Calculus I: MAC2311 **Name:** _____
Fall 2022
Exam 2 A **Section:** _____
10/19/2022
Time Limit: 90 Minutes **UF-ID:** _____

Scantron Instruction: This exam uses a scantron. Follow the instructions listed on this page to fill out the scantron.

A. Sign your scantron **on the back** at the bottom in the white area.

B. Write **and code** in the spaces indicated:

1) Name (last name, first initial, middle initial)

2) UFID Number

3) 4-digit Section Number

C. Under *special codes*, code in the test numbers 2, 1:

1 • 3 4 5 6 7 8 9 0
• 2 3 4 5 6 7 8 9 0

D. At the top right of your scantron, fill in the *Test Form Code* as A.

• B C D E

E. This exam consists of 14 multiple choice questions and 5 free response questions. Make sure you check for errors in the number of questions your exam contains.

F. The time allowed is 90 minutes.

G. WHEN YOU ARE FINISHED:

1) Before turning in your test check for **transcribing errors**. Any mistakes you leave in are there to stay!

2) You must turn in your scantron to your proctor. **Be prepared to show your GatorID with a legible signature.**

It is your responsibility to ensure that your test has **19 questions**. If it does not, show it to your proctor immediately. You will not be permitted to make up any problems omitted from your test after the testing period ends. There are a total of 105 points available on this exam.

Part I Instructions: 14 multiple choice questions. Complete the scantron sheet provided with your information and fill in the appropriate spaces to answer your questions. Only the answer on the scantron sheet will be graded. Each problem is worth five (5) points for a total of 70 points on Part I.

1. Find the slope of the tangent line to $f(x) = \ln\left(\dfrac{e^{2x}\sqrt{x+3}}{x^2+1}\right)$ at $x = 1$.

(A) $\dfrac{5}{4}$ (B) $-\dfrac{3}{8}$ (C) $\dfrac{7}{4}$ (D) $\dfrac{13}{8}$ (E) $\dfrac{9}{8}$

2. Suppose $f(x) = e^{2x} + 2x^3 - \sin(x)$. Calculate $f'''(1)$.

(A) $e^2 - \cos(1) + 12$ (B) $8e^2 + \cos(1) + 12$ (C) $8e^2 - \cos(1) + 12$ (D) $e^2 + \cos(1) + 12$

3. The vertical position of your jetpack-wearing Calculus instructor is given by $s(t) = t^3 - 9t^2 - 21t + 16$ for $t \geq 0$. On which of the following intervals are they slowing down?

(A) $(0, 3)$ (B) $(3, 7)$ (C) $(1, 5)$ (D) $(0, 1)$ (E) $(7, \infty)$

4. Find the derivative of $f(x) = x^2 + \arccos(x + 1)$.

(A) $f'(x) = \dfrac{1}{\sqrt{-x^2 - 2x}}$

(B) $f'(x) = 2x + \dfrac{1}{\sqrt{-x^2 - 2x}}$

(C) $f'(x) = 2x - \dfrac{1}{\sqrt{-x^2 - 2x}}$

(D) $2x - \dfrac{1}{\sqrt{x^2 + 2x}}$

5. Let $f(x) = 4^x + \ln(x) + 4x^3$. what is the value of $f'(\frac{1}{2})$?

(A) 7 (B) $\ln(4) + 5$ (C) $4\ln(4) + 5$ (D) $2\ln(4) + 5$ (E) $2\ln(8) + 5$

6. Use implicit differentiation to find $\frac{dy}{dx}$ for $6x^3 + 7y^3 = 13xy$.

(A) $\frac{dy}{dx} = \frac{13y - 18x^2}{21y^2 - 13x}$

(B) $\frac{dy}{dx} = \frac{18y - 13x^2}{21y^2 - 13x}$

(C) $\frac{dy}{dx} = \frac{13y - 18x^2}{13y^2 - 21x}$

(D) $\frac{dy}{dx} = \frac{13y - 18x^2}{13y - 21x^2}$

7. Let $f(1) = 2$ and $g(x) = \dfrac{f(x) - 2}{f(x) + 1}$. If $g'(1) = 2$, then which of the following is equal to $f'(1)$?

