It is your responsibility to ensure that your test has **19 questions**. If it does not, show it to your proctor immediately. You will not be permitted to make up any problems omitted from your test after the testing period ends. There are a total of 105 points available on this exam.

Part I Instructions: 14 multiple choice questions. Complete the scantron sheet provided with your information and fill in the appropriate spaces to answer your questions. Only the answer on the scantron sheet will be graded. Each problem is worth five (5) points for a total of 70 points on Part I.

$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$
$\ln(a \cdot b) = \ln(a) + \ln(b)$
$\ln(a^b) = b \cdot \ln(a)$

1. Find the slope of the tangent line to $f(x) = \ln\left(\frac{e^{2x}\sqrt{x+3}}{x^2+1}\right)$ at $x = 1$.

(A) $\frac{5}{4}$ (B) $-\frac{3}{8}$ (C) $\frac{7}{4}$ (D) $\frac{13}{8}$ (E) $\frac{9}{8}$

$f(x) = \ln(e^{2x} \cdot \sqrt{x+3}) - \ln(x^2+1)$
$f(x) = \ln(e^{2x}) + \ln(\sqrt{x+3}) - \ln(x^2+1)$
$f(x) = 2x \cdot \cancel{\ln(e)}^{1} + \frac{1}{2} \cdot \ln(x+3) - \ln(x^2+1)$
$f'(x) = 2 + \frac{1}{2} \cdot \frac{1}{x+3} \cdot 1 - \frac{1}{x^2+1} \cdot 2x$
$f'(1) = 2 + \frac{1}{2} \cdot \frac{1}{1+3} - \frac{1}{1^2+1} \cdot 2(1)$
$= 2 + \frac{1}{2}\left(\frac{1}{4}\right) - \frac{1}{2}(2)$
$= 2 + \frac{1}{8} - 1 = 1 + \frac{1}{8}$

$\sqrt{x+3} = (x+3)^{1/2}$

2. Suppose $f(x) = e^{2x} + 2x^3 - \sin(x)$. Calculate $f'''(1)$.

(A) $e^2 - \cos(1) + 12$ (B) $8e^2 + \cos(1) + 12$ (C) $8e^2 - \cos(1) + 12$ (D) $e^2 + \cos(1) + 12$

$f'(x) = e^{2x} \cdot 2 + 2 \cdot 3x^2 - \cos(x)$
$= 2e^{2x} + 6x^2 - \cos(x)$
$f''(x) = 2e^{2x} \cdot 2 + 6 \cdot 2x' - (-\sin(x))$
$= 4e^{2x} + 12x + \sin(x)$
$f'''(x) = 4e^{2x} \cdot 2 + 12 + \cos(x)$
$= 8e^{2x} + 12 + \cos(x)$
$f'''(1) = 8e^{2(1)} + 12 + \cos(1)$
$= 8e^2 + 12 + \cos(1)$

$\frac{d}{dx}\sin(x) = \cos(x)$

$\frac{d}{dx}\cos(x) = -\sin(x)$

$\frac{d}{dx} e^{a} = e^{a} \cdot a'$
$e^x \Rightarrow e^x \cdot 1$

$\cos(0) = 1$
$\cos^{-1}(1) = 0$

3. The vertical position of your jetpack-wearing Calculus instructor is given by $s(t) = t^3 - 9t^2 - 21t + 16$ for $t \geq 0$. On which of the following intervals are they slowing down? vel & acc to have opp. signs

(A) $(0, 3)$ (B) $(3, 7)$ (C) $(1, 5)$ (D) $(0, 1)$ (E) $(7, \infty)$

$v(t) = 3t^2 - 18t - 21$
$3(t^2 - 6t - 7) = 0$
$3(t-7)(t+1) = 0$
$t = -1, 7$

$v(t)$: 0 $- - -$ 3 $- - - - -$ 7 $+++$

$V(1) = 3 - 18 - 21 = -36$
$V(8) = 3(8-7)(8+1) = 3(1)(9) = 27$

$a(t) = 6t - 18$
$6(t-3) = 0$
$t = 3$

$a(t)$: 0 $- - -$ 3 $+++++$ 7 $+++$

$a(1) = 6 - 18 = -12$
$a(4) = 24 - 18 = 6$

$a(t) = v'(t) = 3 \cdot 2 \cdot t - 18$

4. Find the derivative of $f(x) = x^2 + \arccos(x + 1)$.

$2x$

(A) $f'(x) = \dfrac{1}{\sqrt{-x^2 - 2x}}$ (B) $f'(x) = 2x + \dfrac{1}{\sqrt{-x^2 - 2x}}$

