Calculus I: MAC2311 **Name:** _____

Exam 2

Part I Instructions: 14 multiple choice questions

1. Find $f''(x)$ for the function $f(x) = \dfrac{x}{e^x}$

(A) $\dfrac{-x+2}{e^x}$ (B) $\dfrac{x+2}{e^x}$ (C) $\dfrac{-x-2}{e^x}$ (D) $\dfrac{x-2}{e^x}$

2. Suppose $g(x)$ is differentiable for any real number x. Let $f(x) = \dfrac{e^{g(x)}}{x^2+3}$. Find $f'(x)$.

(A) $\dfrac{2xe^{g'(x)} - (x^2+3)e^{g(x)}g'(x)}{(x^2+3)^2}$ (B) $\dfrac{2xe^{g(x)} - 2xe^{g(x)}g'(x)}{(x^2+3)^2}$ (C) $\dfrac{2xe^{g(x)} - (x^2+3)e^{g(x)}g'(x)}{(x^2+3)^2}$

(D) $\dfrac{2xe^{g(x)}g'(x) - 2xe^{g(x)}}{(x^2+3)^2}$ (E) $\dfrac{(x^2+3)e^{g(x)}g'(x) - 2xe^{g(x)}}{(x^2+3)^2}$

3. Determine the derivative of $f(x) = e^{\cos^2(x)}$.

(A) $e^{-2\sin(x)\cos(x)}$ (B) $-2e^{\cos^2(x)}\sin(x)$ (C) $2e^{\cos^2(x)}\cos(x)$

(D) $-2e^{\cos^2(x)}\cos(x)\sin(x)$ (E) $e^{-2\sin(x)\cos(x)}\sin(x)$

4. If $f(x) = \cot^{-1}(x)$ ($f(x) = \text{arccot}(x)$), then find an expression for $f''(x)$.

(A) $f''(x) = \dfrac{2x}{(1+x^2)^2}$ (B) $f''(x) = \dfrac{-2x}{(1+x^2)^2}$

(C) $f''(x) = \dfrac{1-2x+x^2}{(1+x^2)^2}$ (D) $f''(x) = \dfrac{-x^2+2x-1}{(1+x^2)^2}$

5. Use implicit differentiation to find y' for the equation $xy = \sin(y)$.

(A) $y' = \dfrac{-y}{x-\cos(y)}$ (B) $y' = \dfrac{y}{x-\cos(y)}$ (C) $y' = \dfrac{x-\cos(y)}{-y}$ (D) $y' = \dfrac{x-\cos(y)}{y}$

6. Which of the following is equal to the derivative of $g(x) = 3x^2 + \sqrt[3]{x^2} + 4$

(A) $g'(x) = 6x + \dfrac{1}{3\sqrt[3]{x}}$ (B) $g'(x) = 3x + \dfrac{1}{3\sqrt[3]{x}}$ (C) $g'(x) = 3x + \dfrac{2}{3\sqrt[3]{x}} + 4$ (D) $g'(x) = 6x + \dfrac{2}{3\sqrt[3]{x}}$

7. Let $f(x) = (1 + \sqrt{x})^{\frac{1}{3}}$. What is $f'(x)$?

(A) $\dfrac{1}{3}\left(1 + x^{\frac{1}{2}}\right)^{-\frac{2}{3}}$ (B) $\dfrac{1}{6}x^{-\frac{1}{2}}\left(1 + x^{\frac{1}{2}}\right)^{-\frac{2}{3}}$ (C) $\dfrac{1}{3}\left(1 + \frac{1}{2}x^{-\frac{1}{2}}\right)^{-\frac{2}{3}}$ (D) $\dfrac{1}{3}\left(\frac{1}{2}x^{-\frac{1}{2}}\right)^{-\frac{2}{3}}$ (E) $\dfrac{1}{2}x^{-\frac{1}{2}}\left(1 + x^{\frac{1}{2}}\right)^{-\frac{2}{3}}$

8. What is the slope of the line tangent to the graph of $y = 2^x - 3^x + 4^x$ when $x = 0$?

(A) $\ln\left(\dfrac{3}{2}\right)$ (B) $\ln\left(\dfrac{8}{3}\right)$ (C) 1 (D) $\ln(24)$

9. What is an equation for the line tangent to the function $f(x) = 6x\sin(x) + \pi$ at $x = \dfrac{\pi}{2}$?

(A) $y = 6x$ (B) $y = 6x + \pi$ (C) $y = 6x - \pi$ (D) $y = 6x + 6$ (E) $y = 6x - 6$

10. Let $f(x)$ and $g(x)$ be differentiable functions such that $g(3) = 1$. Which of the following is equal to $h'(3)$ where $h(x) = f(x)g(x) + \dfrac{f(x)}{g(x)}$?

(A) 2 (B) $2f(3)$ (C) $2f'(3)$ (D) $2g'(3)$ (E) None of these

11. The mass of a length of wire is $m(x) = x(1 + 2\sqrt{x})$ kilograms, where x is the length of the wire measured in meters. The **linear density** of the wire is the rate of change of the mass m with respect to x. Find the linear density of the wire (expressed in kg/m) when $x = 4$ m.

