Calculus I: MAC2311 **Name:** Exam Review

Exam 1

Part I Instructions: 14 multiple choice questions

1. How many of the following functions are continuous on the interval $(-1, \infty)$?

(i) $f(x) = x^2 + x + 2$ $(-\infty, \infty)$ ✓

(ii) $g(x) = \ln(x)$ $(0, \infty)$

(iii) $h(x) = \sqrt{x-2}$ $x - 2 \geq 0$ $x \geq 2$ $[2, \infty)$

(iv) $k(x) = 3^x$ $(-\infty, \infty)$ ✓

(A) 0 (B) 1 $\boxed{(C)\ 2}$ (D) 3 (E) 4

2. The displacement (in meters) of a particle moving in a straight line is given by $s(t) = t^2 + 8t + 18$, where t is measured in seconds. What is the average velocity of the particle over the time interval $[2, 4]$?

(A) 12 m/sec $\boxed{(B)\ 14\text{ m/sec}}$ (C) 16 m/sec (D) 28 m/sec

$$\frac{s(4) - s(2)}{4 - 2}$$

$$s(4) = (4)^2 + 8(4) + 18$$
$$= 16 + 32 + 18 = 66$$
$$s(2) = (2)^2 + 8(2) + 18$$
$$= 4 + 16 + 18$$
$$= 38$$

$$\frac{66 - 38}{2} = \frac{28}{2} = 14$$

3. Suppose $3x \le f(x) \le 9 - \frac{3}{4}x^2$. What is $\lim_{x\to 2^-} f(x)$?

(*A*) 0 (*B*) 3 $\boxed{(C)\ 6}$ (*D*) 9 (*E*) Does not exist

$$\lim_{x\to 2^-} 3x \le \lim_{x\to 2^-} f(x) \le \lim_{x\to 2^-} 9-\frac{3}{4}x^2$$

$$6 \le \lim_{x\to 2^-} f(x) \le 9-\frac{3}{4}(2)^2$$

$$6 \le \lim_{x\to 2^-} f(x) \le 6 \quad \to \quad \lim_{x\to 2^-} f(x) = 6$$

4. Let $f(x) = \frac{1}{(x+3)^2}$. Which one of the following statements concerning $f(x)$ is correct?

$\boxed{(A)\ \lim_{x\to -3^-} f(x) = \infty \text{ and } x=-3 \text{ is a vertical asymptote of } y=f(x)}$

(*B*) $\lim_{x\to -3^-} f(x) = -\infty$ and $x = -3$ is a vertical asymptote of $y = f(x)$

(*C*) $\lim_{x\to 3^-} f(x) = \infty$ and $y = 0$ is a vertical asymptote of $y = f(x)$

(*D*) $\lim_{x\to 3^-} f(x) = -\infty$ and $y = 0$ is a vertical asymptote of $y = f(x)$

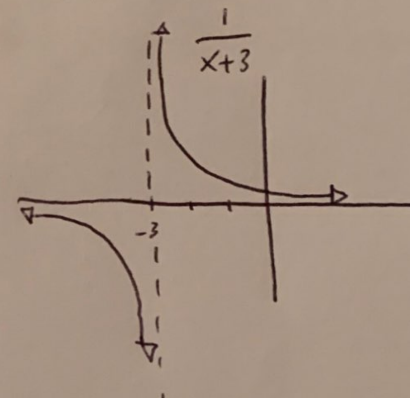
(*E*) None of these

} X because VA @ $x=-3$

$$\lim_{x\to -3^-} \frac{1}{(x+3)^2} = \lim_{x\to -3^-} \frac{1}{(x+3)} \cdot \frac{1}{(x+3)}$$

$$= (-\infty)(-\infty)$$

$$= \infty$$



5. The function $f(x) = x^2 + 3x - 3$ is guaranteed to have a root in the interval $[-2, 2]$ by the Intermediate Value Theorem.