(C) $f'(x) = 2x - \dfrac{1}{\sqrt{-x^2 - 2x}}$ (D) $2x - \dfrac{1}{\sqrt{x^2 + 2x}}$

$x' \cdot \dfrac{-1}{\sqrt{1 - x^2}} \Rightarrow \dfrac{-1}{\sqrt{1 - (x+1)^2}} = \dfrac{-1}{\sqrt{1 - x^2 - 2x - 1}} = \dfrac{-1}{\sqrt{-x^2 - 2x}}$

$(x+1)(x+1)$
$x^2 + 2x + 1$
$1 - (x^2 + 2x + 1)$

$\sin^{-1} = \dfrac{1}{\sqrt{1 - x^2}}$

$\arctan = \dfrac{1}{1 + x^2}$

$\text{arcsec} = \dfrac{1}{|x|\sqrt{x^2 - 1}}$

5. Let $f(x) = 4^x + \ln(x) + 4x^3$. what is the value of $f'(\frac{1}{2})$?

$b^x \Rightarrow \ln(b) \cdot b^x \cdot x'$

(A) 7 (B) $\ln(4) + 5$ (C) $4\ln(4) + 5$ (D) $2\ln(4) + 5$ (E) $2\ln(8) + 5$

$f'(x) = \ln(4) \cdot 4^x + \frac{1}{x} + 4 \cdot 3x^2$

$f'(\frac{1}{2}) = \ln(4) \cdot 4^{1/2} + \frac{1}{1/2} + 12(\frac{1}{2})^2$

$= \ln(4) \cdot 2 + 2 + 12(\frac{1}{4})$

$= 2\ln(4) + 2 + 3$

$= 2\ln(4) + 5$

$4^x \Rightarrow \ln(4) \cdot 4^x \cdot (1)$

$= \ln(4) \cdot 4^x$

6. Use implicit differentiation to find $\frac{dy}{dx}$ for $6x^3 + 7y^3 = 13xy$.

$f(x) = 13x$ $g(x) = y$

$f'(x) = 13$ $g'(x) = \frac{dy}{dx} = y'$

$f'(x) \cdot g(x) + g'(x) \cdot f(x)$

(A) $\frac{dy}{dx} = \frac{13y - 18x^2}{21y^2 - 13x}$ (B) $\frac{dy}{dx} = \frac{18y - 13x^2}{21y^2 - 13x}$

(C) $\frac{dy}{dx} = \frac{13y - 18x^2}{13y^2 - 21x}$ (D) $\frac{dy}{dx} = \frac{13y - 18x^2}{13y - 21x^2}$

$6 \cdot 3x^2 + 7 \cdot 3y^2 \cdot \frac{dy}{dx} = 13y + 13x \cdot \frac{dy}{dx}$

$18x^2 + 21y^2 y' = 13y + 13xy'$

$-13xy'$ $-13xy'$

$18x^2 - 13xy' + 21y^2y' = 13y$

$-18x^2$ $-18x^2$

$-13xy' + 21y^2y' = 13y - 18x^2$

$y'(21y^2 - 13x) = 13y - 18x^2$

$y' = \frac{13y - 18x^2}{21y^2 - 13x} \cdot \frac{-1}{-1}$

$\frac{d}{dx}(y) = y'$

$y^3 \rightarrow 3y^2 \cdot \frac{dy}{dx}$

7. Let $f(1) = 2$ and $g(x) = \dfrac{f(x) - 2}{f(x) + 1}$. If $g'(1) = 2$, then which of the following is equal to $f'(1)$?

