Calculus I: MAC2311 **Name:** _____
Fall 2022
Exam 1 A **Section:** _____
9/22/2022
Time Limit: 90 Minutes **UF-ID:** _____

Scantron Instruction: This exam uses a scantron. Follow the instructions listed on this page to fill out the scantron.

A. Sign your scantron **on the back** at the bottom in the white area.

B. Write **and code** in the spaces indicated:

1) Name (last name, first initial, middle initial)

2) UFID Number

3) 4-digit Section Number

C. Under *special codes*, code in the test numbers 1, 1:

- 2 3 4 5 6 7 8 9 0
- 2 3 4 5 6 7 8 9 0

D. At the top right of your scantron, fill in the *Test Form Code* as A.

- B C D E

E. This exam consists of 14 multiple choice questions and 5 free response questions. Make sure you check for errors in the number of questions your exam contains.

F. The time allowed is 90 minutes.

G. WHEN YOU ARE FINISHED:

1) Before turning in your test check for **transcribing errors**. Any mistakes you leave in are there to stay!

2) You must turn in your scantron to your proctor. **Be prepared to show your GatorID with a legible signature.**

It is your responsibility to ensure that your test has **19 questions**. If it does not, show it to your proctor immediately. You will not be permitted to make up any problems omitted from your test after the testing period ends. There are a total of 105 points available on this exam.

Part I Instructions: 14 multiple choice questions. Complete the scantron sheet provided with your information and fill in the appropriate spaces to answer your questions. Only the answer on the scantron sheet will be graded. Each problem is worth five (5) points for a total of 70 points on Part I.

1. For how many of the following values of a will $y = \frac{1}{2}$ be a horizontal asymptote of the function

$$g(x) = \frac{(x^3+2)^2}{2(x^a-1)}$$

(i) $a = 2$

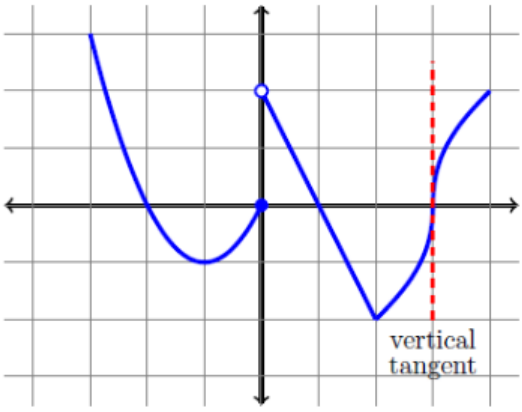
(ii) $a = 3$

(iii) $a = 4$

(iv) $a = 6$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

2. Use the graph of $f(x)$ below:



Which of the following lists contains all x-values where $f(x)$ fails to be differentiable?

(A) $x = 0, 2, 3$ (B) $x = 0, 2$ (C) $x = 2, 3$ (D) $x = -1, 2, 3$ (E) $x = 0, 3$

3. Evaluate $\lim_{x\to 3^+} \dfrac{|x-3|}{2x-6}$

(A) $-\dfrac{1}{2}$ (B) 0 (C) $\dfrac{1}{2}$ (D) 2 (E) Does not exist

4. Evaluate $\lim_{x\to 1} \dfrac{\sqrt{x+3}+2}{x+1}$

(A) 2 (B) 0 (C) -2 (D) 4 (E) Does not exist

5. Suppose $f(x)$ and $g(x)$ are continuous functions for all real numbers x. How many of the following statements are necessarily true?

I. $(f+g)(x)$ is a continuous function for all real numbers x.

II. $cf(x)$ is a continuous function for all real numbers x and for any real constant c.

III. $\left(\frac{f}{g}\right)(x)$ is a continuous function for all real numbers x.

IV. $\lim_{x\to a} f(x)$ exists for every real number a.

(*A*) 0 (*B*) 1 (*C*) 2 (*D*) 3 (*E*) 4

6. If the function $f(x)$ has a vertical asymptote at $x = a$ which of the following must be true?

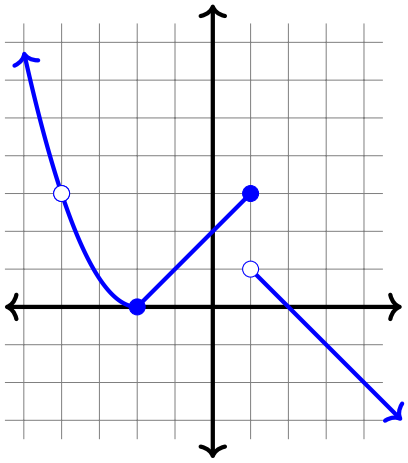
I. $\lim_{x\to a^+} f(x) = \pm\infty$ or $\lim_{x\to a^-} f(x) = \pm\infty$