(A) $-\dfrac{2}{3}$ (B) 4 (C) 3 (D) -3 (E) 6

8. Assume that $f(x)$ and $g(x)$ are differentiable functions such that

$$f'(x) = -g(x) \text{ and } g'(x) = -f(x).$$

Let $h(x) = (f(x))^2 - (g(x))^2$. Which of the following is equal to $h'(x)$?

(A) $4f(x)g(x)$ (B) 0 (C) $4f(x)f'(x)$ (D) $2g(x)f(x)$ (E) $2g'(x)f'(x) + (f(x))^2$

9. How many of the following statements are necessarily true?

- $\frac{d}{dx}\left[f(x)g(x)\right] = \frac{d}{dx}[f(x)] \cdot \frac{d}{dx}[g(x)]$
- $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x) \cdot \frac{d}{dx}[f(x)] - f(x) \cdot \frac{d}{dx}[g(x)]}{[g(x)]^2}$
- $\frac{d}{dx}\left[f(x)g(x)\right] = \frac{d}{dx}[f(x)] \cdot g(x) + f(x) \cdot \frac{d}{dx}[g(x)]$
- $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{\frac{d}{dx}[f(x)]}{\frac{d}{dx}[g(x)]}$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

10. Let $f(x) = \sin(x) + \cos(x) + \tan(x) + \csc(x)$. What is the value of $f'(\frac{\pi}{4})$?

(A) $2 - 2\sqrt{2}$ (B) $\sqrt{2} - 2$ (C) $2 - \sqrt{2}$ (D) $2 + \sqrt{2}$ (E) $2 + 2\sqrt{2}$

11. Which of the following is an equation for the tangent line to the graph of $f(x) = \arctan(2x)$ at $x = 0$?

(A) $y = x$ (B) $y = x + 1$ (C) $y = 2x$ (D) $y = x - 1$ (E) $y = 2x - 1$

12. Consider $f(x) = x^3 \ln(x) + \dfrac{x}{x^2 + 1}$. What is $f'(1)$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

13. If $g(x) = x^{1/5}(x-1)^{3/5}$, find the domain of $g'(x)$.

(A) $(-\infty, 2) \cup (2, 3) \cup (3, \infty)$

(B) $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

(C) $(-\infty, \infty)$

(D) $(0, \infty)$

14. If $h(x) = [f(x)g(x) + x]^2$, what is $h'(0)$ if $f(0) = 2$, $g(0) = 1$, $f'(0) = 0$, and $g'(0) = -1$?

(A) -2 (B) -4 (C) 6 (D) -3 (E) 4

Calculus I: MAC2311
Fall 2022
Exam 2 A
10/19/2022
Time Limit: 90 Minutes

Name: _____

Section: _____

UF-ID: _____

Part II Instructions: 5 free response questions. Neatly give a complete solution to each problem and show all work and intermediate steps. We are grading the work and notation as well as the answer. Each problem is worth seven (7) points. A total of 35 points is possible on Part II. **No credit will given without proper work.** If we cannot read it and follow it, you will receive no credit for the problem.

For Instructor Use Only:

FR 1	
FR 2	
FR 3	
FR 4	
FR 5	
Total Points	

1. Jacques Monod modeled the per capita growth rate R of *Escherichia coli* bacteria by the function

$$R(N) = \frac{N}{3+N}$$

where N is the concentration of the nutrient.

(*a*) Determine $\dfrac{dR}{dN}$.

(*b*) Calculate $\dfrac{dR}{dN}$ at $N = 8$. Using this answer, is the growth rate rising or falling at $N = 8$? Explain why.

2. The vertical position of a yo-yo at time t, for $0 \leq t \leq \pi$, is given by

$$s(t) = \sqrt{3}\sin(t) - \cos(t).$$

(a) What is the displacement of the yo-yo from $t = 0$ to $t = \dfrac{\pi}{2}$?

(b) What is the total distance traveled by the yo-yo from $t = 0$ to $t = \pi$?

3. Let $f(x) = x^{\sin(x)}$. Calculate $f'(x)$.

4. Let $x^4 - x^2y + y^4 = 1$

(a) Find the slope of the tangent line at the point $(-1, 1)$.

(b) Write an equation for the tangent line to the curve at the point $(-1, 1)$.

