Calculus I: MAC2311
Fall 2022
Exam 1 A
9/22/2022
Time Limit: 90 Minutes

Name: Solutions

Section: ______________________

UF-ID: ______________________

Scantron Instruction: This exam uses a scantron. Follow the instructions listed on this page to fill out the scantron.

A. Sign your scantron **on the back** at the bottom in the white area.

B. Write **and code** in the spaces indicated:

1) Name (last name, first initial, middle initial)
2) UFID Number
3) 4-digit Section Number

C. Under *special codes*, code in the test numbers 1, 1:

- • 2 3 4 5 6 7 8 9 0
- • 2 3 4 5 6 7 8 9 0

D. At the top right of your scantron, fill in the *Test Form Code* as A.

- • B C D E

E. This exam consists of 14 multiple choice questions and 5 free response questions. Make sure you check for errors in the number of questions your exam contains.

F. The time allowed is 90 minutes.

G. WHEN YOU ARE FINISHED:

1) Before turning in your test check for **transcribing errors**. Any mistakes you leave in are there to stay!

2) You must turn in your scantron to your proctor. **Be prepared to show your GatorID with a legible signature.**

It is your responsibility to ensure that your test has **19 questions**. If it does not, show it to your proctor immediately. You will not be permitted to make up any problems omitted from your test after the testing period ends. There are a total of 105 points available on this exam.

Part I Instructions: 14 multiple choice questions. Complete the scantron sheet provided with your information and fill in the appropriate spaces to answer your questions. Only the answer on the scantron sheet will be graded. Each problem is worth five (5) points for a total of 70 points on Part I.

1. For how many of the following values of a will $y = \frac{1}{2}$ be a horizontal asymptote of the function

$$g(x) = \frac{(x^3+2)^2}{2(x^a-1)}$$

$$\lim_{x\to\infty} \frac{(x^3+2)^2}{2(x^a-1)} = \lim_{x\to\infty} \frac{x^6+4x^3+4}{2x^a-2}$$

$$= \lim_{x\to\infty} \frac{\frac{x^6}{x^a}+\frac{4x^3}{x^a}+\frac{4}{x^a}}{2-\frac{2}{x^a}}$$

if $a=6$

$$= \lim_{x\to\infty} \frac{1+\frac{4}{x^3}+\frac{4}{x^6}}{2-\frac{2}{x^6}} = \frac{1+0+0}{2-0} = \frac{1}{2}$$

(*i*) $a = 2$

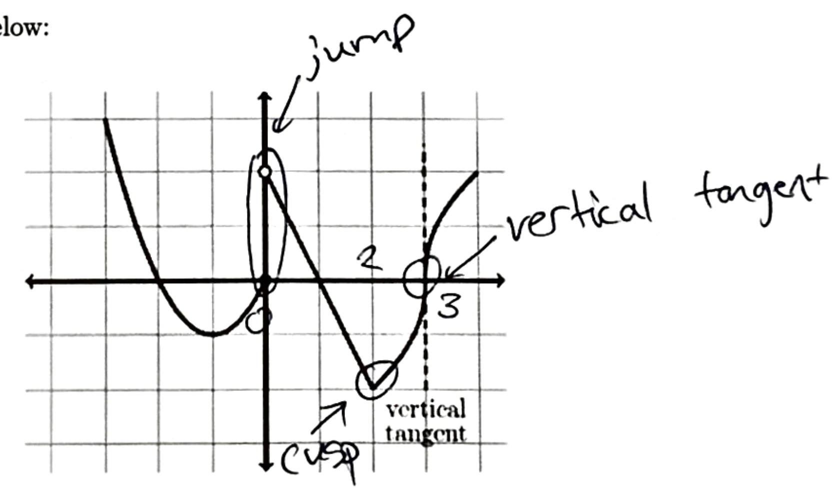
(*ii*) $a = 3$

(*iii*) $a = 4$

(*iv*) $a = 6$

(*A*) 0 (*B*) 1 (*C*) 2 (*D*) 3 (*E*) 4

2. Use the graph of $f(x)$ below:



Which of the following lists contains all x-values where $f(x)$ fails to be differentiable?

