Calculus I: MAC2311 **Name:** _____

Exam 1

Part I Instructions: 14 multiple choice questions

1. How many of the following functions are continuous on the interval $(-1, \infty)$?

(i) $f(x) = x^2 + x + 2$

(ii) $g(x) = \ln(x)$

(iii) $h(x) = \sqrt{x-2}$

(iv) $k(x) = 3^x$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

2. The displacement (in meters) of a particle moving in a straight line is given by $s(t) = t^2 + 8t + 18$, where t is measured in seconds. What is the average velocity of the particle over the time interval $[2, 4]$?

(A) 12 m/sec (B) 14 m/sec (C) 16 m/sec (D) 28 m/sec

3. Suppose $3x \leq f(x) \leq 9 - \frac{3}{4}x^2$. What is $\lim_{x \to 2^-} f(x)$?

(*A*) 0 (*B*) 3 (*C*) 6 (*D*) 9 (*E*) Does not exist

4. Let $f(x) = \frac{1}{(x+3)^2}$. Which one of the following statements concerning $f(x)$ is correct?

(*A*) $\lim_{x \to -3^-} f(x) = \infty$ and $x = -3$ is a vertical asymptote of $y = f(x)$
(*B*) $\lim_{x \to -3^-} f(x) = -\infty$ and $x = -3$ is a vertical asymptote of $y = f(x)$
(*C*) $\lim_{x \to 3^-} f(x) = \infty$ and $y = 0$ is a vertical asymptote of $y = f(x)$
(*D*) $\lim_{x \to 3^-} f(x) = -\infty$ and $y = 0$ is a vertical asymptote of $y = f(x)$
(*E*) None of these

5. The function $f(x) = x^2 + 3x - 3$ is guaranteed to have a root in the interval $[-2, 2]$ by the Intermediate Value Theorem.

(*A*) True (*B*) False

6. Evaluate $\lim_{x\to 1} \dfrac{\sqrt{3x+6}-3}{x-1}$.

$(A)\ \dfrac{1}{2}$ $(B)\ \dfrac{1}{6}$ $(C)\ 0$ $(D)\ \dfrac{1}{3}$ (E) Does not exist

7. Let $f(x) = \dfrac{x+2}{x^2-4}$. Which one of the following statements is correct?

$(A)\ \lim_{x\to -2^+} f(x) = \infty$

$(B)\ x = -2$ is a vertical asymptote of $f(x)$

$(C)\ \lim_{x\to 2^-} f(x) = -\infty$

$(D)\ \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$

(E) None of these

8. Which of the following statements is necessarily true?

(A) If $f(x)$ and $g(x)$ are continuous, then $f(x)+g(x)$ is discontinuous

(B) If $f(x)$ and $g(x)$ are continuous, then $f(x)g(x)$ is continuous

(C) If $f(x)$ is discontinuous and $g(x)$ is continuous, then $f(x)+g(x)$ is continuous

(D) If $f(x)$ is discontinuous, then $cf(x)$ is continuous where c is any real number

9. Let

$$f(x) = \begin{cases} \dfrac{x^2+3x+2}{x^2-1}, & x \neq 1 \\ 3, & x = 1 \end{cases}.$$

Which one of the following statements concerning $f(x)$ is correct?

(A) $f(-1) = -\frac{1}{2}$ and $\lim\limits_{x\to 1} f(x)$ does not exist
(B) $f(-1)$ is undefined and $\lim\limits_{x\to 1} f(x)$ does not exist
(C) $f(-1)$ is undefined and $\lim\limits_{x\to 1} f(x) = 3$
(D) $f(-1) = -\frac{1}{2}$ and $\lim\limits_{x\to 1} f(x) = 3$
(E) None of these

10. For which value of k is the following function continuous for all real numbers?

$$f(x) = \begin{cases} k2^x, & x < 2 \\ x^2 + 4kx, & x \geq 2 \end{cases}$$

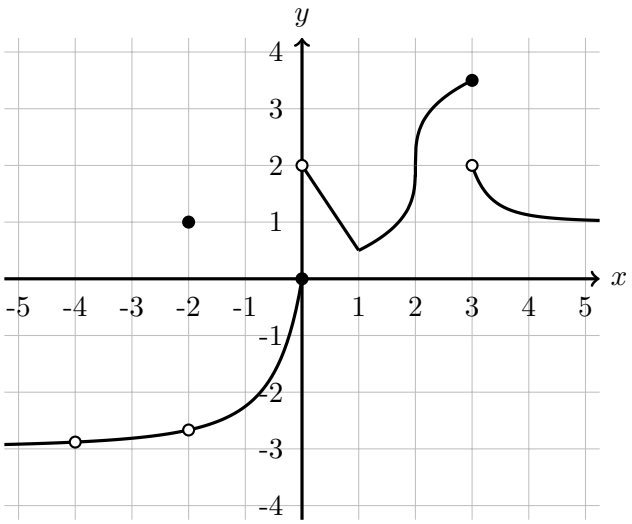
(A) 0 (B) 2 (C) 1 (D) -1 (E) No such value exists

11. Let $f(x) = \sqrt{x-3}$. Which of the following is equal to $f'(4)$?

(A) $\lim\limits_{h\to 0} \dfrac{\sqrt{1+h}-\sqrt{h}}{h}$ (B) $\lim\limits_{h\to 0} \dfrac{\sqrt{1+h}-1}{h}$

(C) $\lim\limits_{h\to 4} \dfrac{\sqrt{1+h}-\sqrt{h}}{h}$ (D) $\lim\limits_{h\to 4} \dfrac{\sqrt{1+h}-1}{h}$

The following graph is a graph of the function $f(x)$ and it will be used for Problems 12-13.



12. **Using the graph above**, at how many points is $f(x)$ **not** differentiable?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

13. **Using the graph above**, what is/are the horizontal asymptote(s) of $y = f(x)$?

(A) No horizontal asymptotes (B) $y = 1$ only (C) $y = -3$ only (D) $y = -3$ and $y = 1$

14. Evaluate $\lim\limits_{t\to 1} \dfrac{10t^2 - 3t + 6}{-2t^4 + 7t^3 + 1}$.

(A) -5 (B) $-\infty$ (C) $\dfrac{13}{6}$ (D) 6 (E) Does not exist

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Exam 1

Part II Instructions: 5 free response questions

For Instructor Use Only:

FR 1	
FR 2	
FR 3	
FR 4	
FR 5	
Total Points	

1. Let $f(x) = \dfrac{2}{x+5}$. Use the **limit definition of the derivative** to find $f'(x)$. (NOTE: NO credit will be given if another method is used.)

2. Find all vertical and horizontal asymptotes of the graph $y = \dfrac{2x^2 - 4x - 6}{x^2 - 1}$.

3. Evaluate $\lim_{x \to 0} x^4 \cos\left(\frac{3}{x}\right)$.

4. Find an interval where the equation $x^3 + 2x + 1 = 0$ has at least one solution and explain why it has such a solution.

5. Consider the function

$$f(x) = \begin{cases} x+2, & x < 0 \\ \dfrac{x-1}{x^2-1}, & 0 \le x < 2 \\ \dfrac{1}{x-3}, & x \ge 2 \end{cases}.$$

Give all the values of x at which each of the following types of discontinuities occur. If no such discontinuity occurs, write NA in the correct space.

Removable Discontinuity:

Jump Discontinuity:

Infinite Discontinuity: