1. How many of the following functions are continuous on the interval $(-1, \infty)$?

(i) $f(x) = x^2 + x + 2$

(ii) $g(x) = \ln(x)$

(iii) $h(x) = \sqrt{x-2}$

(iv) $k(x) = 3^x$

(A) 0 $\qquad$ (B) 1 $\qquad$ (C) 2 $\qquad$ (D) 3 $\qquad$ (E) 4

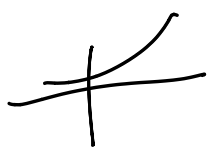
(i) cont. on $(-\infty, \infty)$ ✓

(ii) cont on $(0, \infty)$ X

(iii) cont if $x - 2 \geq 0$

$x \geq 2$ X

(iv) cont on $(-\infty, \infty)$ ✓



2. The displacement (in meters) of a particle moving in a straight line is given by $s(t) = t^2 + 8t + 18$, where t is measured in seconds. What is the average velocity of the particle over the time interval $[2, 4]$?

(A) 12 m/sec $\quad$ (B) 14 m/sec $\quad$ (C) 16 m/sec $\quad$ (D) 28 m/sec

$$\frac{s(4)-s(2)}{4-2} = \frac{[4^2+8(4)+18]-[2^2+8(2)+18]}{2}$$

$$= \frac{[16+32+18]-[4+16+18]}{2}$$

$$= \frac{\cancel{16}+32+\cancel{18}-4-\cancel{16}-\cancel{18}}{2}$$

$$= \frac{28}{2} = 14$$

3. Suppose $3x \le f(x) \le 9 - \frac{3}{4}x^2$. What is $\lim_{x\to 2^-} f(x)$?

(A) 0 (B) 3 (C) 6 (D) 9 (E) Does not exist

$$\lim_{x\to 2^-} 3x \le \lim_{x\to 2^-} f(x) \le \lim_{x\to 2^-} 9 - \frac{3}{4}x^2$$

$$3(2) \le \lim_{x\to 2^-} f(x) \le 9 - \frac{3}{4}(2)^2$$

$$6 \le \lim_{x\to 2^-} f(x) \le 9 - \frac{3(4)}{4}$$

$$6 \le \lim_{x\to 2^-} f(x) \le 9 - 3$$

$$6 \le \lim_{x\to 2^-} f(x) \le 6$$

4. Let $f(x) = \dfrac{1}{(x+3)^2}$. Which one of the following statements concerning $f(x)$ is correct?

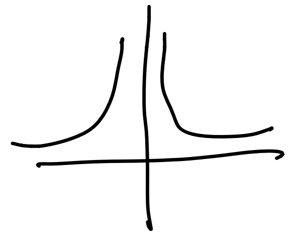
(A) $\lim_{x \to -3^-} f(x) = \infty$ and $x = -3$ is a vertical asymptote of $y = f(x)$

(B) $\lim_{x \to -3^-} f(x) = -\infty$ and $x = -3$ is a vertical asymptote of $y = f(x)$

(C) $\lim_{x \to 3^-} f(x) = \infty$ and $y = 0$ is a vertical asymptote of $y = f(x)$

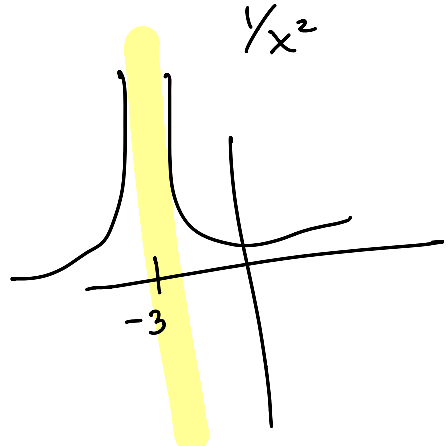
(D) $\lim_{x \to 3^-} f(x) = -\infty$ and $y = 0$ is a vertical asymptote of $y = f(x)$

(E) None of these



$1/x^2$

$\lim_{x \to -3^-} = \infty$



5. The function $f(x) = x^2 + 3x - 3$ is guaranteed to have a root in the interval $[-2, 2]$ by the Intermediate Value Theorem.