II. $\lim_{x\to a} f(x)$ exists

III. $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x)$

(*A*) only *I* (*B*) only *II* (*C*) only *I* and *II* (*D*) only *I* and *III* (*E*) *I*, *II*, and *III*

7. Given the following graph of the function $f(x)$, which of the following statement(s) must be true?



I. $f(x)$ has a non-removable discontinuity at $x = -4$

II. $\lim_{x \to 1^-} f(x) = f(1)$

III. $f(x)$ has a jump discontinuity at $x = 1$

IV. $f(x)$ is not continuous at $x = -2$

(*A*) *I* and *II* only (*B*) *II* and *III* only (*C*) *III* and *IV* only (*D*) *I* and *III* only

8. Let $f(x)$ be a function such that $f'(3) = -2$ and $f(3) = 5$. Which of the following lines is tangent to the graph of $y = f(x)$ at $x = 3$?

(*A*) $y = -2x + 5$ (*B*) $y = -2x + 11$ (*C*) $y = 5x - 17$ (*D*) $y = 5x - 2$

9. Suppose that $0 \leq f(x) \leq 2$ for all x near $x = 0$. What is $\lim_{x \to 0} xf(x)$?

(A) 0 (B) 1 (C) -1 (D) ∞ (E) Does not exist

10. Consider the function $g(x) = 2\tan(x)$. Over which of the following intervals is the Intermediate Value Theorem **not** applicable in showing the existence of a zero (root) of $g(x)$ over that interval?

(A) $\left[\frac{7\pi}{4}, \frac{9\pi}{4}\right]$ (B) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ (C) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ (D) $\left[-\frac{5\pi}{4}, -\frac{3\pi}{4}\right]$

11. For which of the following functions does $f'(2) = \lim_{h\to 0} \dfrac{\dfrac{h+5}{h+1} - 5}{h}$?

(A) $f(x) = -\dfrac{x+5}{x+1}$ (B) $f(x) = -\dfrac{x+3}{x-1}$ (C) $f(x) = \dfrac{x+3}{x-1}$ (D) $f(x) = \dfrac{x+5}{x+1}$

12. Let

$$g(x) = \begin{cases} x^3 + k^2, & x < 2 \\ 0, & x = 2 \\ kx^2 + kx, & x > 2 \end{cases}$$

Find all real values of k such that $\lim_{x\to 2} g(x)$ exists.

(A) $k = 0, 2$ (B) $k = 2, 4$ (C) $k = -2, 4$ (D) $k = 0$ only (E) No such values of k

13. An object moves along a straight line with position function given by $s(t) = t^2 - 2t + 2$, where $s(t)$ is measured in feet and t in seconds. What is the average velocity in feet per second of the object on the interval $[1, 5]$?

(A) 5 (B) -4 (C) 4 (D) -5 (E) $\frac{17}{4}$

14. Evaluate $\lim_{x \to 1^+} \left(\ln(x-1) - \frac{x}{x^2-1} \right)$.

(A) 0 (B) 1 (C) ∞ (D) $-\infty$

Calculus I: MAC2311
Fall 2022
Exam 1 A
9/22/2022
Time Limit: 90 Minutes

Name: _____

Section: _____

UF-ID: _____

Part II Instructions: 5 free response questions. Neatly give a complete solution to each problem and show all work and intermediate steps. We are grading the work and notation as well as the answer. Each problem is worth seven (7) points. A total of 35 points is possible on Part II. **No credit will given without proper work.** If we cannot read it and follow it, you will receive no credit for the problem.

For Instructor Use Only:

FR 1	
FR 2	
FR 3	
FR 4	
FR 5	
Total Points	

1. Let $f(x) = \sqrt{\dfrac{x+8}{x+2}}$. Use the **limit definition of the derivative** to find $f'(0)$. NO CREDIT will be given if the limit definition of the derivative is not used.

2. Show that the function $f(x) = 5x^3 - 2x^2 + 3x - 4$ has a zero (root) in the interval $[0, 1]$.

3. Consider the following piecewise defined function

$$g(x) = \begin{cases} 5 - x, & x < 5 \\ \sqrt{x - 4}, & 5 \leq x \leq 13 \\ \dfrac{x + 5}{6}, & x > 13 \end{cases}.$$

(*a*) Is $g(x)$ continuous at $x = 5$? Justify your answer by calculating the appropriate limits.

