $x = 0$?

(A) $\ln\left(\frac{3}{2}\right)$ (B) $\boxed{\ln\left(\frac{8}{3}\right)}$ (C) 1 (D) $\ln(24)$

$\ln(y) = \ln(2^x)$

$\ln(y) = x\ln(2)$

↳ derivative:

$\frac{1}{y}\frac{dy}{dx} = \ln(2)$

$\frac{dy}{dx} = \ln(2)\cdot y$

$= \ln(2)\cdot 2^x$

$\frac{dy}{dx} = \ln(2)2^x - \ln(3)3^x + \ln(4)4^x$

when $x=0$

$\frac{dy}{dx} = \ln(2)\cancel{2^0} - \ln(3)\cancel{3^0} + \ln(4)\cancel{4^0}$ 1 1 1

$\frac{dy}{dx} = \ln(2) - \ln(3) + \ln(4)$

$\frac{dy}{dx} = \ln\left(\frac{2}{3}\right) + \ln(4)$

$\frac{dy}{dx} = \ln\left(\frac{2}{3}\cdot 4\right) = \ln\left(\frac{8}{3}\right)$

9. What is an equation for the line tangent to the function $f(x) = 6x\sin(x) + \pi$ at $x = \frac{\pi}{2}$?

(A) $y = 6x$ $\boxed{(B)\ y = 6x + \pi}$ (C) $y = 6x - \pi$ (D) $y = 6x + 6$ (E) $y = 6x - 6$

tangent line → point
→ slope

$f\left(\frac{\pi}{2}\right) = 6\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{2}\right) + \pi$ $\left(\frac{\pi}{2}, 4\pi\right)$

$f\left(\frac{\pi}{2}\right) = 3\pi(1) + \pi = 4\pi$

slope: find $f'(x)$

$f'(x) = 6\sin(x) + 6x\cos(x)$ $m = 6$

$f'\left(\frac{\pi}{2}\right) = 6(1) + 6\left(\frac{\pi}{2}\right)(0) = 6$

Use point slope form:

$y - y_1 = m(x - x_1)$

$y - 4\pi = 6\left(x - \frac{\pi}{2}\right)$

$y = 6x - 3\pi + 4\pi$

$y = 6x + \pi$

10. Let $f(x)$ and $g(x)$ be differentiable functions such that $g(3) = 1$. Which of the following is equal to $h'(3)$ where $h(x) = f(x)g(x) + \dfrac{f(x)}{g(x)}$?

(A) 2 (B) $2f(3)$ $\boxed{(C)\ 2f'(3)}$ (D) $2g'(3)$ (E) None of these

$$h(x) = f(x)g(x) + \frac{f(x)}{g(x)}$$

$$h'(x) = f(x)g'(x) + f'(x)g(x) + \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

$$h'(3) = f(3)g'(3) + f'(3)\cancelto{1}{g(3)} + \frac{\cancelto{1}{g(3)}f'(3) - f(3)g'(3)}{\cancelto{1}{(g(3))^2}}$$

$$h'(3) = \cancel{f(3)g'(3)} + f'(3) + f'(3) - \cancel{f(3)g'(3)}$$

$$h'(3) = 2f'(3)$$

11. The mass of a length of wire is $m(x) = x(1+2\sqrt{x})$ kilograms, where x is the length of the wire measured in meters. The **linear density** of the wire is the rate of change of the mass m with respect to x. Find the linear density of the wire (expressed in kg/m) when $x = 4$ m.