(*A*) $x = 0, 2, 3$ (*B*) $x = 0, 2$ (*C*) $x = 2, 3$ (*D*) $x = -1, 2, 3$ (*E*) $x = 0, 3$

3. Evaluate $\lim_{x\to 3^+} \frac{|x-3|}{2x-6}$

(A) $-\frac{1}{2}$ (B) 0 (C) $\frac{1}{2}$ (D) 2 (E) Does not exist

Note $\frac{|x-3|}{2x-6} = \begin{cases} \frac{x-3}{2x-6}, & x-3 \geq 0 \\ \frac{-(x-3)}{2x-6}, & x-3 < 0 \end{cases} = \begin{cases} \frac{x-3}{2(x-3)}, & x \geq 3 \\ \frac{-(x-3)}{2(x-3)}, & x < 3 \end{cases}$

$= \begin{cases} \frac{1}{2}, & x \geq 3 \\ -\frac{1}{2}, & x < 3 \end{cases}$

So $\lim_{\substack{x\to 3^+ \\ x>3}} \frac{|x-3|}{2x-6} = \lim_{x\to 3^+} \frac{1}{2} = \frac{1}{2}$

4. Evaluate $\lim_{x\to 1} \frac{\sqrt{x+3}+2}{x+1}$

(A) 2 (B) 0 (C) -2 (D) 4 (E) Does not exist

$\lim_{x\to 1} \frac{\sqrt{x+3}+2}{x+1} = \frac{\sqrt{1+3}+2}{1+1} = \frac{2+2}{2} = \frac{4}{2} = 2$

5. Suppose $f(x)$ and $g(x)$ are continuous functions for all real numbers x. How many of the following statements are necessarily true?

I. $(f+g)(x)$ is a continuous function for all real numbers x. ✓

II. $cf(x)$ is a continuous function for all real numbers x and for any real constant c. ✓

III. $\left(\frac{f}{g}\right)(x)$ is a continuous function for all real numbers x. X (what if $g(x)=0$?)

IV. $\lim_{x\to a} f(x)$ exists for every real number a. ✓

(*A*) 0 (*B*) 1 (*C*) 2 (*D*) 3 (*E*) 4

6. If the function $f(x)$ has a vertical asymptote at $x=a$ which of the following must be true?

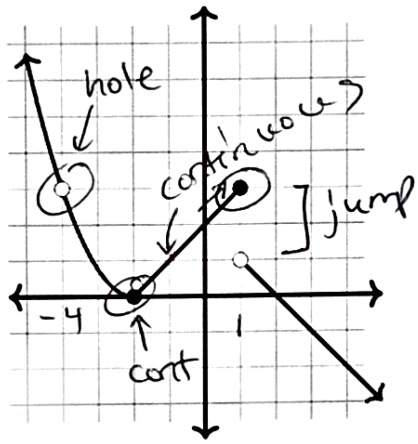
I. $\lim_{x\to a^+} f(x) = \pm\infty$ or $\lim_{x\to a^-} f(x) = \pm\infty$ (Definition) ✓

II. $\lim_{x\to a} f(x)$ exists X

III. $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x)$ X

(*A*) only *I* (*B*) only *II* (*C*) only *I* and *II* (*D*) only *I* and *III* (*E*) *I*, *II*, and *III*

7. Given the following graph of the function $f(x)$, which of the following statement(s) must be true?



$I.$ $f(x)$ has a non-removable discontinuity at $x = -4$ X

$II.$ $\lim_{x \to 1^-} f(x) = f(1)$ ✓

$III.$ $f(x)$ has a jump discontinuity at $x = 1$ ✓

$IV.$ $f(x)$ is not continuous at $x = -2$ X

(A) I and II only (B) II and III only (C) III and IV only (D) I and III only

8. Let $f(x)$ be a function such that $\overbrace{f'(3) = -2}^{\text{slope}}$ and $f(3) = 5$. (3,5) point. Which of the following lines is tangent to the graph of $y = f(x)$ at $x = 3$?