5. The table below gives the values of $f(x)$, $f'(x)$, $g(x)$, and $g'(x)$ for various values of x. Use the table to answer the following questions.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	0	4	2	3
2	1	-1	2	-2
3	6	5	4	3

(*a*) Let $A(x) = \ln(g(x))$. What is $A'(2)$?

(*b*) Let $B(x) = f(x)(g(x))^2$. What is $B'(3)$?

(*c*) Let $C(x) = \dfrac{(f(x))^3}{g(x)}$. What is $C'(2)$?

Calculus I: MAC2311 **Name:** _____
Fall 2022
Exam 2 B **Section:** _____
10/19/2022
Time Limit: 90 Minutes **UF-ID:** _____

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B. Write **and code** in the spaces indicated:

1) Name (last name, first initial, middle initial)

2) UFID Number

3) 4-digit Section Number

C. Under *special codes*, code in the test numbers 2, 2:

1 • 3 4 5 6 7 8 9 0
1 • 3 4 5 6 7 8 9 0

D. At the top right of your scantron, fill in the *Test Form Code* as B.

A • C D E

E. This exam consists of 14 multiple choice questions and 5 free response questions. Make sure you check for errors in the number of questions your exam contains.

F. The time allowed is 90 minutes.

G. WHEN YOU ARE FINISHED:

1) Before turning in your test check for **transcribing errors**. Any mistakes you leave in are there to stay!

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Part I Instructions: 14 multiple choice questions. Complete the scantron sheet provided with your information and fill in the appropriate spaces to answer your questions. Only the answer on the scantron sheet will be graded. Each problem is worth five (5) points for a total of 70 points on Part I.

1. Let $f(x) = \sin(x) + \cos(x) + \cot(x) + \sec(x)$. What is the value of $f'(\frac{\pi}{4})$?

(A) $2 - 2\sqrt{2}$ (B) $\sqrt{2} - 2$ (C) $2 - \sqrt{2}$ (D) $2 + \sqrt{2}$ (E) $2 + 2\sqrt{2}$

2. Suppose $f(x) = e^{3x} + x^3 + \cos(x)$. Calculate $f'''(1)$.

(A) $27e^3 + 6 - \sin(1)$ (B) $27e^3 + 6 + \sin(1)$ (C) $e^3 + 6 - \sin(1)$ (D) $e + 6 + \sin(1)$

3. If $h(x) = [f(x)g(x) + x]^2$, what is $h'(0)$ if $f(0) = 2$, $g(0) = 1$, $f'(0) = 0$, and $g'(0) = -2$?

(A) -12 (B) 8 (C) -6 (D) -3 (E) 6

4. Which of the following is an equation for the tangent line to the graph of $f(x) = \arcsin(2x)$ at $x = 0$?

(A) $y = x$ (B) $y = x + 1$ (C) $y = 2x$ (D) $y = x - 1$ (E) $y = 2x - 1$

5. Use implicit differentiation to find $\frac{dy}{dx}$ for $4x^3 + 5y^3 = 11xy$.

(A) $\frac{dy}{dx} = \frac{12x^2 - 11y}{15y^2 - 11x}$

(B) $\frac{dy}{dx} = \frac{11y - 12x^2}{15y^2 - 13x}$

(C) $\frac{dy}{dx} = \frac{11y - 12x^2}{15y^2 - 11x}$

(D) $\frac{dy}{dx} = \frac{11y - 12x^2}{11y^2 - 21x}$

6. If $g(x) = (x+1)^{1/5}(x+2)^{3/5}$, find the domain of $g'(x)$.

(A) $(-\infty, 1) \cup (1, 2) \cup (2, \infty)$

(B) $(-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$

(C) $(-\infty, \infty)$

(D) $(0, \infty)$

7. Assume that $f(x)$ and $g(x)$ are differentiable functions such that

$$f'(x) = -\frac{g(x)}{2} \text{ and } g'(x) = -\frac{f(x)}{2}.$$

Let $h(x) = (f(x))^2 - (g(x))^2$. Which of the following is equal to $h'(x)$?

