Calculus I: MAC2311 **Name:** ____________________

Exam 1

Part I Instructions: 14 multiple choice questions

Exam 1
MC Key

1	C
2	B
3	C
4	A
5	A
6	A
7	C
8	B
9	B
10	D
11	B
12	D
13	D
14	C

Calculus I: MAC2311 **Name:** ____________________

Exam 1

Part II Instructions: 5 free response questions

For Instructor Use Only:

FR 1	
FR 2	
FR 3	
FR 4	
FR 5	
Total Points	

1. Let $f(x) = \dfrac{2}{x+5}$. Use the **limit definition of the derivative** to find $f'(x)$. (NOTE: NO credit will be given if another method is used.)

Solution

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{\frac{2}{x+h+5} - \frac{2}{x+5}}{h} \\
&= \lim_{h\to 0} \left(\frac{2}{x+h+5} \cdot \frac{x+5}{x+5} - \frac{2}{x+5} \cdot \frac{x+h+5}{x+h+5} \right) \cdot \frac{1}{h} \\
&= \lim_{h\to 0} \frac{2x+10-2x-2h-10}{(x+h+5)(x+5)} \cdot \frac{1}{h} \\
&= \lim_{h\to 0} \frac{-2h}{(x+h+5)(x+5)} \cdot \frac{1}{h} \\
&= \lim_{h\to 0} \frac{-2}{(x+h+5)(x+5)} \\
&= \frac{-2}{(x+5)^2}
\end{aligned}$$

2. Find all vertical and horizontal asymptotes of the graph $y = \dfrac{2x^2 - 4x - 6}{x^2 - 1}$.

Solution

Vertical Asymptote:

$$y = \frac{2(x^2 - 2x - 3)}{x^2 - 1} = \frac{2(x-3)(x+1)}{(x-1)(x+1)} = \frac{2(x-3)}{x-1}, x \neq -1$$

Vertical asymptote is $x = 1$

Horizontal Asymptote:

$$\lim_{x\to\infty} \frac{2x^2 - 4x - 6}{x^2 - 1} = \lim_{x\to\infty} \frac{2x^2 - 4x - 6}{x^2 - 1} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to\infty} \frac{2 - \frac{4}{x} - \frac{6}{x^2}}{1 - \frac{1}{x^2}} = \frac{2 - 0 - 0}{1 - 0} = 2$$

Horizontal asymptote is $y = 2$

3. Evaluate $\lim_{x\to 0} x^4 \cos\left(\frac{3}{x}\right)$.

Solution

$$\begin{aligned} -1 &\leq \cos\left(\frac{3}{x}\right) \leq 1 \\ -x^4 &\leq x^4\cos\left(\frac{3}{x}\right) \leq x^4 \end{aligned}$$

Because $\lim_{x\to 0} x^4 = 0$ and $\lim_{x\to 0} -x^4 = 0$, $\lim_{x\to 0} x^4 \cos\left(\frac{3}{x}\right) = 0$ by the Squeeze Theorem.

4. Find an interval where the equation $x^3 + 2x + 1 = 0$ has at least one solution and explain why it has such a solution.

Solution Consider the interval $[-1, 0]$. Let $f(x) = x^3 + 2x + 1$. Notice that $f(x)$ is a polynomial and is therefore continuous on its domain of all real numbers. $f(x)$ is therefore continuous on $[-1, 0]$. $f(-1) = -2$ and $f(0) = 1$. Because $f(-1) < 0 < f(0)$, there exists a number c in $(-1, 0)$ such that $f(c) = 0$ by the Intermediate Value Theorem. That is, $x^3 + 2x + 1 = 0$ has a solution in the interval $(-1, 0)$.

5. Consider the function

$$f(x) = \begin{cases} x+2, & x<0 \\ \dfrac{x-1}{x^2-1}, & 0 \le x < 2 \\ \dfrac{1}{x-3}, & x \ge 2 \end{cases}.$$

Give all the values of x at which each of the following types of discontinuities occur. If no such discontinuity occurs, write NA in the correct space.

Removable Discontinuity:

Jump Discontinuity:

Infinite Discontinuity:

Solution $\lim_{x\to 0^-} f(x) = 2$ and $\lim_{x\to 0^+} f(x) = 1$. Because the limits are not equal, there is a jump discontinuity at $x = 0$. $\lim_{x\to 2^-} f(x) = \dfrac{1}{3}$ and $\lim_{x\to 2^+} f(x) = -1$. Because the limits are not equal, there is a jump discontinuity at $x = 2$.

When $0 \le x < 2$, $f(x) = \dfrac{x-1}{(x-1)(x+1)} = \dfrac{1}{x+1}, x \neq 1$. Because 1 is in $[0,2)$, there is a removable discontinuity at $x = 1$. Because -1 is not in $[0,2)$, there is not an infinite discontinuity at $x = -1$.

When $x \ge 2$, $f(x) = \dfrac{1}{x-3}$. Because 3 is in $[2,\infty)$, there is an infinite discontinuity at $x = 3$.