(A) $y = -2x + 5$ (B) $y = -2x + 11$ (C) $y = 5x - 17$ (D) $y = 5x - 2$

point-slope: $y - 5 = -2(x-3)$

$\Rightarrow y = -2x + 6 + 5$

$\Rightarrow y = -2x + 11$

9. Suppose that $0 \leq f(x) \leq 2$ for all x near $x = 0$. What is $\lim_{x\to 0} x f(x)$?

(A) 0 (B) 1 (C) -1 (D) ∞ (E) Does not exist

$0 \leq f(x) \leq 2$

$\Rightarrow 0 \leq x f(x) \leq 2x$ for $x > 0$

or $0 \geq x f(x) \geq 2x$ for $x < 0$

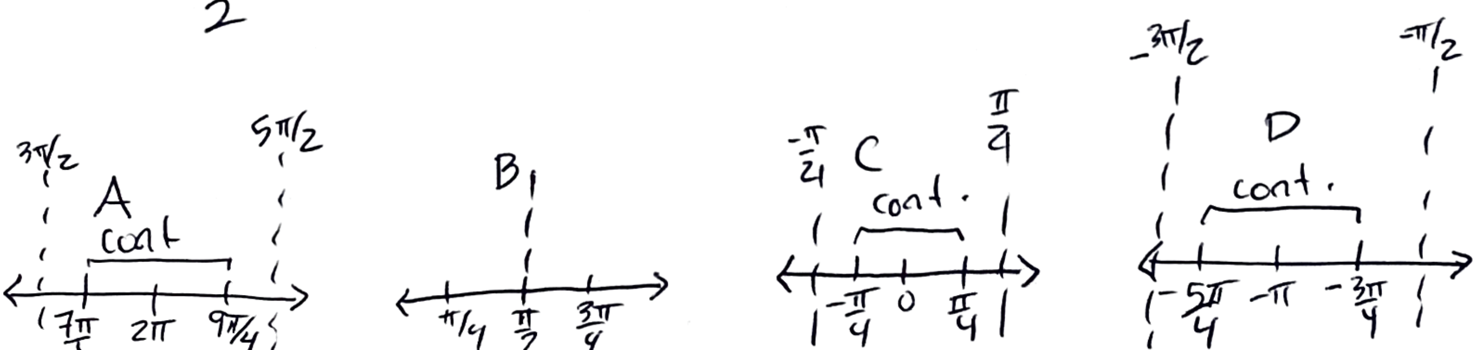
both times $\lim_{\substack{x\to 0^+ \\ x \to 0^-}} 0 = 0 = \lim_{\substack{x\to 0^+ \\ x \to 0^-}} 2x$

So by the Squeeze Theorem $\lim_{x\to 0} x f(x) = 0$

10. Consider the function $g(x) = 2\tan(x)$. Over which of the following intervals is the Intermediate Value Theorem **not** applicable in showing the existence of a zero (root) of $g(x)$ over that interval?

(A) $\left[\frac{7\pi}{4}, \frac{9\pi}{4}\right]$ (B) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ (C) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ (D) $\left[-\frac{5\pi}{4}, -\frac{3\pi}{4}\right]$

$g(x) = 2\tan(x)$ has vertical asymptotes (discontinuities) at $x = (2n+1)\cdot\frac{\pi}{2}$ for n an integer $(\ldots, -3, -2, -1, 0, 1, 2, \ldots)$ so at any odd multiple of $\frac{\pi}{2}$. We see that $\frac{\pi}{2}$ is in the interval $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$



$+5 = f(2)$

11. For which of the following functions does $f'(2) = \lim_{h\to 0} \dfrac{\frac{h+5}{h+1} - 5}{h}$?

(A) $f(x) = -\dfrac{x+5}{x+1}$ (B) $f(x) = -\dfrac{x+3}{x-1}$ (C) $f(x) = \dfrac{x+3}{x-1}$ (D) $f(x) = \dfrac{x+5}{x+1}$

Recall $f'(2) = \lim_{h\to 0} \dfrac{f(2+h) - f(2)}{h}$ now

A: $f(2) = -\dfrac{2+5}{2+1} = -\dfrac{7}{3}$ ✗

B: $f(2) = -\dfrac{2+3}{2-1} = -\dfrac{5}{1} = -5$ ✗

C: $f(2) = \dfrac{2+3}{2-1} = \dfrac{5}{1} = 5$ ✓

D: $f(2) = \dfrac{2+5}{2+1} = \dfrac{7}{3}$ ✗

12. Let

$$g(x) = \begin{cases} x^3 + k^2, & x < 2 \\ 0, & x = 2 \\ kx^2 + kx, & x > 2 \end{cases}$$

Find all real values of k such that $\lim_{x\to 2} g(x)$ exists.