One needs to have a Positive y, one needs to have a negative y

$\boxed{(A)\ \text{True}}$ (*B*) False

$$f(-2) = (-2)^2 + 3(-2) - 3$$
$$= 4-6-3$$
$$= -2-3$$
$$= -5$$

$$f(2) = (2)^2 + 3(2) - 3$$
$$= 4+6-3$$
$$= 7$$

6. Evaluate $\lim\limits_{x \to 1} \dfrac{\sqrt{3x+6}-3}{x-1}$.

$\dfrac{\sqrt{9}-3}{1-1} = \dfrac{0}{0}$

$\boxed{(A)\ \frac{1}{2}}$ (B) $\frac{1}{6}$ (C) 0 (D) $\frac{1}{3}$ (E) Does not exist

$\lim\limits_{x \to 1} \dfrac{\sqrt{3x+6}-3}{x-1} \cdot \dfrac{(\sqrt{3x+6}+3)}{(\sqrt{3x+6}+3)} = \lim\limits_{x \to 1} \dfrac{3x+6-9}{(x-1)(\sqrt{3x+6}+3)} = \lim\limits_{x \to 1} \dfrac{3x-3}{(x-1)(\sqrt{3x+6}+3)} = \lim\limits_{x \to 1} \dfrac{3(x-1)}{(x-1)(\sqrt{3x+6}+3)}$

$= \lim\limits_{x \to 1} \dfrac{3}{\sqrt{3x+6}+3} = \dfrac{3}{\sqrt{9}+3} = \dfrac{3}{6} = \dfrac{1}{2}$

$f(x) = \dfrac{x+2}{(x^2-4)} = \dfrac{\cancel{x+2}}{(\cancel{x+2})(x-2)} = \dfrac{1}{x-2}$

$x \neq -2$

7. Let $f(x) = \dfrac{x+2}{x^2-4}$. Which one of the following statements is correct?

$\lim\limits_{x \to -2^+} \dfrac{1}{x-2} = \dfrac{1}{-2-2} = \dfrac{-1}{4}$

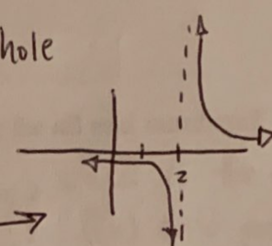
~~(A)~~ $\lim\limits_{x \to -2^+} f(x) = \infty$

~~(B)~~ $x = -2$ is a vertical asymptote of $f(x)$ → It's a hole

$\boxed{(C)\ \lim\limits_{x \to 2^-} f(x) = -\infty}$ ✓

$\lim\limits_{x \to 2^-} \dfrac{1}{x-2} \longrightarrow$

$= -\infty$



(D) $\lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^+} f(x)$ $\longrightarrow$

$-\infty$ $\quad \infty$

(E) None of these

8. Which of the following statements is necessarily true?

~~(A)~~ If $f(x)$ and $g(x)$ are continuous, then $f(x) + g(x)$ is discontinuous

$\boxed{\text{(B) If } f(x) \text{ and } g(x) \text{ are continuous, then } f(x)g(x) \text{ is continuous}}$

~~(C)~~ If $f(x)$ is discontinuous and $g(x)$ is continuous, then $f(x) + g(x)$ is continuous

~~(D)~~ If $f(x)$ is discontinuous, then $cf(x)$ is continuous where c is any real number

9. Let

$$\begin{array}{c|l} 2 & 3 \\ 2,1 & \end{array}$$

$$f(x)=\begin{cases}\dfrac{x^2+3x+2}{x^2-1}, & x\neq 1\\ 3, & x=1\end{cases}.$$

$$\frac{x^2+3x+2}{x^2-1}=\frac{(x+2)\cancel{(x+1)}}{(x-1)\cancel{(x+1)}}=\frac{x+2}{x-1};x\neq-1$$

Which one of the following statements concerning $f(x)$ is correct?

$$\cancel{f(-1)=\frac{(-1)+2}{(-1)-1}}=\frac{1}{-2}$$

f(-1) is undefined

~~(A)~~ $f(-1)=-\frac{1}{2}$ and $\lim_{x\to1} f(x)$ does not exist

$\boxed{(B)\ f(-1) \text{ is undefined and } \lim_{x\to1} f(x) \text{ does not exist}}$