(A) 1 (B) 4 (C) 7 (D) 10

12. Let $h(x) = f(g(x))$. Based on the following table of values for $f(x)$, $f'(x)$, $g(x)$, and $g'(x)$, find $h'(2)$.

x	0	1	2	3
$f(x)$	0	2	1	1
$f'(x)$	2	3	1	2
$g(x)$	2	1	3	3
$g'(x)$	0	1	3	2

(A) 0 (B) 1 (C) 2 (D) 3 (E) 6

13. Let $f(x) = \sec^2(x)$. What is $f'(x)$?

(A) $\dfrac{1}{\sin^2(x)}$ (B) $-2\sec^3(x)$ (C) $2\sec(x)$ (D) $2\sec^2(x)\tan(x)$ (E) None of these

14. For which of the following x-values does the curve $x(x-1) = y^3$ have a vertical tangent line?

(A) $x = 0$ only (B) $x = 1$ only (C) $x = 0, 1$ only (D) $x = 0, 1, -1$ (E) No such value

Calculus I: MAC2311 **Name:** _____

Exam 2

Part II Instructions: 5 free response questions

For Instructor Use Only:

FR 1	
FR 2	
FR 3	
FR 4	
FR 5	
Total Points	

1. Complete both parts of the problem concerning the function $f(x) = 2x - 4\cos(x)$.

   (a) How many tangent lines to the function $f(x)$ are horizontal in the interval $[0, 2\pi]$?

   (b) Write down the equations of these horizontal tangent lines.

2. Find an equation for the line tangent to the graph $y = \dfrac{(x^2 + 3x + 1)e^x}{\cos(x)}$ at $x = 0$.

3. Find all points on the curve $x^2 - xy + y^2 = 3$ at which there is a vertical tangent line. Write your answer(s) in the form (x, y) for each coordinate pair.

4. Evaluate the following:

- $\frac{d}{dx}\left(\sin^{-1}(x)\right)$, $\left(\frac{d}{dx}(\arcsin(x))\right)$
- $\frac{d}{dx}\left(\csc^{-1}(x^2)\right)$, $\left(\frac{d}{dx}(\operatorname{arccsc}(x^2))\right)$
- $\frac{d}{dx}\left(\tan^{-1}(3x)\right)$, $\left(\frac{d}{dx}(\arctan(3x))\right)$
- $\frac{d}{dx}\left(\cos^{-1}(2x+1)\right)$, $\left(\frac{d}{dx}(\arccos(2x+1))\right)$

5. A stone is thrown upward from a 60 meter tall cliff so that its height above the ground is $h(t) = 60 + 4t - t^2$ for $t \geq 0$.

(a) When does the stone reach its highest point?

(b) When does the stone hit the ground?

(c) What is the total vertical distance traveled by the stone from when it is thrown to when it hits the ground?

Calculus I: MAC2311 **Name:** _____

Part III Instructions: 6 multiple choice questions

1. The linearization of $f(x) = \sqrt[3]{1+3x}$ at $a = 0$ is given by $L(x) = x + 1$. Use this to approximate $\sqrt[3]{1.03}$.

(A) 2.03 (B) 1.01 (C) 1.03 (D) 1.003 (E) None of the above

2. Suppose that $A = B + C$ and A, B, and C are functions of t. If $\frac{dB}{dt} = 3$ and $\frac{dC}{dt} = -4$ what is $\frac{dA}{dt}$?

(A) 0 (B) 1 (C) 7 (D) -1 (E) None of the above

3. The elevation h (in feet above the ground) of a stone dropped from a height of 1000 ft is modeled by the equation $h(t) = 1000 - 16t^2$, where t is measured in seconds and air resistance is neglected. Use differentials to approximate the change in elevation over the interval $3 \leq t \leq 3.1$ seconds.

(A) $\Delta h \approx -9.6$ ft $\quad (B)$ $\Delta h \approx -4.8$ ft $\quad (C)$ $\Delta h \approx -846.4$ ft $\quad (D)$ $\Delta h \approx -118$ ft $\quad (E)$ None of the above

7. If $f(x) = \ln(\sqrt{e^{\sin(x)}})$, then $f'(\frac{\pi}{3})$ equals:

(A) $\dfrac{1}{4}$ $\qquad (B)$ $\dfrac{\sqrt{2}}{\ln(\frac{\pi}{3})}$ $\qquad (C)$ $\dfrac{\sqrt{3}}{2}$ $\qquad (D)$ $\dfrac{1}{2}\ln(e^{\pi})$ $\qquad (E)$ $\dfrac{\sqrt{2}}{4}$

13. If $f(x) = 5\ln(3x^3 + 18x)$, at how many different points on the graph does there exist a horizontal tangent line?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

14. The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$ with radius r. Suppose the sphere expands as time passes. Which of the following gives $\frac{dV}{dt}$ in terms of $\frac{dr}{dt}$?

(A) $\frac{dV}{dt} = \frac{4}{3}\pi r^3 \frac{dr}{dt}$ (B) $\frac{dV}{dt} = 4\pi r^2$ (C) $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

(D) $\frac{dV}{dt} = 4\pi \left(\frac{dr}{dt}\right)^2$ (E) None of the above

Calculus I: MAC2311 **Name:** _____

Part IV Instructions: 2 free response questions

For Instructor Use Only:

FR 1	
FR 2	
Total Points	

1. (7 pts) Use logarithmic differentiation to find the derivative of $f(x) = \sqrt{\dfrac{x^2 \cos^3(x)}{e^x \sqrt{x}}}$.

2. (7 pts) Suppose a 13 foot ladder rests against a wall. If the bottom of the ladder slides away from the wall at a rate of 3 feet per second, at what rate does the angle the ladder makes with the ground change when the top of the ladder is 5 feet from the ground?