**Calculus I: MAC2311**
**Fall 2022**
**Exam 2 A**
**10/19/2022**
**Time Limit: 90 Minutes**

**Name:** Solution

**Section:** ____________________

**UF-ID:** ____________________

**Scantron Instruction**: This exam uses a scantron. Follow the instructions listed on this page to fill out the scantron.

A. Sign your scantron **on the back** at the bottom in the white area.

B. Write **and code** in the spaces indicated:

1) Name (last name, first initial, middle initial)
2) UFID Number
3) 4-digit Section Number

C. Under *special codes*, code in the test numbers 2, 1:

1 ● 3 4 5 6 7 8 9 0
● 2 3 4 5 6 7 8 9 0

D. At the top right of your scantron, fill in the *Test Form Code* as A.

● B C D E

E. This exam consists of 14 multiple choice questions and 5 free response questions. Make sure you check for errors in the number of questions your exam contains.

F. The time allowed is 90 minutes.

G. WHEN YOU ARE FINISHED:

1) Before turning in your test check for **transcribing errors**. Any mistakes you leave in are there to stay!

2) You must turn in your scantron to your proctor. **Be prepared to show your GatorID with a legible signature.**

It is your responsibility to ensure that your test has **19 questions**. If it does not, show it to your proctor immediately. You will not be permitted to make up any problems omitted from your test after the testing period ends. There are a total of 105 points available on this exam.

Part I Instructions: 14 multiple choice questions. Complete the scantron sheet provided with your information and fill in the appropriate spaces to answer your questions. Only the answer on the scantron sheet will be graded. Each problem is worth five (5) points for a total of 70 points on Part I.

1. Find the slope of the tangent line to $f(x) = \ln\left(\frac{e^{2x}\sqrt{x+3}}{x^2+1}\right)$ at $x = 1$.

(A) $\frac{5}{4}$ (B) $-\frac{3}{8}$ (C) $\frac{7}{4}$ (D) $\frac{13}{8}$ [✓ (E) $\frac{9}{8}$]

$f(x) = \ln(e^{2x}) + \ln(x+3)^{1/2} - \ln(x^2+1)$

$f(x) = 2x + \frac{1}{2}\ln(x+3) - \ln(x^2+1)$

$f'(x) = 2 + \frac{1}{2(x+3)} - \frac{2x}{x^2+1}$

Slope at $x=1$ : $f'(1) = 2 + \frac{1}{8} - \frac{2}{2} = \boxed{\frac{9}{8}}$

2. Suppose $f(x) = e^{2x} + 2x^3 - \sin(x)$. Calculate $f'''(1)$.

(A) $e^2 - \cos(1) + 12$ [✓ (B) $8e^2 + \cos(1) + 12$] (C) $8e^2 - \cos(1) + 12$ (D) $e^2 + \cos(1) + 12$

$f'(x) = 2e^{2x} + 6x^2 - \cos(x)$

$f''(x) = 4e^{2x} + 12x + \sin(x)$

$f'''(x) = 8e^{2x} + 12 + \cos(x)$

$f'''(1) = 8e^2 + 12 + \cos(1)$

3. The vertical position of your jetpack-wearing Calculus instructor is given by $s(t) = t^3 - 9t^2 - 21t + 16$ for $t \geq 0$. On which of the following intervals are they slowing down?

(A) $(0, 3)$ [✓ (B) $(3, 7)$] (C) $(1, 5)$ (D) $(0, 1)$ (E) $(7, \infty)$

$v(t) = 3t^2 - 18t - 21$

Let $0 = 3t^2 - 18t - 21$

$0 = t^2 - 6t - 7$

$0 = (t-7)(t+1)$

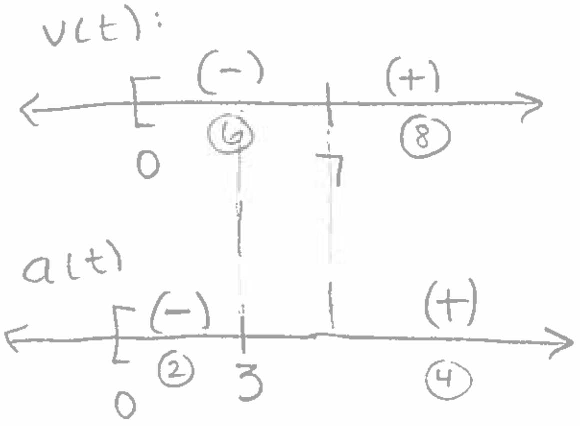
$t = 7$, ~~$t = -1$~~

$a(t) = 6t - 18$

Let $0 = 6t - 18$

$18 = 6t$

$t = 3$



We need v(t) and a(t) to have opposite signs. This occurs on (3,7)