$(A)\ -\dfrac{2}{3}$ $(B)\ 4$ $(C)\ 3$ $(D)\ -3$ $(E)\ 6$

$a = f(x) - 2$ $b = f(x) + 1$ $\frac{a}{b}$

$a' = f'(x)$ $b' = f'(x)$

$$\frac{a' \cdot b - b' \cdot a}{b^2} = \frac{f'(x)(f(x)+1) - f'(x)(f(x)-2)}{(f(x)+1)^2} = g'(x)$$

$$g'(1) = \frac{f'(1)(f(1)+1) - f'(1)(f(1)-2)}{(f(1)+1)^2} = 2$$

$$2 = \frac{f'(1)(2+1) - f'(1)(2-2)}{(2+1)^2}$$

$$2 = \frac{f'(1)(3) - 0}{3^2} \Rightarrow 2 = \frac{3f'(1)}{9} \Rightarrow 18 = 3f'(1)$$

$$f'(1) = 6$$

8. Assume that $f(x)$ and $g(x)$ are differentiable functions such that

$$f'(x) = -g(x) \text{ and } g'(x) = -f(x).$$

Let $h(x) = (f(x))^2 - (g(x))^2$. Which of the following is equal to $h'(x)$?

$(A)\ 4f(x)g(x)$ $(B)\ 0$ $(C)\ 4f(x)f'(x)$ $(D)\ 2g(x)f(x)$ $(E)\ 2g'(x)f'(x) + (f(x))^2$

$$h'(x) = 2f(x) \cdot f'(x) - 2g(x) \cdot g'(x)$$

$$h'(x) = 2f(x) \cdot (-g(x)) - 2g(x) \cdot (-f(x))$$

$$= -2f(x)g(x) + 2f(x)g(x)$$

$$= 0$$

$$g(x^2) \Rightarrow g'(x^2) \cdot 2x$$

$$(g(x^2))^2 \Rightarrow 2 \cdot g(x^2) \cdot g'(x^2) \cdot 2x$$

9. How many of the following statements are necessarily true?

- $\frac{d}{dx}[f(x)g(x)] = \frac{d}{dx}[f(x)] \cdot \frac{d}{dx}[g(x)]$
- $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x) \cdot \frac{d}{dx}[f(x)] - f(x) \cdot \frac{d}{dx}[g(x)]}{[g(x)]^2}$
- $\frac{d}{dx}[f(x)g(x)] = \frac{d}{dx}[f(x)] \cdot g(x) + f(x) \cdot \frac{d}{dx}[g(x)]$ $= f(x) \cdot \frac{d}{dx} g(x) + \frac{d}{dx} f(x) \cdot g(x)$
- $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{\frac{d}{dx}[f(x)]}{\frac{d}{dx}[g(x)]}$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

10. Let $f(x) = \sin(x) + \cos(x) + \tan(x) + \csc(x)$. What is the value of $f'(\frac{\pi}{4})$?

(A) $2 - 2\sqrt{2}$ (B) $\sqrt{2} - 2$ (C) $2 - \sqrt{2}$ (D) $2 + \sqrt{2}$ (E) $2 + 2\sqrt{2}$

$f'(x) = \cos(x) - \sin(x) + \sec^2(x) - \csc(x) \cdot \cot(x)$

$f'(\frac{\pi}{4}) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} + \left(\frac{2}{\sqrt{2}}\right)^2 - \frac{2}{\sqrt{2}}(1)$

$= 0 + \frac{4}{2} - \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= 2 - \frac{\cancel{2}\sqrt{2}}{\cancel{2}}$

$= 2 - \sqrt{2}$

$\sin(x) \Rightarrow \cos(x)$: $\cos(x) = -\sin(x)$

$\tan(x) \Rightarrow \sec^2(x)$: $\cot(x) = -\csc^2(x)$

$\sec(x) \Rightarrow \sec\tan$: $\csc \Rightarrow -\csc\cot$

$\frac{11\pi}{4} - 2\pi$

$\frac{11\pi}{4} - \frac{8\pi}{4} = \frac{3\pi}{4}$

$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

11. Which of the following is an equation for the tangent line to the graph of $f(x) = \arctan(2x)$ at $x = 0$?

(A) $y = x$ (B) $y = x + 1$ (C) $y = 2x$ (D) $y = x - 1$ (E) $y = 2x - 1$

$f'(x) = \frac{1}{(2x)^2+1} \cdot 2$

$f'(x) = \frac{1}{4x^2+1} \cdot 2$

$f'(0) = \frac{1}{4(0)+1} \cdot 2 = 2$ = slope

$y - y_1 = f'(x)(x - x_1)$

$f(0) = \tan^{-1}(0) = 0$

$y - 0 = 2(x-0)$

$y = 2x$

$\tan^{-1}(1) = \frac{\pi}{4}$

$\tan^{-1}(\sqrt{3}) = \frac{\sin}{\cos}$ $\frac{\frac{\sqrt{3}}{2}}{1/2}$ $= \frac{\pi}{3}$