(A) True

(B) False

We can use IVT since $f(x)$ is cont. on $(-\infty, \infty)$

Check:

$f(-2) = (-2)^2 + 3(-2) - 3$

$= 4 - 6 - 3 = -5$

$f(2) = (2)^2 + 3(2) - 3$

$= 4 + 6 - 3$

$= 7$

Since $f(-2) < 0$ and $f(2) > 0$

6. Evaluate $\lim_{x \to 1} \frac{\sqrt{3x+6}-3}{x-1}$.

(A) $\frac{1}{2}$ (B) $\frac{1}{6}$ (C) 0 (D) $\frac{1}{3}$ (E) Does not exist

$$\lim_{x \to 1} \frac{\sqrt{3x+6}-3}{x-1} \cdot \frac{\sqrt{3x+6}+3}{\sqrt{3x+6}+3}$$

$$\lim_{x \to 1} \frac{3x+6-3\sqrt{3x+6}+3\sqrt{3x+6}-9}{(x-1)(\sqrt{3x+6}+3)}$$

$$\lim_{x \to 1} \frac{3x+6-9}{(x-1)(\sqrt{3x+6}+3)}$$

$$\lim_{x \to 1} \frac{3x-3}{(x-1)(\sqrt{3x+6}+3)}$$

$$\lim_{x \to 1} \frac{3\cancel{(x-1)}}{\cancel{(x-1)}(\sqrt{3x+6}+3)}$$

$$\lim_{x \to 1} \frac{3}{\sqrt{3x+6}+3} = \frac{3}{\sqrt{3(1)+6}+3} = \frac{3}{\sqrt{9}+3}$$

$$= \frac{3}{3+3} = \frac{3}{6} = \frac{1}{2}$$

7. Let $f(x) = \dfrac{x+2}{x^2-4}$. Which one of the following statements is correct?

(A) $\lim\limits_{x\to -2^+} f(x) = \infty$ $= \frac{-1}{4}$ X

(B) $x = -2$ is a vertical asymptote of $f(x)$ X at $x=2$

(C) $\lim\limits_{x\to 2^-} f(x) = -\infty$

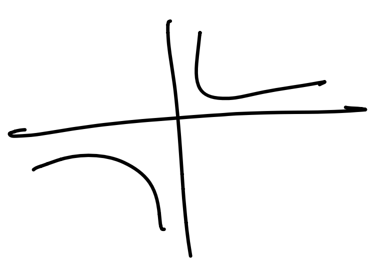
(D) $\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^+} f(x)$ $-\infty \neq \infty$ X

(E) None of these

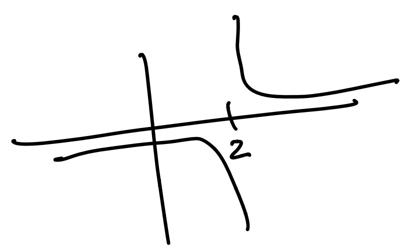
$$f(x) = \frac{x+2}{x^2-4} = \frac{x+2}{(x-2)(x+2)} = \frac{1}{x-2}$$

$$\lim_{x\to -2^+} \frac{1}{x-2} = \frac{1}{-2-2} = \frac{1}{-4}$$

$$\lim_{x\to 2^-} f(x) = -\infty \qquad \lim_{x\to 2^+} f(x) = \infty$$



$1/x$



8. Which of the following statements is necessarily true?

(A) If $f(x)$ and $g(x)$ are continuous, then $f(x)+g(x)$ is discontinuous ✗

(B) If $f(x)$ and $g(x)$ are continuous, then $f(x)g(x)$ is continuous

(C) If $f(x)$ is discontinuous and $g(x)$ is continuous, then $f(x)+g(x)$ is continuous ✗

(D) If $f(x)$ is discontinuous, then $cf(x)$ is continuous where c is any real number ✗

A function is cont. if

$$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$$

For our examples let the interval be $(-\infty, \infty)$

(A) $f(x) = x^3$, $g(x) = x^2$ > both are cont.