(A) $f(x)g(x)$ (B) 0 (C) $f(x)f'(x)$ (D) $\frac{g(x)f(x)}{4}$ (E) $2g'(x)f'(x) + (f(x))^2$

8. Find the derivative of $f(x) = x^2 + \text{arccot}(x+1)$.

(A) $f'(x) = -\frac{1}{x^2+2x+2}$

(B) $f'(x) = 2x + \frac{1}{x^2+2x+2}$

(C) $f'(x) = 2x - \frac{1}{x^2+2x+2}$

(D) $2x + \frac{1}{x^2-2x+2}$

9. How many of the following statements are necessarily true?

- $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{\dfrac{d}{dx}[f(x)]}{\dfrac{d}{dx}[g(x)]}$
- $\dfrac{d}{dx}\left[f(x)g(x)\right] = \dfrac{d}{dx}[f(x)] \cdot g(x) + f(x) \cdot \dfrac{d}{dx}[g(x)]$
- $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{g(x) \cdot \dfrac{d}{dx}[f(x)] + f(x) \cdot \dfrac{d}{dx}[g(x)]}{[g(x)]^2}$
- $\dfrac{d}{dx}\left[f(x)g(x)\right] = \dfrac{d}{dx}[f(x)] \cdot \dfrac{d}{dx}[g(x)]$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

10. Let $f(1) = 2$ and $g(x) = \dfrac{f(x)+2}{f(x)-1}$. If $g'(1) = 12$, then which of the following is equal to $f'(1)$?

(A) 6 (B) $\dfrac{2}{3}$ (C) 2 (D) -4 (E) -6

11. Find the slope of the tangent line to $f(x) = \ln\left(\dfrac{e^{3x}\sqrt{x}}{x^2+1}\right)$ at $x = 1$.

(A) $\dfrac{5}{2}$ (B) $-\dfrac{3}{4}$ (C) $\dfrac{7}{2}$ (D) $\dfrac{13}{4}$ (E) $\dfrac{9}{2}$

12. Let $f(x) = 9^x + \ln(x) + 4x^3$. what is the value of $f'(\frac{1}{2})$?

(A) $\ln(9) + 5$ (B) $3\ln(9) + 5$ (C) $6\ln(9) + 5$ (D) $\ln(9) + 5$ (E) 7

13. The vertical position of your jetpack-wearing Calculus instructor is given by $s(t) = t^3 - 12t^2 + 21t - 13$ for $t \geq 0$. On which of the following intervals are they slowing down?

(A) $(0, 4)$ (B) $(1, 7)$ (C) $(4, 7)$ (D) $(0, 7)$ (E) $(7, \infty)$

14. Consider $f(x) = x^2 \ln(x) + \dfrac{4x^2}{x+1}$. What is $f'(1)$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Calculus I: MAC2311
Fall 2022
Exam 2 B
10/19/2022
Time Limit: 90 Minutes

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For Instructor Use Only:

FR 1	
FR 2	
FR 3	
FR 4	
FR 5	
Total Points	

1. Jacques Monod modeled the per capita growth rate R of *Escherichia coli* bacteria by the function

$$R(N) = \frac{2N}{2+N}$$

where N is the concentration of the nutrient, S is its saturation level, and c is a positive constant.

(a) Determine $\dfrac{dR}{dN}$.

(b) Calculate $\dfrac{dR}{dN}$ at $N = 10$. Using this answer, is the growth rate rising or falling at $N = 10$? Explain why.

2. The vertical position of a yo-yo at time t, for $0 \leq t \leq \pi$, is given by

$$s(t) = \sin(t) - \sqrt{3}\cos(t).$$

(a) What is the displacement of the yo-yo from $t = 0$ to $t = \dfrac{\pi}{2}$?

(b) What is the total distance traveled by the yo-yo from $t = 0$ to $t = \pi$?

3. Let $f(x) = x^{\cos(x)}$. Calculate $f'(x)$.

4. Let $y^4 + x^2y - x^4 = 1$

(a) Find the slope of the tangent line at the point $(-1, 1)$.

(b) Write an equation for the tangent line to the curve at the point $(-1, 1)$.

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x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	0	4	2	3
2	1	-1	2	-2
3	6	5	4	3

(*a*) Let $A(x) = \ln(g(x))$. What is $A'(1)$?

(*b*) Let $B(x) = f(x)(g(x))^2$. What is $B'(2)$?

(*c*) Let $C(x) = \dfrac{(f(x))^3}{g(x)}$. What is $C'(3)$?