(A) $k = 0, 2$ (B) $k = 2, 4$ (C) $k = -2, 4$ (D) $k = 0$ only (E) No such values of k

We set $\lim_{\substack{x\to 2^- \\ x<2}} g(x) = \lim_{\substack{x\to 2^+ \\ x>2}} g(x)$

$\Rightarrow \lim_{x\to 2^-} (x^3 + k^2) = \lim_{x\to 2^+} kx^2 + kx$

$\Rightarrow 8 + k^2 = 4k + 2k$

$\Rightarrow 8 + k^2 = 6k \Rightarrow k^2 - 6k + 8 = 0$

$\Rightarrow (k-2)(k-4) = 0$

$\Rightarrow k = 2, 4$

13. An object moves along a straight line with position function given by $s(t) = t^2 - 2t + 2$, where $s(t)$ is measured in feet and t in seconds. What is the average velocity in feet per second of the object on the interval $[1, 5]$?

(A) 5 (B) -4 (C) 4 (D) -5 (E) $\frac{17}{4}$

$$\text{average velocity} = \frac{s(5) - s(1)}{5 - 1}$$

$$= \frac{(5^2 - 2(5) + 2) - (1^2 - 2(1) + 2)}{4}$$

$$= \frac{25 - 10 + 2 - 1 \cancel{+2} \cancel{-2}}{4}$$

$$= \frac{16}{4} = 4$$

14. Evaluate $\lim_{x \to 1^+} \left(\ln(x-1) - \frac{x}{x^2 - 1} \right)$.

(A) 0 (B) 1 (C) ∞ (D) $-\infty$

$$\lim_{x \to 1^+} \left(\ln(x-1) - \frac{x}{x^2-1} \right) \sim -\infty - (\infty) \sim -\infty$$

Note $\lim_{x \to 1^+} \ln(x-1) = -\infty$

$$\lim_{x \to 1^+} \frac{x}{x^2-1} = \lim_{x \to 1^+} \frac{x}{(x-1)(x+1)} = \infty$$

1^+

$-1 \quad 0 \quad 1 \quad 2$

$$\frac{2}{(2-1)(2+1)} > 0$$

So $\lim_{x \to 1^+} \left(\ln(x-1) - \frac{x}{x^2-1} \right) = -\infty$

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A • C D E

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Part I Instructions: 14 multiple choice questions. Complete the scantron sheet provided with your information and fill in the appropriate spaces to answer your questions. Only the answer on the scantron sheet will be graded. Each problem is worth five (5) points for a total of 70 points on Part I.

1. If the function $f(x)$ has a vertical asymptote at $x = a$ which of the following must be true?

I $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x)$ X

II $\lim_{x\to a^+} f(x) = \pm\infty$ or $\lim_{x\to a^-} f(x) = \pm\infty$ ✓ (Definition)

III $\lim_{x\to a} f(x) = f(a)$ X

(A) only I (B) only II (C) only I and II (D) only I and III (E) I, II, and III

2. For which of the following functions does $f'(4) = \lim_{h\to 0} \dfrac{\frac{h+8}{h+2} - 4}{h}$?

$\to = f(4) = 4$

(A) $f(x) = -\dfrac{x+8}{x+2}$ (B) $f(x) = \dfrac{x+8}{x+2}$ (C) $f(x) = -\dfrac{x+4}{x-2}$ (D) $f(x) = \dfrac{x+4}{x-2}$

Recall $f'(4) = \lim_{h\to 0} \dfrac{f(4+h) - f(4)}{h}$

A: $f(4) = -\dfrac{4+8}{4+2} = -\dfrac{12}{6} = -2$ X

B: $f(4) = \dfrac{4+8}{4+2} = \dfrac{12}{6} = 2$ X

C: $f(4) = -\dfrac{4+4}{4-2} = -\dfrac{8}{2} = -4$ X

D: $f(4) = \dfrac{4+4}{4-2} = \dfrac{8}{2} = 4$ ✓

3. Let $f(x)$ be a function such that $f'(1) = -2$ and $f(1) = 5$. Which of the following lines is tangent to the graph of $y = f(x)$ at $x = 1$?

slope (1,5) point

(A) $y = -2x + 7$ (B) $y = -2x + 5$ (C) $y = 5x - 7$ (D) $y = 5x - 2$

point slope: $y - 5 = -2(x-1)$

$\Rightarrow y = -2x + 2 + 5 = -2x + 7$

4. Consider the function $g(x) = -3\tan(x)$. Over which of the following intervals is the Intermediate Value Theorem **not** applicable in showing the existence of a zero (root) of $g(x)$ over that interval?