4. Find the derivative of $f(x) = x^2 + \arccos(x+1)$.

(A) $f'(x) = \frac{1}{\sqrt{-x^2 - 2x}}$ (B) $f'(x) = 2x + \frac{1}{\sqrt{-x^2 - 2x}}$

[✓ (C) $f'(x) = 2x - \frac{1}{\sqrt{-x^2 - 2x}}$] (D) $2x - \frac{1}{\sqrt{x^2 + 2x}}$

$$f'(x) = 2x - \frac{1}{\sqrt{1-(x+1)^2}} = 2x - \frac{1}{\sqrt{1-(x^2+2x+1)}}$$

$$= 2x - \frac{1}{\sqrt{1-x^2-2x-1}}$$

$$= 2x - \frac{1}{\sqrt{-x^2-2x}}$$

5. Let $f(x) = 4^x + \ln(x) + 4x^3$. what is the value of $f'(\frac{1}{2})$?

(A) 7 (B) $\ln(4) + 5$ (C) $4\ln(4) + 5$ [✓ (D) $2\ln(4) + 5$] (E) $2\ln(8) + 5$

$$f'(x) = 4^x \ln(4) + \frac{1}{x} + 12x^2$$

$$f'(\tfrac{1}{2}) = 4^{\frac{1}{2}} \ln(4) + \frac{1}{(\frac{1}{2})} + 12\left(\tfrac{1}{2}\right)^2$$

$$f'(\tfrac{1}{2}) = 2\ln(4) + 2 + 12\left(\tfrac{1}{4}\right)$$

$$f'(\tfrac{1}{2}) = 2\ln(4) + 5$$

6. Use implicit differentiation to find $\frac{dy}{dx}$ for $6x^3 + 7y^3 = 13xy$.

[✓ (A) $\frac{dy}{dx} = \frac{13y - 18x^2}{21y^2 - 13x}$]

(B) $\frac{dy}{dx} = \frac{18y - 13x^2}{21y^2 - 13x}$

(C) $\frac{dy}{dx} = \frac{13y - 18x^2}{13y^2 - 21x}$

(D) $\frac{dy}{dx} = \frac{13y - 18x^2}{13y - 21x^2}$

$$\frac{d}{dx}(6x^3 + 7y^3) = \frac{d}{dx}(13xy)$$

$$18x^2 + 21y^2 \cdot y' = 13(y + xy')$$

$$18x^2 + 21y^2 \cdot y' = 13y + 13xy'$$

$$21y^2 \cdot y' - 13xy' = 13y - 18x^2$$

$$\boxed{y' = \frac{13y - 18x^2}{21y^2 - 13x}}$$

7. Let $f(1) = 2$ and $g(x) = \frac{f(x) - 2}{f(x) + 1}$. If $g'(1) = 2$, then which of the following is equal to $f'(1)$?

(A) $-\frac{2}{3}$ (B) 4 (C) 3 (D) $-3$ [✓ (E) 6]

$$g'(x) = \frac{(f(x)+1)(f'(x)) - (f(x)-2)(f'(x))}{(f(x)+1)^2}$$

use given info:

$$2 = g'(1) = \frac{(f(1)+1)(f'(1)) - (f(1)-2)(f'(1))}{(f(1)+1)^2}$$

$$\therefore \quad 2 = \frac{(3)f'(1) - 0}{9} \qquad 18 = 3f'(1)$$

$$\boxed{f'(1) = 6}$$

8. Assume that $f(x)$ and $g(x)$ are differentiable functions such that

$$f'(x) = -g(x) \text{ and } g'(x) = -f(x).$$

Let $h(x) = (f(x))^2 - (g(x))^2$. Which of the following is equal to $h'(x)$?

Chain rule twice.

(A) $4f(x)g(x)$ [✓ (B) 0] (C) $4f(x)f'(x)$ (D) $2g(x)f(x)$ (E) $2g'(x)f'(x) + (f(x))^2$

$$h'(x) = 2f(x) \cdot f'(x) - 2g(x)g'(x)$$

Plug in given.

$$h'(x) = 2f(x)[-g(x)] - 2g(x)[-f(x)]$$

$$= -2f(x)g(x) + 2g(x)f(x)$$

= ☺

9. How many of the following statements are necessarily true?

- ✗ $\cancel{\frac{d}{dx}[f(x)g(x)]} = \frac{d}{dx}[f(x)] \cdot \frac{d}{dx}[g(x)]$
- ✓ $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x) \cdot \frac{d}{dx}[f(x)] - f(x) \cdot \frac{d}{dx}[g(x)]}{[g(x)]^2}$ (quotient rule)
- ✓ $\frac{d}{dx}[f(x)g(x)] = \frac{d}{dx}[f(x)] \cdot g(x) + f(x) \cdot \frac{d}{dx}[g(x)]$ (product rule)
- ✗ $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{\frac{d}{dx}[f(x)]}{\frac{d}{dx}[g(x)]}$ nope.

(A) 0 (B) 1 [✓ (C) 2] (D) 3 (E) 4

10. Let $f(x) = \sin(x) + \cos(x) + \tan(x) + \csc(x)$. What is the value of $f'(\frac{\pi}{4})$?

(A) $2 - 2\sqrt{2}$ (B) $\sqrt{2} - 2$ [✓ (C) $2 - \sqrt{2}$] (D) $2 + \sqrt{2}$ (E) $2 + 2\sqrt{2}$

$$f'(x) = \cos(x) - \sin(x) + \sec^2(x) - \csc(x)\cot(x)$$

$$f'\left(\tfrac{\pi}{4}\right) = \cos\left(\tfrac{\pi}{4}\right) - \sin\left(\tfrac{\pi}{4}\right) + \frac{1}{\left[\cos\left(\frac{\pi}{4}\right)\right]^2} - \frac{1}{\sin\left(\frac{\pi}{4}\right)} \cdot \frac{\cos\left(\frac{\pi}{4}\right)}{\sin\left(\frac{\pi}{4}\right)}$$

$$= \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} + \frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2} - \frac{1}{\frac{\sqrt{2}}{2}} \cdot \frac{\cancel{\frac{\sqrt{2}}{2}}}{\cancel{\frac{\sqrt{2}}{2}}}$$

$$= 0 + 2 - \frac{2}{\sqrt{2}} \cdot (1)$$

$$= 2 - \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{2 - \sqrt{2}}$$

11. Which of the following is an equation for the tangent line to the graph of $f(x) = \arctan(2x)$ at $x = 0$?

(A) $y = x$ (B) $y = x + 1$ [✓ (C) $y = 2x$] (D) $y = x - 1$ (E) $y = 2x - 1$

$$f'(x) = \frac{1}{1+(2x)^2} \cdot (2) = \frac{2}{1+4x^2}$$

$$f'(0) = \frac{2}{1+0} = 2 \text{ (slope)}$$

$$f(0) = \arctan(0) = 0. \quad \overset{x_1,\ y_1}{(0, 0)}$$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = 2(x - 0)$$

$$\boxed{y = 2x}$$

12. Consider $f(x) = x^3 \ln(x) + \frac{x}{x^2+1}$. What is $f'(1)$? Product and Quotient rule

(A) 0 [✓ (B) 1] (C) 2 (D) 3 (E) 4

$$f'(x) = (3x^2)\ln(x) + x^3\left(\frac{1}{x}\right) + \frac{(x^2+1)(1) - x(2x)}{(x^2+1)^2}$$

$$f'(1) = (3)(0) + 1 + \frac{(2)(1) - (1)(2)}{2^2}$$

$$= 0 + 1 + 0$$

$$\boxed{= 1}$$

13. If $g(x) = x^{1/5}(x-1)^{3/5}$, find the domain of $g'(x)$. Product rule

(A) $(-\infty, 2) \cup (2, 3) \cup (3, \infty)$

[✓ (B) $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$]

(C) $(-\infty, \infty)$

(D) $(0, \infty)$

$$g'(x) = \frac{1}{5}x^{-4/5}(x-1)^{3/5} + x^{1/5}\cdot\frac{3}{5}(x-1)^{-2/5}$$ Factor.

$$g'(x) = \frac{1}{5}x^{-4/5}(x-1)^{-2/5}\left[x-1+3x\right]$$

$$g'(x) = \frac{4x-1}{5\left[\sqrt[5]{x^4}\cdot\sqrt[5]{(x-1)^2}\right]}$$

Where is $g'(x)$ defined?