(*b*) Is $g(x)$ continuous at $x = 13$? Justify your answer by calculating the appropriate limits.

4. Find all vertical and horizontal asymptotes of the function

$$k(x) = \frac{-3x^2 + 6x}{x^2 - 2x + 1}.$$

5. Consider the following table of expressions for functions:

$\dfrac{\sin(x)}{x}$	$x^2 \cos\left(\dfrac{1}{x}\right)$	$\ln(x^2)$
$\dfrac{1-\cos(x)}{x}$	$\sin\left(\dfrac{1}{x}\right)$	$\dfrac{1}{x}$
$\arctan(x)$	$e^{1/x}$	$\dfrac{x}{\lvert x\rvert}$

Use the table above to give examples of each of the following. Only list one function for each part, even if there are multiple functions that have the desired property.

(*a*) A function $f(x)$ where $\lim\limits_{x\to 0} f(x) = 0$

(*b*) A function $f(x)$ where $\lim\limits_{x\to 0} f(x) = 1$

(*c*) A function $f(x)$ where $\lim\limits_{x\to 0} f(x) = \infty$ or $-\infty$

(*d*) A function $f(x)$ where $\lim\limits_{x\to 0} f(x)$ does not exist, but $\lim\limits_{x\to 0^-} f(x)$ and $\lim\limits_{x\to 0^+} f(x)$ both exist (either finite or infinite)

(*e*) A function $f(x)$ where neither $\lim\limits_{x\to 0^-} f(x)$ nor $\lim\limits_{x\to 0^+} f(x)$ exists (Neither finite nor infinite)

Calculus I: MAC2311 **Name:** _____
Fall 2021
Exam 1 B **Section:** _____
9/15/2021
Time Limit: 1 Hour 30 Minutes **UF-ID:** _____

Scantron Instruction: This exam uses a scantron. Follow the instructions listed on this page to fill out the scantron.

A. Sign your scantron **on the back** at the bottom in the white area.

B. Write **and code** in the spaces indicated:

1) Name (last name, first initial, middle initial)

2) UFID Number

3) 4-digit Section Number

C. Under *special codes*, code in the test numbers 1, 3:

• 2 3 4 5 6 7 8 9 0
1 2 • 4 5 6 7 8 9 0

D. At the top right of your scantron, fill in the *Test Form Code* as B .

A • C D E

E. This exam consists of 14 multiple choice questions and 5 free response questions. Make sure you check for errors in the number of questions your exam contains.

F. The time allowed is 90 minutes.

G. WHEN YOU ARE FINISHED:

1) Before turning in your test check for **transcribing errors**. Any mistakes you leave in are there to stay!

2) You must turn in your scantron to your proctor. **Be prepared to show your GatorID with a legible signature.**

It is your responsibility to ensure that your test has **19 questions**. If it does not, show it to your proctor immediately. You will not be permitted to make up any problems omitted from your test after the testing period ends. There are a total of 105 points available on this exam.

Part I Instructions: 14 multiple choice questions. Complete the scantron sheet provided with your information and fill in the appropriate spaces to answer your questions. Only the answer on the scantron sheet will be graded. Each problem is worth five (5) points for a total of 70 points on Part I.

1. If $f(x) = \dfrac{x^2 - 5x}{|x|}$, which of the following statements is/are true?

   I. $\lim\limits_{x \to 0^-} f(x) = 5$.

   II. $f(x)$ has a removable discontinuity at $x = 0$.

   III. $\lim\limits_{x \to -\infty} f(x) = \infty$.

(A) I and III only (B) I and II only (C) I only (D) II only (E) III only

2. What is $\lim\limits_{x \to 1^-} \dfrac{3\ln(x)}{4 - x}$?

(A) 0 (B) ∞ (C) $-\infty$ (D) -1 (E) Does not exist

3. Suppose that as an object falls from the top of a cliff its position in feet above the ground after t seconds is given by

$$s(t) = -30 - 20t^2.$$

Find the average velocity of the object from $t = 1$ to $t = 1 + h$ seconds, where $h \neq 0$.

(A) $-40 - 20h$ (B) $\dfrac{-100 - 40h - 20h^2}{h}$ (C) $-20 - 20h$ (D) $\dfrac{-100 - 20h + 20h^2}{h}$ (E) $-20h + 40h^2$

4. If $f(x) = \dfrac{ax^4 - 5x^3 + 7x - 1}{bx^4 + 5x^3 - 2x^2 - x + 5}$, for which of the following values of a and b will $\lim\limits_{x\to\infty} f(x) = -\dfrac{1}{3}$?