(A) $\left[\frac{7\pi}{4}, \frac{9\pi}{4}\right]$ (B) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ (C) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ (D) $\left[-\frac{5\pi}{4}, -\frac{3\pi}{4}\right]$

$g(x) = -3\tan(x)$ has vertical asymptotes at all odd multiples of $\frac{\pi}{2}$ / $(2n+1)\frac{\pi}{2}$ where n is an integer

The only interval which contains such a multiple is B $[\frac{\pi}{4}, \frac{3\pi}{4}]$ which has $\frac{\pi}{2}$

5. Evaluate $\lim_{x\to 3^-} \frac{3x-9}{|x-3|}$

(A) 3 (B) 0 (C) −3 (D) $\frac{1}{3}$ (E) Does not exist

We have $\frac{3x-9}{|x-3|} = \begin{cases} \frac{3x-9}{x-3}, & x-3 \geq 0 \\ \frac{3x-9}{-(x-3)}, & x-3 < 0 \end{cases} = \begin{cases} \frac{3(x-3)}{x-3}, & x \geq 3 \\ \frac{3(x-3)}{-(x-3)}, & x < 3 \end{cases}$

$= \begin{cases} 3, & x \geq 3 \\ -3, & x < 3 \end{cases}$

So $\lim_{x\to 3^-} \frac{3x-9}{|x-3|} = \lim_{x\to 3^-} -3 = -3$

$x<3$

6. Evaluate $\lim_{x\to 2} \frac{\sqrt{x+2}+1}{x+1}$

(A) −1 (B) 0 (C) 1 (D) 2 (E) Does not exist

$\lim_{x\to 2} \frac{\sqrt{x+2}+1}{x+1} = \frac{\sqrt{2+2}+1}{2+1} = \frac{2+1}{3} = \frac{3}{3} = 1$

7. An object moves along a straight line with position function given by $s(t) = t^2 - 2t + 1$, where $s(t)$ is measured in feet and t in seconds. What is the average velocity in feet per second of the object on the interval $[0, 3]$?

(A) 3 (B) -1 (C) 2 (D) 1 (E) $\frac{17}{4}$

$$\text{average velocity} = \frac{s(3)-s(0)}{3-0}$$

$$= \frac{(3^2-2(3)+1)-(0^2-2(0)+1)}{3}$$

$$= \frac{9-6+1-1}{3} = \frac{9-6}{3} = \frac{3}{3} = 1$$

8. Suppose that $1 \leq f(x) \leq 4$ for all x near $x = 0$. What is $\lim_{x\to 0} xf(x)$?

(A) -4 (B) -1 (C) 0 (D) ∞ (E) Does not exist

$$1 \leq f(x) \leq 4$$

$$\Rightarrow \begin{cases} x \leq xf(x) \leq 4x & , x > 0 \\ x \geq xf(x) \geq 4x & , x < 0 \end{cases}$$

Yet $\lim_{\substack{x\to 0^+ \\ x\to 0^-}} x = 0 = \lim_{\substack{x\to 0^+ \\ x\to 0^-}} 4x$

So by the squeeze Theorem $\lim_{x\to 0} xf(x) = 0$

9. Suppose $f(x)$ and $g(x)$ are continuous functions for all real numbers x. How many of the following statements are necessarily true?

I. $\left(\frac{f}{g}\right)(x)$ is a continuous function for all real numbers x. (what if $g(x)=0$?) X

II. $cf(x)$ is a continuous function for all real numbers x and for any real constant c. ✓

III. $\lim_{x\to a} f(x)$ exists for every real number a. ✓

IV. $(f+g)(x)$ is a continuous function for all real numbers x. ✓

(*A*) 0 (*B*) 1 (*C*) 2 (*D*) 3 (*E*) 4

10. Let

$$g(x) = \begin{cases} x^3 + k^2, & x < -2 \\ 0, & x = -2 \\ kx^2 + kx, & x > -2 \end{cases}$$

Find all real values of k such that $\lim_{x\to -2} g(x)$ exists.