$$\boxed{(-\infty, 0) \cup (0, 1) \cup (1, \infty)}$$

14. If $h(x) = [f(x)g(x) + x]^2$, what is $h'(0)$ if $f(0) = 2$, $g(0) = 1$, $f'(0) = 0$, and $g'(0) = -1$?

Chain rule

(A) $-2$ [✓ (B) $-4$] (C) $6$ (D) $-3$ (E) $4$

$$h'(x) = 2\left[f(x)g(x) + x\right]\cdot\left[f'(x)g(x) + f(x)g'(x) + 1\right]$$

$$h'(0) = 2\left[f(0)g(0) + 0\right]\cdot\left[f'(0)g(0) + f(0)g'(0) + 1\right]$$

$$h'(0) = 2\left[2(1) + 0\right]\cdot\left[0 + 2(-1) + 1\right]$$

$$= 2\left[2\right]\left[-1\right] = \boxed{-4}$$

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**Fall 2022**
**Exam 2 B**
**10/19/2022**
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**UF-ID:** ________________

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A ● C D E

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Part I Instructions: 14 multiple choice questions. Complete the scantron sheet provided with your information and fill in the appropriate spaces to answer your questions. Only the answer on the scantron sheet will be graded. Each problem is worth five (5) points for a total of 70 points on Part I.

1. Let $f(x) = \sin(x) + \cos(x) + \cot(x) + \sec(x)$. What is the value of $f'(\frac{\pi}{4})$?

$(A)\ 2 - 2\sqrt{2}$ [✓ $(B)\ \sqrt{2} - 2$] $(C)\ 2 - \sqrt{2}$ $(D)\ 2 + \sqrt{2}$ $(E)\ 2 + 2\sqrt{2}$

$$f'(x) = \cos x - \sin x - \csc^2(x) + \sec x \tan x$$

$$f'(\tfrac{\pi}{4}) = \cos\tfrac{\pi}{4} - \sin\tfrac{\pi}{4} - \frac{1}{(\sin\frac{\pi}{4})^2} + \frac{1}{\cos\frac{\pi}{4}} \cdot \frac{\sin\frac{\pi}{4}}{\cos\frac{\pi}{4}}$$

$$f'(\tfrac{\pi}{4}) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} - \frac{1}{(\frac{\sqrt{2}}{2})^2} + \frac{1}{\frac{\sqrt{2}}{2}} \cdot \frac{\sqrt{2}/2}{\sqrt{2}/2}$$

$$= 0 - 2 + \frac{2}{\sqrt{2}} \cdot (1)$$

$$= \frac{2}{\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2}} - 2 = \boxed{\sqrt{2} - 2}$$

2. Suppose $f(x) = e^{3x} + x^3 + \cos(x)$. Calculate $f'''(1)$.

$(A)\ 27e^3 + 6 - \sin(1)$ [✓ $(B)\ 27e^3 + 6 + \sin(1)$] $(C)\ e^3 + 6 - \sin(1)$ $(D)\ e + 6 + \sin(1)$

$$f'(x) = 3e^{3x} + 3x^2 - \sin(x)$$

$$f''(x) = 9e^{3x} + 6x - \cos(x)$$

$$f'''(x) = 27e^{3x} + 6 + \sin(x)$$

$$f'''(1) = 27e^3 + 6 + \sin(1)$$

3. If $h(x) = [f(x)g(x) + x]^2$, what is $h'(0)$ if $f(0) = 2$, $g(0) = 1$, $f'(0) = 0$, and $g'(0) = -2$?

Chain rule.

[✓ (A) $-12$] (B) $8$ (C) $-6$ (D) $-3$ (E) $6$

$$h'(x) = 2[f(x)g(x)+x][f'(x)g(x)+f(x)g'(x)+1]$$

$$h'(0) = 2[f(0)g(0)+0][f'(0)g(0)+f(0)g'(0)+1]$$

$$= 2[(2)(1)][0+(2)(-2)+1]$$

$$= 2(2)(-3)$$

$$= \boxed{-12}$$

4. Which of the following is an equation for the tangent line to the graph of $f(x) = \arcsin(2x)$ at $x = 0$?

(A) $y = x$ (B) $y = x + 1$ [✓ (C) $y = 2x$] (D) $y = x - 1$ (E) $y = 2x - 1$

$$f'(x) = \frac{1}{\sqrt{1-(2x)^2}} \cdot (2) = \frac{2}{\sqrt{1-4x^2}}$$

$$f'(0) = \frac{2}{\sqrt{1-0}} = 2 \text{ (slope)} \qquad (\overset{x_1}{0}, \overset{y_1}{0})$$

$$\arcsin(0) = 0$$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = 2(x - 0)$$

$$\boxed{y = 2x}$$

5. Use implicit differentiation to find $\frac{dy}{dx}$ for $4x^3 + 5y^3 = 11xy$.

(A) $\frac{dy}{dx} = \frac{12x^2 - 11y}{15y^2 - 11x}$

(B) $\frac{dy}{dx} = \frac{11y - 12x^2}{15y^2 - 13x}$

[✓ (C) $\frac{dy}{dx} = \frac{11y - 12x^2}{15y^2 - 11x}$]