(*A*) 1 (*B*) 4 $\boxed{(C)\ 7}$ (*D*) 10

lin. density $= \frac{dm}{dx}$ and $x=4$

$$m(x) = x(1+2\sqrt{x}) = x(1+2x^{1/2})$$
$$= x + 2x^{3/2}$$

$$m'(x) = 1 + 2\left(\frac{3}{2}\right)x^{1/2}$$

$$m'(x) = 1 + 3x^{1/2} = 1 + 3\sqrt{x}$$

$$m'(4) = 1 + 3\sqrt{4}$$
$$= 1 + 3(2) = 7$$

12. Let $h(x) = f(g(x))$. Based on the following table of values for $f(x)$, $f'(x)$, $g(x)$, and $g'(x)$, find $h'(2)$.

x	0	1	2	3
$f(x)$	0	2	1	1
$f'(x)$	2	3	1	2
$g(x)$	2	1	3	3
$g'(x)$	0	1	3	2

(A) 0 (B) 1 (C) 2 (D) 3 $\boxed{(E)\ 6}$

$h(x) = f(g(x))$

$h'(x) = f'(g(x)) \cdot g'(x)$

$h'(2) = f'(g(2)) \cdot g'(2)$

$= f'(3) \cdot 3$

$= 2 \cdot 3 = 6$

13. Let $f(x) = \sec^2(x)$. What is $f'(x)$?

(*A*) $\dfrac{1}{\sin^2(x)}$ (*B*) $-2\sec^3(x)$ (*C*) $2\sec(x)$ $\boxed{(D)\ 2\sec^2(x)\tan(x)}$ (*E*) None of these

$$f(x) = \sec^2(x)$$

$$= \frac{1}{(\cos(x))^2} = (\cos(x))^{-2}$$

$$f'(x) = -2(\cos(x))^{-3} \cdot (-\sin(x))$$

$$f'(x) = \frac{2\sin(x)}{(\cos(x))^3}$$

$$= 2\,\frac{\sin(x)}{\cos(x)} \cdot \frac{1}{(\cos(x))^2}$$

$$= 2\tan(x)\sec^2(x)$$

slope undefined

14. For which of the following x-values does the curve $x(x-1) = y^3$ have a vertical tangent line?

(A) $x = 0$ only $\quad$ (B) $x = 1$ only $\quad$ $\boxed{(C)\ x = 0, 1 \text{ only}}$ $\quad$ (D) $x = 0, 1, -1$ $\quad$ (E) No such value

$$x(x-1) = y^3$$

$$\frac{d}{dx}(x^2 - x) = \frac{d}{dx}(y^3)$$

(Distribute to avoid Prod. Rule)

$$2x - 1 = 3y^2 \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2x-1}{3y^2}$$

undefined if $3y^2 = 0 \rightarrow y = 0$

Plug $y = 0$ into original

$$x(x-1) = 0^3$$

$$x(x-1) = 0$$

$$x = 0, \quad x = 1$$

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Part III Instructions: 6 multiple choice questions

2. Suppose that $A = B + C$ and A, B, and C are functions of t. If $\dfrac{dB}{dt} = 3$ and $\dfrac{dC}{dt} = -4$ what is $\dfrac{dA}{dt}$?

(A) 0 (B) 1 (C) 7 $\boxed{(D)\ -1}$ (E) None of the above

$$\frac{d}{dt}(A) = \frac{d}{dt}(B + C)$$

$$1\frac{dA}{dt} = 1\frac{dB}{dt} + 1\frac{dC}{dt}$$

$$\frac{dA}{dt} = 3 - 4 = -1$$

7. If $f(x) = \ln(\sqrt{e^{\sin(x)}})$, then $f'(\frac{\pi}{3})$ equals:

$\boxed{(A)\ \frac{1}{4}}$ (B) $\frac{\sqrt{2}}{\ln(\frac{\pi}{3})}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{2}\ln(e^{\pi})$ (E) $\frac{\sqrt{2}}{4}$

$$f(x) = \frac{1}{\sqrt{e^{\sin(x)}}} \cdot \underline{\frac{d}{dx}\left(\sqrt{e^{\sin(x)}}\right)}$$

$$\boxed{\begin{aligned}\frac{d}{dx} &= \left(e^{\sin(x)}\right)^{1/2} \\ &= \frac{1}{2}\left(e^{\sin(x)}\right)^{-1/2} \cdot e^{\sin(x)} \cdot \cos(x)\end{aligned}}$$

$$f(x) = \frac{1}{2\cancel{e^{\sin(x)}}} \cdot \cancel{e^{\sin(x)}} \cdot \cos(x)$$

$$= \frac{1}{2}\cos(x)$$

$$f'\left(\frac{\pi}{3}\right) = \frac{1}{2}\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}\left(\frac{1}{2}\right) = \frac{1}{4}$$