(*A*) $k = 2, 4$ (*B*) $k = -2, 4$ (*C*) $k = -2, 0$ (*D*) $k = 0$ only (*E*) No such values of k

Set $\lim_{\substack{x\to -2^- \\ x<-2}} g(x) = \lim_{\substack{x\to -2^+ \\ x>-2}} g(x)$

$\Rightarrow \lim_{x\to -2^-} (x^3+k^2) = \lim_{x\to -2^+} (kx^2+kx)$

$\Rightarrow -8+k^2 = 4k-2k \Rightarrow k^2-2k-8=0$

$\Rightarrow (k-4)(k+2)=0$

$\Rightarrow k=-2,4$

11. Evaluate $\lim_{x\to 2^-}\left(\ln(2-x) - \frac{x}{4-x^2}\right)$. $\sim (-\infty - (\infty)) = -\infty$

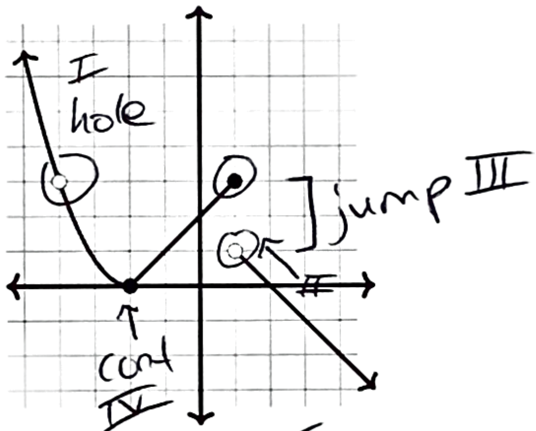
(A) 0 (B) 1 (C) ∞ (D) $-\infty$

Note $\lim_{x\to 2^-} \ln(2-x) = -\infty$ and $\lim_{x\to 2^-} \frac{x}{4-x^2} = \lim_{x\to 2^-} \frac{x}{(2-x)(2+x)} = \infty$

So $\lim_{x\to 2^-}\left(\ln(2-x) - \frac{x}{4-x^2}\right) = -\infty$

$\frac{1}{(2-1)(2+1)} > 0$

12. Given the following graph of the function $f(x)$, which of the following statement(s) must be true?



I. $f(x)$ has a removable discontinuity at $x = -4$

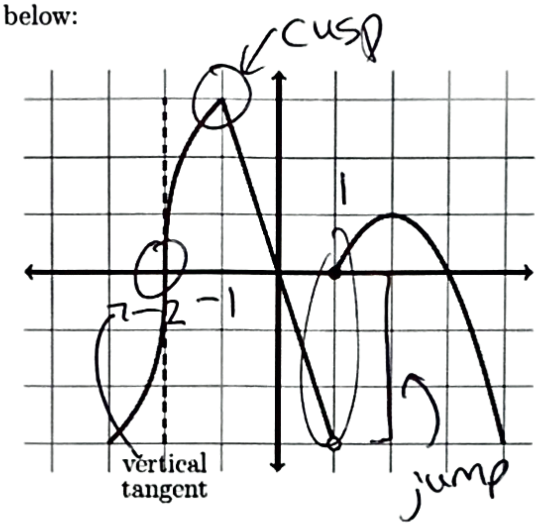
II. $\lim_{x\to 1^+} f(x) = f(1)$

III. $f(x)$ has a jump discontinuity at $x = 1$

IV. $f(x)$ is not continuous at $x = -2$

(A) *I* and *II* only (B) *II* and *III* only (C) *III* and *IV* only (D) *I* and *III* only

13. Use the graph of $f(x)$ below:



Which of the following lists contains all x-values where $f(x)$ fails to be differentiable?

(A) $x = -2, -1, 1$ (B) $x = -2, -1$ (C) $x = -1, 1$ (D) $x = -1, 0, 1$ (E) $x = -2, 0$

14. For how many of the following values of a will $y = \frac{2}{3}$ be a horizontal asymptote of the function

$$g(x) = \frac{2(x^2+2)^2}{3(x^a-3)} = \frac{2(x^4+2x^2+4)}{3x^a-9}$$

$$= \frac{\frac{2x^4}{x^a} - \frac{4x^2}{x^a} + \frac{8}{x^a}}{3 - \frac{9}{x^a}}$$

(i) $a = 1$

(ii) $a = 2$

(iii) $a = 3$

(iv) $a = 6$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

If $\underline{a=4}$ then $\lim\limits_{x\to\infty} g(x) = \lim\limits_{x\to\infty} \frac{2 - \frac{4}{x^2} + \frac{8}{x^4}}{3 - \frac{9}{x^4}} = \frac{2-0+0}{3-0}$

$= \frac{2}{3}$ ✓