$x^3 + x^2$ is still cont. since it's a polynomial

(B) $f(x) = x^3$, $g(x) = x^2$ > both are cont.

$f(x)\,g(x) = (x^3)(x^2) = x^5$ which is cont.

(C) $f(x) = \ln(x)$ is discont on $(-\infty, \infty)$ since it's domain is $(0, \infty)$

$g(x) = x^2$

$f(x) + g(x) = \ln(x) + x^2$ is discont. on $(-\infty, \infty)$

(D) $f(x) = \ln(x)$ is discont

$c = 2$ (some constant)

$c f(x) = 2\ln(x)$ which is discont. on $(-\infty, \infty)$

9. Let

$$f(x) = \begin{cases} \dfrac{x^2+3x+2}{x^2-1}, & x \neq 1 \\ 3, & x = 1 \end{cases}.$$

Which one of the following statements concerning $f(x)$ is correct?

(A) $f(-1) = -\frac{1}{2}$ and $\lim_{x\to 1} f(x)$ does not exist ✗

(B) $f(-1)$ is undefined and $\lim_{x\to 1} f(x)$ does not exist

(C) $f(-1)$ is undefined and $\lim_{x\to 1} f(x) = 3$

(D) $f(-1) = -\frac{1}{2}$ and $\lim_{x\to 1} f(x) = 3$ ✗

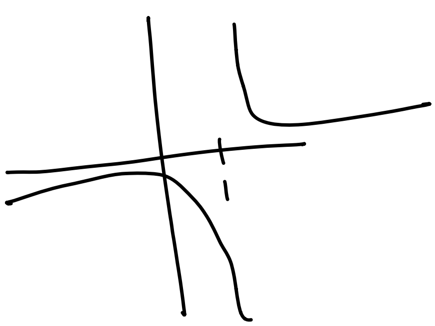
(E) None of these

f(-1) is undefined since this makes a hole

$$\frac{x^2+3x+2}{x^2-1} = \frac{(x+2)\cancel{(x+1)}}{\cancel{(x+1)}(x-1)} = \frac{x+2}{x-1}$$

$$f(x) = \begin{cases} \dfrac{x+2}{x-1}, & x \neq 1 \\ 3, & x = 1 \end{cases}$$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1} \frac{x+2}{x-1} = \frac{1+2}{1-1} = \frac{3}{0} \to \text{check graph}$$



$$\lim_{x\to 1^-} f(x) = -\infty$$

$$\lim_{x\to 1^+} f(x) = \infty$$

so $\lim_{x\to 1^-} \neq \lim_{x\to 1^+}$

and $\lim_{x\to 1}$ DNE

10. For which value of k is the following function continuous for all real numbers?

$$f(x) = \begin{cases} k2^x, & x < 2 \\ x^2 + 4kx, & x \geq 2 \end{cases}$$

(A) 0 (B) 2 (C) 1 (D) -1 (E) No such value exists

Cont if $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} k2^x = k2^2 = 4k$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} x^2 + 4kx = 2^2 + 4k(2) = 4 + 8k$

$$4k = 4 + \cancel{8k}$$
$$-8k \quad \cancel{-8k}$$
$$\frac{-4k}{-4} = \frac{4}{-4}$$

$k = -1$

11. Let $f(x) = \sqrt{x-3}$. Which of the following is equal to $f'(4)$?

(A) $\lim_{h\to 0} \frac{\sqrt{1+h}-\sqrt{h}}{h}$

(B) $\lim_{h\to 0} \frac{\sqrt{1+h}-1}{h}$

(C) $\lim_{h\to 4} \frac{\sqrt{1+h}-\sqrt{h}}{h}$

(D) $\lim_{h\to 4} \frac{\sqrt{1+h}-1}{h}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