(C) $f(-1)$ is undefined and $\lim_{x\to1} f(x)=3$

~~(D)~~ $f(-1)=-\frac{1}{2}$ and $\lim_{x\to1} f(x)=3$

(E) None of these

VA at x=1, so $\lim_{x\to1} f(x)$ does not exist

10. For which value of k is the following function continuous for all real numbers?

$$f(x)=\begin{cases}k2^x, & x<2\\ x^2+4kx, & x\geq 2\end{cases}$$

x=2 is the only place where f(x) could possibly be discontinuous

(A) 0 (B) 2 (C) 1 $\boxed{(D)\ -1}$ (E) No such value exists

$k2^x=x^2+4kx$ @ x=2

$k2^{(2)}=(2)^2+4k(2)$

$4k=4+8k$

$-4k=4$

$k=-1$

11. Let $f(x)=\sqrt{x-3}$. Which of the following is equal to $f'(4)$?

h approaches 0

(A) $\lim_{h\to0}\dfrac{\sqrt{1+h}-\sqrt{h}}{h}$

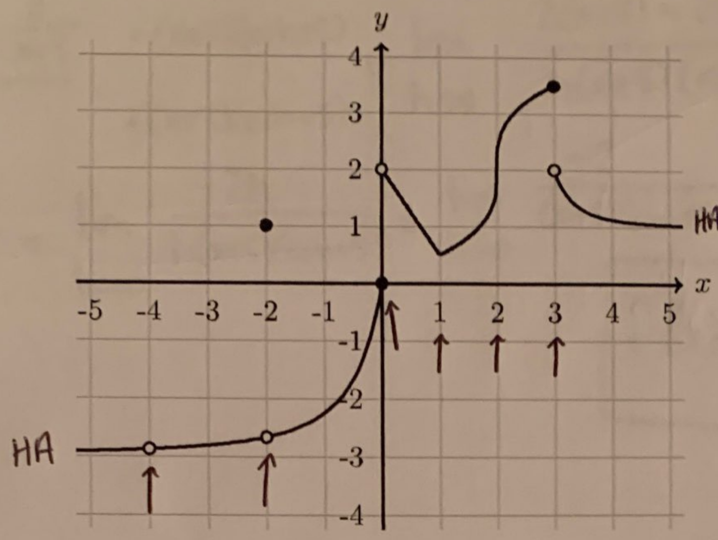
$\boxed{(B)\ \lim_{h\to0}\dfrac{\sqrt{1+h}-1}{h}}$

~~(C)~~ $\lim_{h\to4}\dfrac{\sqrt{1+h}-\sqrt{h}}{h}$

~~(D)~~ $\lim_{h\to4}\dfrac{\sqrt{1+h}-1}{h}$

$$\lim_{h\to0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to0}\frac{\sqrt{4+h-3}-\sqrt{4-3}}{h}=\lim_{h\to0}\frac{\sqrt{h+1}-1}{h}$$

The following graph is a graph of the function $f(x)$ and it will be used for Problems 12-13.



12. **Using the graph above**, at how many points is $f(x)$ **not** differentiable?

(*A*) 3 (*B*) 4 (*C*) 5 $\boxed{(D)\ 6}$ (*E*) 7

13. **Using the graph above**, what is/are the horizontal asymptote(s) of $y = f(x)$?

(*A*) No horizontal asymptotes (*B*) $y = 1$ only (*C*) $y = -3$ only $\boxed{(D)\ y = -3 \text{ and } y = 1}$

14. Evaluate $\lim_{t \to 1} \frac{10t^2 - 3t + 6}{-2t^4 + 7t^3 + 1}$.

$$\frac{10(1)^2 - 3(1) + 6}{-2(1)^4 + 7(1)^3 + 1} = \frac{10 - 3 + 6}{-2 + 7 + 1} = \frac{13}{6}$$

(*A*) -5 (*B*) $-\infty$ $\boxed{(C)\ \frac{13}{6}}$ (*D*) 6 (*E*) Does not exist

1. Let $f(x) = \frac{2}{x+5}$. Use the **limit definition of the derivative** to find $f'(x)$. (NOTE: NO credit will be given if another method is used.)

$$\lim_{h \to 0} \frac{\frac{2}{x+h+5} - \frac{2}{x+5}}{h} \cdot \frac{(x+5)(x+h+5)}{(x+5)(x+h+5)} = \lim_{h \to 0} \frac{2(x+5) - 2(x+h+5)}{h(x+5)(x+h+5)}$$

$$= \lim_{h \to 0} \frac{2x+10-2x-2h-10}{h(x+5)(x+h+5)} = \lim_{h \to 0} \frac{-2h}{h(x+5)(x+h+5)} = \lim_{h \to 0} \frac{-2}{(x+5)(x+h+5)} = \frac{-2}{(x+5)(x+0+5)}$$

$$\boxed{f'(x) = \frac{-2}{(x+5)^2}}$$

2. Find all vertical and horizontal asymptotes of the graph $y = \frac{2x^2 - 4x - 6}{x^2 - 1}$.

$$\begin{array}{c|c} -12 & -4 \\ -6, 2 & \end{array}$$

$$y = \frac{2x^2-4x-6}{x^2-1} = \frac{2x^2-6x+2x-6}{x^2-1} = \frac{2x(x-3)+2(x-3)}{(x-1)(x+1)} = \frac{(2x+2)(x-3)}{(x-1)(x+1)}$$