12. Consider $f(x) = x^3 \ln(x) + \dfrac{x}{x^2+1}$. What is $f'(1)$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

$f(x) = x^3$ $g(x) = \ln(x)$ $f(x) = x$ $g(x) = x^2+1$

$f'(x) = 3x^2$ $g'(x) = \frac{1}{x}$ $f'(x) = 1$ $g'(x) = 2x$

$f'(x) = 3x^2 \cdot \ln(x) + \frac{x^3}{x} + \frac{(x^2+1)\cdot 1 - x(2x)}{(x^2+1)^2}$

$f'(1) = 3(1) \cdot \cancel{\ln(1)} \rightarrow 0 + \frac{1}{1} + \frac{(1+1) - 1(2)}{(1+1)^2}$

$= 1 + \frac{2-2}{2^2}$

$= 1 + \frac{0}{4}$

$f(x) \cdot g'(x) = x^3 \cdot \frac{1}{x} = \frac{x^3}{x}$

13. If $g(x) = x^{1/5}(x-1)^{3/5}$, find the domain of $g'(x)$.

(A) $(-\infty, 2) \cup (2, 3) \cup (3, \infty)$

(B) $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

(C) $(-\infty, \infty)$

(D) $(0, \infty)$

$f(x) = x^{1/5}$

$f'(x) = \frac{1}{5} x^{-4/5}$

$= \frac{1}{5} \cdot \frac{1}{x^{4/5}}$

$h(x) = (x-1)^{3/5}$

$h'(x) = \frac{3}{5}(x-1)^{-2/5} \cdot 1$

$= \frac{3}{5} \cdot \frac{1}{(x-1)^{2/5}}$

$f'(x)h(x) + h'(x)f(x)$

$g'(x) = \frac{1}{5(x^{4/5})}(x-1)^{3/5} + \frac{3}{5(x-1)^{2/5}} x^{1/5}$

$g'(x) = \frac{(x-1)^{3/5}}{5x^{4/5}} + \frac{3x^{1/5}}{5(x-1)^{2/5}}$

$5 \cdot x^{4/5} = 0$

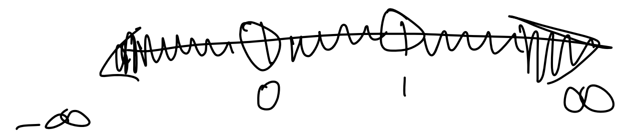
$x \neq 0$

$5(x-1)^{2/5} = 0$

$(x-1)^{2/5} = 0$

$x - 1 = 0$

$x \neq 1$



14. If $h(x) = [f(x)g(x) + x]^2$, what is $h'(0)$ if $f(0) = 2$, $g(0) = 1$, $f'(0) = 0$, and $g'(0) = -1$?

(A) -2 (B) -4 (C) 6 (D) -3 (E) 4

$h'(x) = 2(f(x)g(x) + x)[f'(x)g(x) + f(x)g'(x) + 1]$

$h'(0) = 2(f(0)g(0) + 0)(f'(0)g(0) + f(0)g'(0) + 1]$

$= 2[2(1)][0(1) + 2(-1) + 1]$

$= 2(2)(0 - 2 + 1)$

$= 4(-1)$

$= -4$

1. Jacques Monod modeled the per capita growth rate R of *Escherichia coli* bacteria by the function

$$R(N) = \frac{N}{3+N}$$

where N is the concentration of the nutrient.

(a) Determine $\frac{dR}{dN}$.

$f(N) = N$ $\quad$ $g(N) = 3+N$

$f'(N) = 1$ $\quad$ $g'(N) = 1$

$$\frac{dR}{dN} = \frac{(3+N)(1) - N(1)}{(3+N)^2}$$

$$= \frac{3+N-N}{(3+N)^2}$$

$$= \frac{3}{(3+N)^2}$$

(b) Calculate $\frac{dR}{dN}$ at $N = 8$. Using this answer, is the growth rate rising or falling at $N = 8$? Explain why.

$$R'(8) = \frac{3}{(3+8)^2}$$

$$= \frac{3}{11^2} = \frac{3}{121}$$

since $\frac{dR}{dN}$ @ $N=8$ is pos., the growth rate is rising @ $N=8$.