(A) $a = 1$ and $b = 3$

(B) $a = 3$ and $b = -1$

(C) $a = 0$ and $b = 0$

(D) $a = 0$ and $b = -3$

(E) $a = -2$ and $b = 6$

5. Suppose $\lim\limits_{x\to a} f(x) = \dfrac{9}{2}$, $\lim\limits_{x\to a} h(x) = 27$, and $2f(x) \leq g(x) \leq \dfrac{1}{3}h(x)$ for x near a. What is $\lim\limits_{x\to a} g(x)$?

(A) -1 (B) 1 (C) 3 (D) 9 (E) Does not exist

6. Evaluate $\lim_{x\to -25} \sqrt{\ln(25+x+e^{-x})}$.

(A) 5 (B) -5 (C) 25 (D) -25 (E) Does not exist

7. Suppose that the line tangent to the graph of the function $f(x)$ when $x = 3$ has equation $y = -2x+5$. Which of the following is true?

(A) $f(3) = -1$ and $f'(3) = 3$ (B) $f(3) = 5$ and $f'(3) = 3$

(C) $f(3) = -1$ and $f'(3) = -2$ (D) $f(3) = 5$ and $f'(3) = -2$

8. Consider the piecewise-defined function $f(x)$ below. For what value of a does $\lim_{x\to 3} f(x)$ exist?

$$f(x) = \begin{cases} x^2 + ax - 3, & x < 3 \\ 5, & x = 3 \\ \sqrt{5x+2a}, & x > 3 \end{cases}$$

(A) $-\frac{4}{3}$ (B) $-\frac{7}{9}$ (C) $-\frac{1}{3}$ (D) 5 (E) No such value exists

9. How many of the following functions are continuous on the interval $(-2, \infty)$?

- $f(x) = \dfrac{x-3}{x+3}$
- $g(x) = e^x$
- $h(x) = \ln(x)$
- $k(x) = \sqrt{x-4}$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

10. How many of the following functions have vertical asymptotes?

- $f(x) = \dfrac{x^2-9}{x^2+9}$
- $g(x) = \dfrac{(x-2)(x+1)}{(x-2)(x+5)}$
- $h(x) = \dfrac{3+x}{3+e^x}$
- $k(x) = \dfrac{2+x}{\sin(x)}$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

11. Which of the following statements must be true?

(*A*) If $f(x)$ and $g(x)$ are continuous at $x = a$, then $f(x) + g(x)$ is discontinuous at $x = a$.

(*B*) If $f(x)$ is discontinuous at $x = a$, then $cf(x)$ is continuous at $x = a$ for any real number c.

(*C*) If $f(x)$ and $g(x)$ are continuous at $x = a$, then $f(x)g(x)$ is continuous at $x = a$.

(*D*) If $f(x)$ and $g(x)$ are continuous at $x = a$, then $\dfrac{f(x)}{g(x)}$ is continuous at $x = a$.

12. Evaluate $\lim\limits_{x\to -4} \dfrac{\sqrt{x^2+9}-5}{x+4}$.

(*A*) $-\dfrac{4}{5}$ (*B*) $\dfrac{4}{5}$ (*C*) $-\infty$ (*D*) ∞ (*E*) Does not exist

13. How many of the following are true?

- A function can not have more than two horizontal asymptotes.
- Horizontal asymptotes are lines of the form $x = c$ where c is a constant.
- A function always has a discontinuity at a vertical asymptote.
- If $f(x) = \dfrac{h(x)}{g(x)}$ and $g(a) = 0$, then there is a vertical asymptote at $x = a$.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

14. Let $f(x)$ be a function and let a be a real number. Which of the following statements must be true?

(A) If $\lim_{x\to a} f(x)$ exists, then $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ exist.

(B) If $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ exist, then $\lim_{x\to a} f(x)$ exists.

(C) If $\lim_{x\to a} f(x)$ exists, then $f(a)$ is defined.

(D) If $f(a)$ is defined, then $\lim_{x\to a} f(x)$ exists.

Calculus I: MAC2311
Fall 2021
Exam 1 B
9/15/2021
Time Limit: 1 Hour 30 Minutes

Name: _____

Section: _____

UF-ID: _____

Part II Instructions: 5 free response questions. Neatly give a complete solution to each problem and show all work and intermediate steps. We are grading the work and notation as well as the answer. Each problem is worth seven (7) points. A total of 35 points is possible on Part II. **No credit will given without proper work.** If we cannot read it and follow it, you will receive no credit for the problem.