13. If $f(x) = 5\ln(3x^3 + 18x)$, at how many different points on the graph does there exist a horizontal tangent line?

when $f'(x) = 0$

$\boxed{(A)\ 0}$ $\quad (B)\ 1 \quad (C)\ 2 \quad (D)\ 3 \quad (E)\ 4$

$$f(x) = 5\ln(3x^3 + 18x)$$

chain rule
deriv. of inside

$$f'(x) = 5\left(\frac{1}{3x^3+18x}\right)\overbrace{\left(9x^2+18\right)}$$

$$f'(x) = 5\left(\frac{9x^2+18}{3x^3+18x}\right)$$

$$f'(x) = 5\left(\frac{9(x^2+2)}{3x(x^2+6)}\right)$$

$$f'(x) = \frac{45(x^2+2)}{3x(x^2+6)} = 0$$

$$45(x^2+2) = 0$$

$$x^2+2 = 0$$

$$x^2 = -2$$

X Impossible

14. The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$ with radius r. Suppose the sphere expands as time passes. Which of the following gives $\frac{dV}{dt}$ in terms of $\frac{dr}{dt}$?

(A) $\frac{dV}{dt} = \frac{4}{3}\pi r^3 \frac{dr}{dt}$

(B) $\frac{dV}{dt} = 4\pi r^2$

$\boxed{(C)\ \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}}$

(D) $\frac{dV}{dt} = 4\pi \left(\frac{dr}{dt}\right)^2$

(E) None of the above

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \cancel{3}\left(\frac{4}{\cancel{3}}\right)\pi r^2 \frac{dr}{dt}$$

$$= 4\pi r^2 \frac{dr}{dt}$$

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Exam 2

Part II Instructions: 5 free response questions

For Instructor Use Only:

FR 1	
FR 2	
FR 3	
FR 4	
FR 5	
Total Points	

1. Complete both parts of the problem concerning the function $f(x) = 2x - 4\cos(x)$.

(a) How many tangent lines to the function $f(x)$ are horizontal in the interval $[0, 2\pi]$?

$f'(x) = 0$

$$f'(x) = 2 + 4\sin(x) = 0$$

So there are 2 horizontal tangent lines

$$2 + 4\sin(x) = 0$$
$$4\sin(x) = -2$$
$$\sin(x) = -\frac{2}{4}$$
$$\sin(x) = -\frac{1}{2} \rightarrow x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

(b) Write down the equations of these horizontal tangent lines.

Horiz. tangent for $x = \frac{7\pi}{6}$

$$f\left(\frac{7\pi}{6}\right) = 2\left(\frac{7\pi}{6}\right) - 4\cos\left(\frac{7\pi}{6}\right)$$
$$= \frac{7\pi}{3} - 4\left(-\frac{\sqrt{3}}{2}\right)$$
$$= \frac{7\pi}{3} + 2\sqrt{3}$$

$$y = \frac{7\pi}{3} + 2\sqrt{3}$$

Horiz tang. for $x = \frac{11\pi}{6}$

$$f\left(\frac{11\pi}{6}\right) = 2\left(\frac{11\pi}{6}\right) - 4\cos\left(\frac{11\pi}{6}\right)$$
$$= \frac{11\pi}{3} - 4\left(\frac{\sqrt{3}}{2}\right)$$
$$= \frac{11\pi}{3} - 2\sqrt{3}$$

$$y = \frac{11\pi}{3} - 2\sqrt{3}$$

2. Find an equation for the line tangent to the graph $y = \dfrac{(x^2+3x+1)e^x}{\cos(x)}$ at $x = 0$.

tangent line → point
→ slope

Point $\dfrac{(0^2+3(0)+1)\,e^0}{\cos(0)} = \dfrac{1(1)}{1} = 1$

$(0,1)$

Slope Find y' (need Quotient Rule: $\dfrac{gf' - fg'}{g^2}$

$f = (x^2+3x+1)e^x$ $\qquad g = \cos(x)$

$f' = (x^2+3x+1)e^x + (2x+3)e^x$ $\qquad g' = -\sin(x)$

$$y' = \frac{\cos(x)\left[(x^2+3x+1)(e^x) + (2x+3)(e^x)\right] - \left[(x^2+3x+1)e^x - (-\sin(x))\right]}{(\cos(x))^2}$$