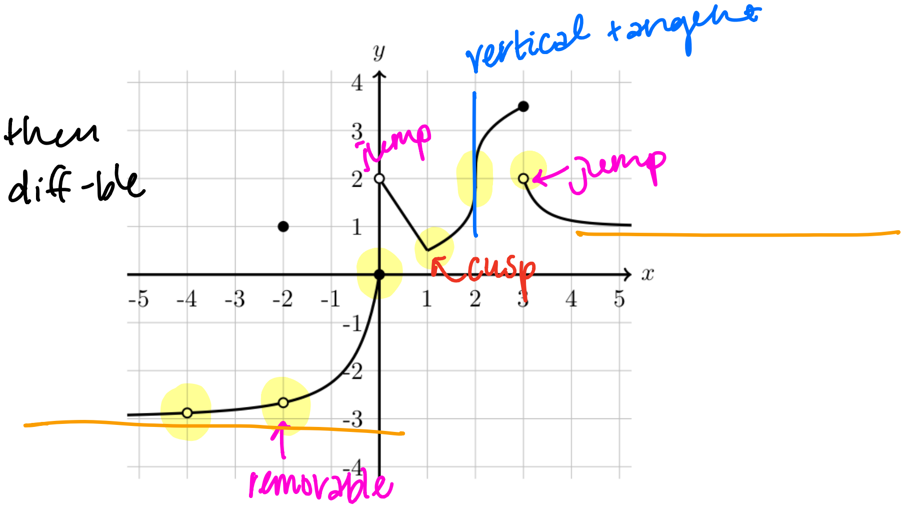
$$f'(x) = \lim_{h\to 0} \frac{\sqrt{x+h-3} - \sqrt{x-3}}{h}$$

$$f'(4) = \lim_{h\to 0} \frac{\sqrt{4+h-3} - \sqrt{4-3}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{1+h} - 1}{h}$$

The following graph is a graph of the function $f(x)$ and it will be used for Problems 12-13.

if f(x) is not cont. then it is not diff-ble



12. **Using the graph above**, at how many points is $f(x)$ **not** differentiable?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

13. **Using the graph above**, what is/are the horizontal asymptote(s) of $y = f(x)$?

(A) No horizontal asymptotes (B) $y = 1$ only (C) $y = -3$ only (D) $y = -3$ and $y = 1$

14. Evaluate $\lim_{t\to 1} \dfrac{10t^2 - 3t + 6}{-2t^4 + 7t^3 + 1}$.

(A) -5 (B) $-\infty$ (C) $\dfrac{13}{6}$ (D) 6 (E) Does not exist

$$\lim_{t \to 1} \frac{10t^2 - 3t + 6}{-2t^4 + 7t^3 + 1}$$

$$\frac{10(1)^2 - 3(1) + 6}{-2(1)^4 + 7(1)^3 + 1}$$

$$\frac{10 - 3 + 6}{-2 + 7 + 1} = \frac{7+6}{8-2} = \frac{13}{6}$$

1. Let $f(x) = \dfrac{2}{x+5}$. Use the **limit definition of the derivative** to find $f'(x)$. (NOTE: NO credit will be given if another method is used.)

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \left(\frac{2}{x+h+5} - \frac{2}{x+5} \right) \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \left[\frac{2}{(x+h+5)} \cdot \frac{(x+5)}{(x+5)} - \frac{2}{(x+5)} \cdot \frac{(x+h+5)}{(x+h+5)} \right] \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \left[\frac{2(x+5) - 2(x+h+5)}{(x+h+5)(x+5)} \right] \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \left[\frac{\cancel{2x} \cancel{+10} - \cancel{2x} - 2h \cancel{-10}}{(x+h+5)(x+5)} \right] \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \left[\frac{-2\cancel{h}}{(x+h+5)(x+5)} \right] \cdot \frac{1}{\cancel{h}} = \frac{-2}{(x+0+5)(x+5)}$$