2. The vertical position of a yo-yo at time t, for $0 \le t \le \pi$, is given by

$$s(t) = \sqrt{3}\sin(t) - \cos(t).$$

(a) What is the displacement of the yo-yo from $t = 0$ to $t = \frac{\pi}{2}$?

$s(\frac{\pi}{2}) - s(0)$

$\sqrt{3}(1) - 0 - (\sqrt{3}(0) - 1)$

$\sqrt{3} + 1$

(b) What is the total distance traveled by the yo-yo from $t = 0$ to $t = \pi$?

$|s(\pi) - s(0)|$

$|s(\pi) - s(\frac{2\pi}{3})| + |s(\frac{2\pi}{3}) - s(0)|$

$s(\pi): \sqrt{3}\sin(\pi) - \cos(\pi)$

$= \sqrt{3}(0) - (-1)$

$= 1$

$s(\frac{2\pi}{3}): \sqrt{3}\sin(\frac{2\pi}{3}) - \cos(\frac{2\pi}{3})$

$= \sqrt{3}\cdot\frac{\sqrt{3}}{2} - (-\frac{1}{2})$

$= \frac{3}{2} + \frac{1}{2} = \frac{4}{2} = 2$

$s(0) = \sqrt{3}\cdot\sin(0) - \cos(0)$

$= 0 - 1$

$= -1$

$|1 - 2| + |2 + 1|$

$|-1| + 3 = 4$

$s(t) = \sqrt{3}\sin(t) - \cos(t).$

$v(t) = \sqrt{3}\cos(t) + \sin(t)$

$0 = \sqrt{3}\cos(t) + \sin(t)$

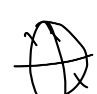
$-\sqrt{3}\cos(t) = \sin(t)$

$-\sqrt{3} = \frac{\sin(t)}{\cos(t)}$

$-\sqrt{3} = \tan(t)$

$\frac{\pi}{3}, \boxed{\frac{2\pi}{3}}$

$\frac{5\pi}{3}$ $\frac{4\pi}{3}$



$(\frac{\sqrt{3}}{2}, \frac{1}{2})$ or $(\frac{1}{2}, \frac{\sqrt{3}}{2})$

$\frac{\pi}{6}$ $\frac{\pi}{3}$

$\frac{\frac{\sqrt{3}}{2}}{1/2} = \frac{\sqrt{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{1}$

3. Let $f(x) = x^{\sin(x)}$. Calculate $f'(x)$.

$$\ln(y) = \ln(x^{\sin(x)})$$

$$\ln(y) = \sin(x) \cdot \ln(x)$$

$$y \cdot \frac{1}{y} \cdot y' = \left(\cos(x) \cdot \ln(x) + \sin(x) \cdot \frac{1}{x} \right) \cdot y$$

$$f'(x) = \left[\cos(x) \cdot \ln(x) + \frac{\sin(x)}{x} \right] \cdot x^{\sin(x)}$$

4. Let $x^4 - x^2y + y^4 = 1$

(a) Find the slope of the tangent line at the point $(-1, 1)$.

$4x^3 - (2xy + x^2y') + 4y^3y' = 0$

$4x^3 - 2xy - x^2y' + 4y^3y' = 0$

$-x^2y' + 4y^3y' = 2xy - 4x^3$

$y'(-x^2 + 4y^3) = 2xy - 4x^3$

$y' = \frac{2xy - 4x^3}{4y^3 - x^2}$

$@(-1,1):\ y' = \frac{2(-1)(1) - 4(-1)^3}{4(1)^3 - (-1)^2} = \frac{-2+4}{4-1} = \frac{2}{3}$

(b) Write an equation for the tangent line to the curve at the point $(-1, 1)$.