Plug in $x=0$

$$y' = \frac{\cos(0)\left[((0^2+3(0)+1)e^0) + (2(0)+3)(e^0)\right] - \left[(0^2+3(0)+1)e^0(-\sin(0))\right]}{(\cos(0))^2}$$

$$y' = \frac{(1)\left[(1)(1) + (0+3)(1)\right] - ((1)(1)(0))}{1^2}$$

$$y' = \frac{(1+3)}{1} = \frac{4}{1} = 4$$

Use point-slope form

$y - y_1 = m(x - x_1)$

$(x_1, y_1) = (0, 1)$

$m = 4$

$y - 1 = 4(x - 0)$

$y - 1 = 4x$

$y = 4x + 1$

3. Find all points on the curve $x^2 - xy + y^2 = 3$ at which there is a vertical tangent line. Write your answer(s) in the form (x, y) for each coordinate pair.

slope undefined

$$\frac{d}{dx}(x^2 - xy + y^2) = \frac{d}{dx}(3)$$

$$2x - \underbrace{\left(x\frac{dy}{dx} + y\right)}_{\text{Prod. Rule}} + 2y\frac{dy}{dx} = 0$$

$$2x - x\frac{dy}{dx} - y + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} - x\frac{dy}{dx} = y - 2x$$

$$\frac{dy}{dx}(2y - x) = y - 2x$$

$$\frac{dy}{dx} = \frac{y - 2x}{2y - x}$$

undefined when $2y - x = 0 \rightarrow 2y = x$

$$(2y)^2 - (2y)y + y^2 = 3$$

$$4y^2 - 2y^2 + y^2 = 3$$

$$3y^2 = 3$$

$$y^2 = 1 \rightarrow y = \pm 1$$

$2y = x$

when $y=1 \rightarrow x = 2(1) = 2$

when $y=-1 \rightarrow x = 2(-1) = -2$

$(2,1)$ and $(-2,-1)$

4. Evaluate the following:

* Memorize *

- $\frac{d}{dx}\left(\sin^{-1}(x)\right), \left(\frac{d}{dx}(\arcsin(x))\right)$

$$= \frac{1}{\sqrt{1-x^2}}$$

- $\frac{d}{dx}\left(\csc^{-1}(x^2)\right), \left(\frac{d}{dx}(\operatorname{arccsc}(x^2))\right)$

$$(\operatorname{arccsc}(x))' = \frac{-1}{|x|\sqrt{x^2-1}}$$

$$= \frac{-1}{|x^2|\sqrt{x^4-1}} \cdot (2x) = \frac{-2}{x\sqrt{x^4-1}}$$

- $\frac{d}{dx}\left(\tan^{-1}(3x)\right), \left(\frac{d}{dx}(\arctan(3x))\right)$

$$(\arctan(x))' = \frac{1}{1+x^2}$$

$$= \frac{1}{1+(3x)^2} \cdot (3) = \frac{3}{1+9x^2}$$

- $\frac{d}{dx}\left(\cos^{-1}(2x+1)\right), \left(\frac{d}{dx}(\arccos(2x+1))\right)$

$$(\arccos(x))' = \frac{-1}{\sqrt{1-x^2}}$$

$$= \frac{-1}{\sqrt{1-(2x+1)^2}} \cdot (2) = \frac{-2}{\sqrt{1-(2x+1)^2}}$$

5. A stone is thrown upward from a 60 meter tall cliff so that its height above the ground is $h(t) = 60 + 4t - t^2$ for $t \geq 0$.