(D) $\frac{dy}{dx} = \frac{11y - 12x^2}{11y^2 - 21x}$

$$\frac{d}{dx}(4x^3 + 5y^3) = \frac{d}{dx}(11xy)$$

$$12x^2 + 15y^2 \cdot y' = 11(y + xy')$$

$$12x^2 + 15y^2y' = 11y + 11xy'$$

$$15y^2y' - 11xy' = 11y - 12x^2$$

$$y' = \frac{11y - 12x^2}{15y^2 - 11x}$$

6. If $g(x) = (x+1)^{1/5}(x+2)^{3/5}$, find the domain of $g'(x)$. Product Rule.

(A) $(-\infty, 1) \cup (1, 2) \cup (2, \infty)$

[✓ (B) $(-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$]

(C) $(-\infty, \infty)$

(D) $(0, \infty)$

$$g'(x) = \frac{1}{5}(x+1)^{-\frac{4}{5}}(x+2)^{\frac{3}{5}} + \frac{3}{5}(x+2)^{-\frac{2}{5}}(x+1)^{\frac{1}{5}}$$ (Factor)

$$g'(x) = \frac{1}{5}(x+1)^{-\frac{4}{5}}(x+2)^{-\frac{2}{5}}\left[x + 2 + 3(x+1)\right]$$

$$g'(x) = \frac{4x + 5}{5 \cdot \sqrt[5]{(x+1)^4} \cdot \sqrt[5]{(x+2)^2}}$$

Where is $g'(x)$ defined?

$(-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$

7. Assume that $f(x)$ and $g(x)$ are differentiable functions such that

$$f'(x) = -\frac{g(x)}{2} \text{ and } g'(x) = -\frac{f(x)}{2}.$$

Let $h(x) = (f(x))^2 - (g(x))^2$. Which of the following is equal to $h'(x)$?

Chain rule twice.

(A) $f(x)g(x)$ [✓ (B) 0] (C) $f(x)f'(x)$ (D) $\frac{g(x)f(x)}{4}$ (E) $2g'(x)f'(x) + (f(x))^2$

$$h'(x) = 2(f(x))\cdot f'(x) - 2(g(x))g'(x)$$

Plug in given info:

$$h'(x) = 2f(x)\cdot\left[-\frac{g(x)}{2}\right] - 2g(x)\cdot\left[-\frac{f(x)}{2}\right]$$

$$= -f(x)g(x) + f(x)g(x)$$

= ☺

8. Find the derivative of $f(x) = x^2 + \operatorname{arccot}(x+1)$.

(A) $f'(x) = -\frac{1}{x^2+2x+2}$

(B) $f'(x) = 2x + \frac{1}{x^2+2x+2}$

[✓ (C) $f'(x) = 2x - \frac{1}{x^2+2x+2}$]

(D) $2x + \frac{1}{x^2-2x+2}$

$$f'(x) = 2x - \frac{1}{1+(x+1)^2}$$

$$= 2x - \frac{1}{1+x^2+2x+1} = \boxed{2x - \frac{1}{x^2+2x+2}}$$

9. How many of the following statements are necessarily true?

- ✗ $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{\frac{d}{dx}[f(x)]}{\frac{d}{dx}[g(x)]}$ No.
- ✓ $\frac{d}{dx}[f(x)g(x)] = \frac{d}{dx}[f(x)] \cdot g(x) + f(x) \cdot \frac{d}{dx}[g(x)]$ Product Rule.
- ✗ $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x) \cdot \frac{d}{dx}[f(x)] + f(x) \cdot \frac{d}{dx}[g(x)]}{[g(x)]^2}$ (should be subtraction)
- ✗ $\frac{d}{dx}[f(x)g(x)] = \frac{d}{dx}[f(x)] \cdot \frac{d}{dx}[g(x)]$