$y - 1 = \frac{2}{3}(x - -1)$

$y = \frac{2}{3}x + \frac{2}{3} + 1$

$y = \frac{2}{3}x + \frac{5}{3}$

5. The table below gives the values of $f(x)$, $f'(x)$, $g(x)$, and $g'(x)$ for various values of x. Use the table to answer the following questions.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	0	4	2	3
2	1	−1	2	−2
3	6	5	4	3

(a) Let $A(x) = \ln(g(x))$. What is $A'(2)$?

$$A'(x) = \frac{1}{g(x)} \cdot g'(x)$$

$$A'(2) = \frac{1}{g(2)} \cdot g'(2)$$

$$= \frac{1}{2} \cdot -2 = -1$$

(b) Let $B(x) = f(x)(g(x))^2$. What is $B'(3)$?

$$B'(x) = f'(x)(g(x))^2 + f(x) \cdot 2g(x) \cdot g'(x)$$

$$B'(3) = f'(3) \cdot (g(3))^2 + f(3) \cdot 2g(3) \cdot g'(3)$$

$$= 5 \cdot (4)^2 + 6 \cdot 2 \cdot 4 \cdot 3$$

$$= 5(16) + 12 \cdot 12$$

$$= 80 + 144$$

$$= 224$$

(c) Let $C(x) = \dfrac{(f(x))^3}{g(x)}$. What is $C'(2)$?

top $= (f(x))^3$

top' $= 3(f(x))^2 \cdot f'(x)$

bot $= g(x)$

bot' $= g'(x)$

$$C'(x) = \frac{3(f(x))^2 \cdot f'(x) \cdot g(x) - (f(x))^3 \cdot g'(x)}{(g(x))^2}$$

$$C'(2) = \frac{3(f(2))^2 \cdot f'(2) \cdot g(2) - (f(2))^3 \cdot g'(2)}{(g(2))^2}$$

$$= \frac{3(1)^2 \cdot (-1) \cdot 2 - (1)^3 \cdot (-2)}{2^2}$$

$$= \frac{3(-2) - (-2)}{4}$$

$$= \frac{-6+2}{4}$$

$$= \frac{-4}{4}$$

$$= 1$$

14. The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$ with radius r. Suppose the sphere expands as time passes. Which of the following gives $\frac{dV}{dt}$ in terms of $\frac{dr}{dt}$?

(A) $\frac{dV}{dt} = \frac{4}{3}\pi r^3 \frac{dr}{dt}$

(B) $\frac{dV}{dt} = 4\pi r^2$

(C) $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

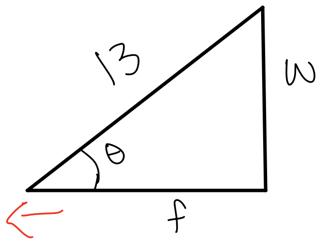
(D) $\frac{dV}{dt} = 4\pi \left(\frac{dr}{dt}\right)^2$

(E) None of the above

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \cdot \frac{dr}{dt}$$

$$= 4\pi r^2 \cdot \frac{dr}{dt}$$

2. (7 pts) Suppose a 13 foot ladder rests against a wall. If the bottom of the ladder slides away from the wall at a rate of 3 feet per second, at what rate does the angle the ladder makes with the ground change when the top of the ladder is 5 feet from the ground?



$\cos(\theta) = \frac{f}{13} \Rightarrow \frac{1}{13} \cdot f \Rightarrow \frac{1}{13} \cdot \frac{df}{dt}$

$-\sin(\theta) \cdot \theta' = \frac{f'}{13}$

$-\frac{w}{13} \cdot \theta' = \frac{f'}{13}$

$f' = 3$

$\theta' = ?$

$w = 5$

$\frac{-13}{5} \cdot \frac{-5}{13} \theta' = \frac{3}{13} \cdot \frac{-13}{5}$

$\theta' = \frac{-3}{5}$ rad/sec

12. The demand function for a certain product is given by
$p(x) = -0.02x + 400$, $0 \le x \le 20,000$, where p is the unit price when x items are sold. The cost function for the product is
$C(x) = 100x + 300,000$.

(a) Find the marginal profit of the product when $x = 2000$.

revenue $= x \cdot p(x)$
$= x(-0.02x + 400)$
$= -0.02x^2 + 400x$

profit = revenue − cost

$f(x) = -0.02x^2 + 400x - (100x + 300,000)$

$f(x) = -0.02x^2 + 400x - 100x - 300,000$

$= -0.02x^2 + 300x - 300,000$

$f'(x) = -0.04x + 300$

$f'(2000) = \frac{-4}{1\cancel{00}} \cdot 20\cancel{00} + 300 = -80 + 300 = 220$