(a) When does the stone reach its highest point?

$$v(t) = h'(t) = 4 - 2t = 0$$
$$4 = 2t$$
$$t = 2$$

(b) When does the stone hit the ground?

$$h(t) = 0$$
$$60 + 4t - t^2 = 0$$
$$t^2 - 4t - 60 = 0$$
$$(t-10)(t+6) = 0$$

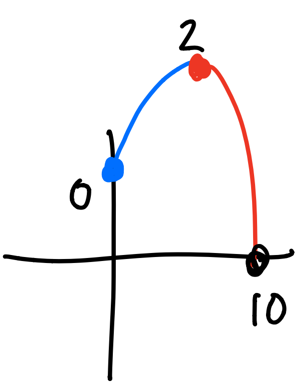
$t = 10$

$\cancel{t = -6}$ since $t \geq 0$

(c) What is the total vertical distance traveled by the stone from when it is thrown to when it hits the ground?

|highest pt - start| + |ground - highest pt|

$$\underbrace{|h(2) - h(0)|} + \underbrace{|h(10) - h(2)|}$$



$$|64 - 60| + |0 - 64|$$

$$4 + 64 = 68 \text{ m}$$

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Part IV Instructions: 2 free response questions

For Instructor Use Only:

FR 1	
FR 2	
Total Points	

1. (7 pts) Use logarithmic differentiation to find the derivative of $f(x) = \sqrt{\frac{x^2 \cos^3(x)}{e^x \sqrt{x}}}$.

$$\ln(y) = \ln\left(\sqrt{\frac{x^2\cos^3(x)}{e^x\sqrt{x}}}\right)$$

$$\ln(y) = \ln\left(\frac{x^2\cos^3(x)}{e^x\sqrt{x}}\right)^{1/2}$$

$$\ln(y) = \frac{1}{2}\ln\left(\frac{x^2\cos^3(x)}{e^x\sqrt{x}}\right)$$

$$\ln(y) = \frac{1}{2}\left[\ln\left(x^2\cos^3(x)\right) - \ln\left(e^x\sqrt{x}\right)\right]$$

$$\ln(y) = \frac{1}{2}\left[\ln(x^2) + \ln(\cos^3(x)) - \left(\ln(e^x) + \ln(\sqrt{x})\right)\right]$$

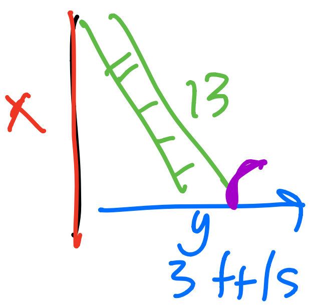
$$\frac{d}{dx}\ln(y) = \frac{d}{dx}\frac{1}{2}\left[2\ln(x) + 3\ln(\cos(x)) - x\ln(e) - \frac{1}{2}\ln(x)\right]$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left[\frac{2}{x} + \frac{-3\sin(x)}{\cos(x)} - 1 - \frac{1}{2x}\right]$$

$$\frac{dy}{dx} = \left(\frac{1}{x} - \frac{3}{2}\tan(x) - \frac{1}{2} - \frac{1}{4x}\right)\cdot y$$

$$\frac{dy}{dx} = \left(\frac{1}{x} - \frac{3}{2}\tan(x) - \frac{1}{2} - \frac{1}{4x}\right)\left(\sqrt{\frac{x^2\cos^3(x)}{e^x\sqrt{x}}}\right)$$

2. (7 pts) Suppose a 13 foot ladder rests against a wall. If the bottom of the ladder slides away from the wall at a rate of 3 feet per second, at what rate does the angle the ladder makes with the ground change when the top of the ladder is 5 feet from the ground?



$\frac{d\theta}{dt} = ?$ when $x = 5$ ft

SOH
CAH
TOA

$$\cos\theta = \frac{y}{13}$$

$$\sin\theta = \frac{x}{13} = \frac{5}{13}$$

$$-\sin\theta \frac{d\theta}{dt} = \frac{1}{13}\frac{dy}{dt}$$

$$-\sin\theta \frac{d\theta}{dt} = \frac{1}{13}(3)$$

$$-\left(\frac{5}{13}\right)\frac{d\theta}{dt} = \frac{3}{13}$$

$$\frac{d\theta}{dt} = -\frac{3}{5} \text{ rad/s}$$

We can't do:

$$\sin(\theta) = \frac{x}{13}$$

$$\cos\theta = \frac{1}{13}\frac{dx}{dt}$$

we don't know $\frac{dx}{dt}$

or $\cos\theta = \frac{y}{13}$ since we don't know y