$$y = \frac{2\cancel{(x+1)}(x-3)}{(x-1)\cancel{(x+1)}} = \frac{2(x-3)}{(x-1)} \; ; x \neq -1$$

$$\lim_{x \to \infty} \frac{2(x-3)}{(x-1)} = \lim_{x \to \infty} \frac{\frac{2(x-3)}{x}}{\frac{x-1}{x}}$$

$$= \lim_{x \to \infty} \frac{2-\frac{6}{x}}{1-\frac{1}{x}} = \frac{2-0}{1-0} = 2$$

$$\lim_{x \to -\infty} \frac{2-\frac{6}{x}}{1-\frac{1}{x}} = \frac{2-0}{1-0} = 2$$

$$\boxed{\begin{array}{l} \underline{\text{VA:}}\ x = 1 \\ \underline{\text{HA:}}\ y = 2 \end{array}}$$

3. Evaluate $\lim_{x\to 0} x^4 \cos\left(\frac{3}{x}\right)$.

Squeeze Theorem

$-1 \le \cos(x) \le 1$

$-1 \le \cos\left(\frac{3}{x}\right) \le 1$

$-x^4 \le x^4 \cos\left(\frac{3}{x}\right) \le x^4$

$\lim_{x\to 0} -x^4 \le \lim_{x\to 0} x^4 \cos\left(\frac{3}{x}\right) \le \lim_{x\to 0} x^4$

$0 \le \lim_{x\to 0} x^4 \cos\left(\frac{3}{x}\right) \le 0$

$\boxed{\text{By the squeeze theorem, } \lim_{x\to 0} x^4 \cos\left(\frac{3}{x}\right) = 0}$

4. Find an interval where the equation $x^3 + 2x + 1 = 0$ has at least one solution and explain why it has such a solution.

You can choose any interval you'd like.

$(-1,1)$

Using ~~IVT~~

@ -1

$(-1)^3 + 2(-1) + 1 = y$

$-1 - 2 + 1 = y$

$-3 + 1 = y$

$-2 = y$

@ 1

$(1)^3 + 2(1) + 1 = y$

$1 + 2 + 1 = y$

$4 = y$

$x^3 + 2x + 1 = 0$ has at least one solution on the interval $(-1,1)$ because $x^3 + 2x + 1$ is a continuous function & $f(-1) < 0$ and $f(1) > 0$.

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5. Consider the function

@0 Left: 2
Right: 1

@2 Left: $\frac{1}{3}$
Right: -1

$$f(x) = \begin{cases} x+2, & x<0 \\ \dfrac{x-1}{x^2-1}, & 0 \le x < 2 \\ \dfrac{1}{x-3}, & x \ge 2 \end{cases}$$

$$\frac{X-1}{X^2-1} = \frac{\cancel{X-1}}{(X+1)\cancel{(X-1)}} = \frac{1}{X+1};\ X \neq 1$$

hole

$X \neq 3$ (Infinite because VA)

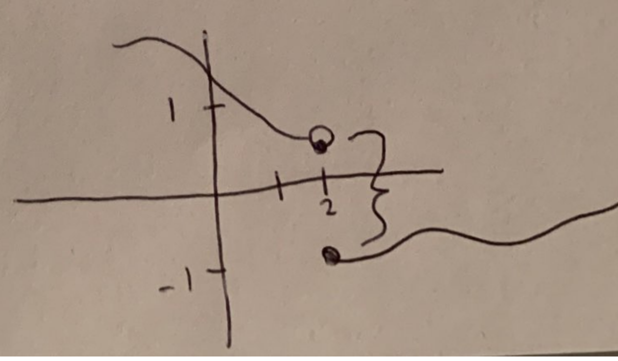
Give all the values of x at which each of the following types of discontinuities occur. If no such discontinuity occurs, write NA in the correct space.

Removable Discontinuity: $X=1$

Jump Discontinuity: $X=0$
$X=2$

0: Left: $(0)+2=2$ Jump
Right: $\frac{1}{(0)+1} = 1$

2: Left: $\frac{1}{(2)+1} = \frac{1}{3}$ Jump
Right: $\frac{1}{(2)-3} = \frac{1}{-1} = -1$



Infinite Discontinuity: $X=3$