(A) 0 [✓ (B) 1] (C) 2 (D) 3 (E) 4

10. Let $f(1) = 2$ and $g(x) = \frac{f(x)+2}{f(x)-1}$. If $g'(1) = 12$, then which of the following is equal to $f'(1)$?

(A) 6 (B) $\frac{2}{3}$ (C) 2 [✓ (D) $-4$] (E) $-6$

$$g'(x) = \frac{(f(x)-1)f'(x) - (f(x)+2)f'(x)}{(f(x)-1)^2}$$

$$12 = g'(1) = \frac{(f(1)-1)f'(1) - (f(1)+2)f'(1)}{(f(1)-1)^2}$$

$$\therefore 12 = \frac{(1)f'(1) - (4)f'(1)}{1^2} = \frac{-3f'(1)}{1}$$

$$\therefore \boxed{-4 = f'(1)}$$

11. Find the slope of the tangent line to $f(x) = \ln\left(\frac{e^{3x}\sqrt{x}}{x^2+1}\right)$ at $x = 1$.

[✓ (A) $\frac{5}{2}$] (B) $-\frac{3}{4}$ (C) $\frac{7}{2}$ (D) $\frac{13}{4}$ (E) $\frac{9}{2}$

$$f(x) = \ln e^{3x} + \ln x^{1/2} - \ln(x^2+1)$$

$$f(x) = 3x + \frac{1}{2}\ln x - \ln(x^2+1)$$

$$f'(x) = 3 + \frac{1}{2x} - \frac{2x}{x^2+1}$$

$$f'(1) = 3 + \frac{1}{2} - \frac{2}{2} = \boxed{\frac{5}{2}}$$

12. Let $f(x) = 9^x + \ln(x) + 4x^3$. what is the value of $f'(\frac{1}{2})$?

(A) $\ln(9) + 5$ [✓ (B) $3\ln(9) + 5$] (C) $6\ln(9) + 5$ (D) $\ln(9) + 5$ (E) 7

$$f'(x) = 9^x \ln(9) + \frac{1}{x} + 12x^2$$

$$f'(\tfrac{1}{2}) = 9^{1/2}\ln(9) + \frac{1}{(\frac{1}{2})} + 12(\tfrac{1}{2})^2$$

$$f'(\tfrac{1}{2}) = 3\ln(9) + 2 + 3$$

$$f'(\tfrac{1}{2}) = 5 + 3\ln(9)$$

13. The vertical position of your jetpack-wearing Calculus instructor is given by $s(t) = t^3 - 12t^2 + 21t - 13$ for $t \geq 0$. On which of the following intervals are they slowing down?

(A) (0, 4) (B) (1, 7) [✓ (C) (4, 7)] (D) (0, 7) (E) (7, ∞)

$v(t) = 3t^2 - 24t + 21$

Let $v(t) = 0$.

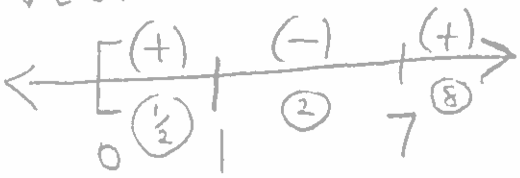
$0 = 3t^2 - 24t + 21$

$0 = t^2 - 8t + 7$

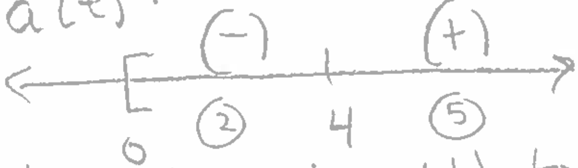
$0 = (t-7)(t-1)$

$t = 7, \; t = 1$

v(t):



a(t):



$a(t) = 6t - 24$

Let $a(t) = 0$

$0 = 6t - 24$

$t = 4$

We need v(t) and a(t) to have opposing signs to be slowing down: $\boxed{(0,1) \cup (4,7)}$

14. Consider $f(x) = x^2 \ln(x) + \dfrac{4x^2}{x+1}$. What is $f'(1)$?

Product and Quotient Rule.

(A) 0 (B) 1 (C) 2 (D) 3 [✓ (E) 4]

$$f'(x) = (2x)\ln(x) + x^2\left(\frac{1}{x}\right) + \frac{(x+1)(8x) - 4x^2(1)}{(x+1)^2}$$

$$f'(1) = 0 + 1 + \frac{(2)(8) - 4}{4}$$

$$= 1 + \frac{16-4}{4} = 1 + 3 = \boxed{4}$$