For Instructor Use Only:

FR 1	
FR 2	
FR 3	
FR 4	
FR 5	
Total Points	

1. Let $f(x) = \dfrac{1}{x^2+4}$. Use the limit definition of the derivative to find $f'(1)$. NO CREDIT will be given if the limit definition of the derivative is not used.

2. Below is a table of some value for a continuous function $g(x)$:

x	$g(x)$
-2	-5
-1	3
0	-10
1	5
2	2

(a) Is $g(x)$ guaranteed to have a root between $x = -2$ and $x = -1$? Why or why not?

(b) Is $g(x)$ guaranteed to have a root between $x = 1$ and $x = 2$? Why or why not?

3. Let

$$f(x) = \begin{cases} -\dfrac{1}{x+1}, & x < -1 \\ 3, & -1 < x < 2 \\ |x| + 1, & x > 2 \end{cases}$$

(a) Evaluate the following limits; use the symbols ∞, $-\infty$, or DNE when appropriate:

$\lim_{x \to -1^-} f(x) =$ $\lim_{x \to -1^+} f(x) =$ $\lim_{x \to -1} f(x) =$

$\lim_{x \to 2^-} f(x) =$ $\lim_{x \to 2^+} f(x) =$ $\lim_{x \to 2} f(x) =$

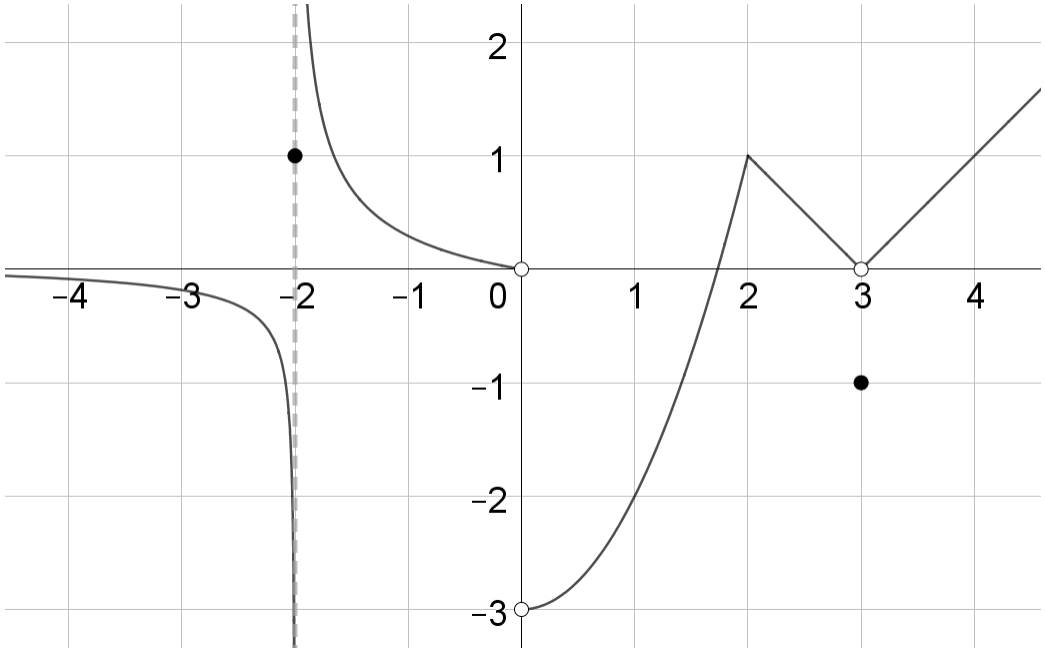
(b) List each value of x at which $f(x)$ is discontinuous, and describe each either a removable, jump, or infinite discontinuity. If the discontinuity is removable, state how you can redefine $f(x)$ to make the function continuous there.

4. Find the value(s) of a and b so that the function

$$f(x) = \begin{cases} x^2 + 2ax + b\sqrt{x^2 + 21} & x \leq 2 \\ a\cos(\frac{\pi}{2}x) + bx & 2 < x \leq 5 \\ 5\sqrt{x-1} + 2bx & x > 5 \end{cases}$$

is continuous everywhere.

5. Use the graph of the function $f(x)$ below to evaluate the following limits; use the symbols ∞, $-\infty$, or DNE when appropriate:



(a) $\lim_{x\to-\infty} f(x) =$

(b) $\lim_{x\to-2^+} f(x) =$

(c) $\lim_{x\to0^-} f(x) =$

(d) $\lim_{x\to0^+} f(x) =$

(e) $\lim_{x\to0} f(x) =$

(f) $\lim_{x\to2} f(x) =$

(g) $\lim_{x\to3} f(x) =$