$$= \frac{-2}{(x+5)^2}$$

2. Find all vertical and horizontal asymptotes of the graph $y = \dfrac{2x^2 - 4x - 6}{x^2 - 1}$.

$$y = \frac{2x^2-4x-6}{x^2-1} = \frac{2(x^2-2x-3)}{(x+1)(x-1)}$$

$$= \frac{2\cancel{(x+1)}(x-3)}{\cancel{(x+1)}(x-1)} = \frac{2(x-3)}{x-1}$$

V.A. denom. $= 0$

$x - 1 = 0$

$\boxed{x = 1}$

H. A $\lim_{x\to\infty} \dfrac{2x^2-4x-6}{x^2-1} = \dfrac{2}{1}$

$\boxed{y = 2}$

3. Evaluate $\lim_{x \to 0} x^4 \cos\left(\frac{3}{x}\right)$.

$x^4 \left[-1 \le \cos\left(\frac{3}{x}\right) \le 1 \right]$

$-x^4 \le x^4 \cos\left(\frac{3}{x}\right) \le x^4$

$\lim_{x \to 0} -x^4 \le \lim_{x \to 0} x^4 \cos\left(\frac{3}{x}\right) \le \lim_{x \to 0} x^4$

$0^4 \le \lim_{x \to 0} x^4 \cos\left(\frac{3}{x}\right) \le 0^4$

so $\lim_{x \to 0} x^4 \cos\left(\frac{3}{x}\right) = 0$ by the Squeeze Thm

f

4. Find an interval where the equation $x^3 + 2x + 1 = 0$ has at least one solution and explain why it has such a solution.

Use IVT since x^3+2x+1 is cont. on $(-\infty,\infty)$

Try $[-1,0]$

$f(-1) = (-1)^3 + 2(-1) + 1$

$= -1 - 2 + 1 = -2$

$f(0) = 0^3 + 2(0) + 1$

$= 1$

Since $f(-1) < 0$ and $f(0) > 0$

$f(-1) < 0 < f(0)$

there exists a number c in $(-1,0)$ such that $f(c) = 0$ by the IVT.

This means $x^3 + 2x + 1 = 0$ has a solution in the interval $(-1,0)$

* talk about "sol" vs. "root"

5. Consider the function

Check 0, 2
1, -1
3

$$f(x) = \begin{cases} x+2, & x < 0 \\ \dfrac{x-1}{x^2-1}, & 0 \le x < 2 \\ \dfrac{1}{x-3}, & x \ge 2 \end{cases}.$$

Give all the values of x at which each of the following types of discontinuities occur. If no such discontinuity occurs, write NA in the correct space.

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}(x+2) = 0+2 = 2$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{x-1}{x^2-1} = \frac{0-1}{0^2-1} = \frac{-1}{-1} = 1$$

Since $\lim_{x\to 0^+} f(x) \neq \lim_{x\to 0^-} f(x)$ there is a jump discontinuity at $x=0$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \frac{x-1}{x^2-1} = \frac{2-1}{2^2-1} = \frac{1}{4-1} = \frac{1}{3}$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \frac{1}{x-3} = \frac{1}{2-3} = \frac{1}{-1} = -1$$

Since $\lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x)$ there is a jump discontinuity at $x=2$

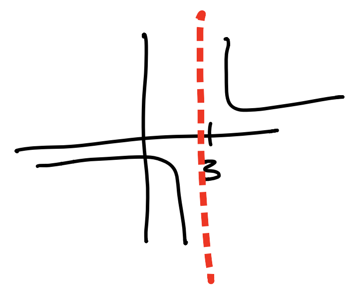
---

when $0 \le x < 2$ $f(x) = \dfrac{x-1}{(x-1)(x+1)} = \dfrac{1}{x+1}$

since $x=1$ $0 \le x < 2$ $x=1$ is a removable discontinuity

And since $x=-1$ is not $0 \le x < 2$
$x=-1$ is not an infinite discontinuity

$$f(x) = \begin{cases} x+2, & x<0 \\ \dfrac{x-1}{x^2-1}, & 0 \le x < 2 \\ \dfrac{1}{x-3}, & x \ge 2 \end{cases}.$$



when $x \geq 2$ $\quad f(x) = \dfrac{1}{x-3}$

Because $3 \geq 2$, there is an infinite discontinuity at $x=3$